Electronics B

Joel Voldman

Massachusetts Institute of Technology

Outline

> **Elements of circuit analysis**

> **Elements of semiconductor physics**
- **Semiconductor diodes and resistors**
- **The MOSFET and the MOSFET inverter/amplifier**

> **Op-amps**

TODAY

Elements of semiconductor physics

> Discrete molecules (e.g., hydrogen atom) have discrete energy levels that the electrons can occupy
  - Determined via quantum mechanics

> Adding a *discrete* amount of energy to the electrons (via light, thermal energy, etc.) can excite an electron from its ground state to an excited state

> Different atoms have different number of filled states
  - All the action typically happens at the highest unoccupied state

> Two distinct molecules have identical and *independent* energy-level structures

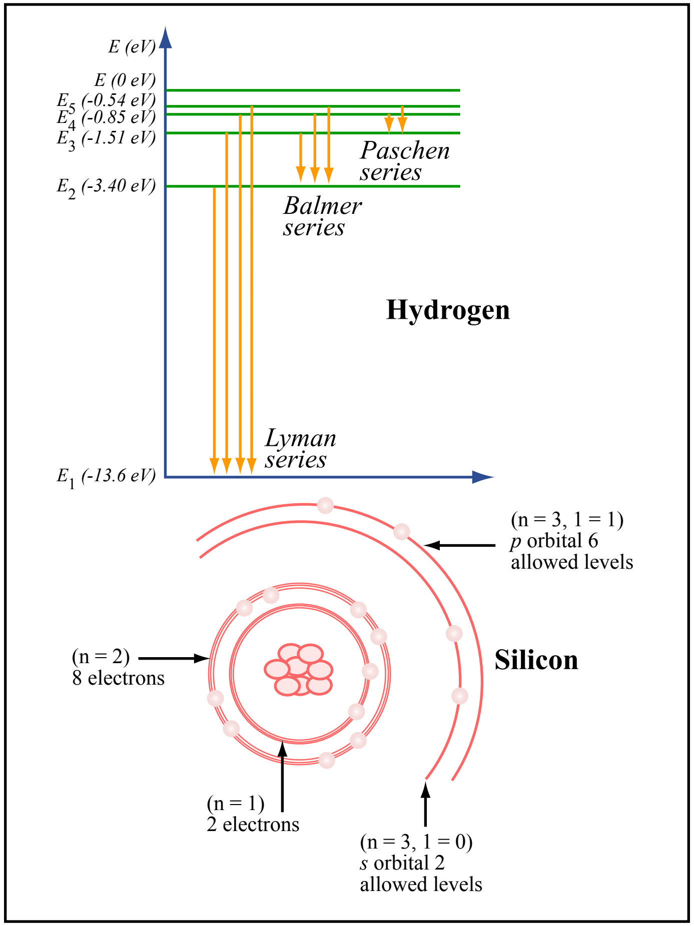


Image by MIT OpenCourseWare.

Adapted from Figures 2.1 and 2.6 in Razeghi, M. *Fundamentals of Solid State Engineering*. 2nd ed. New York, NY: Springer, 2006, pp. 48 and 59. ISBN: 9780387281520.

Razeghi, *Fundamentals of Solid State Engineering*

Elements of semiconductor physics

- Atoms packed into a lattice behave differently than discrete atoms
- Their *discrete* energy levels coalesce into “continuous” energy *bands*
- There may be many energy bands for the molecule
- We don’t care about the ones that are filled and inaccessible
- We care about the highest one with electrons

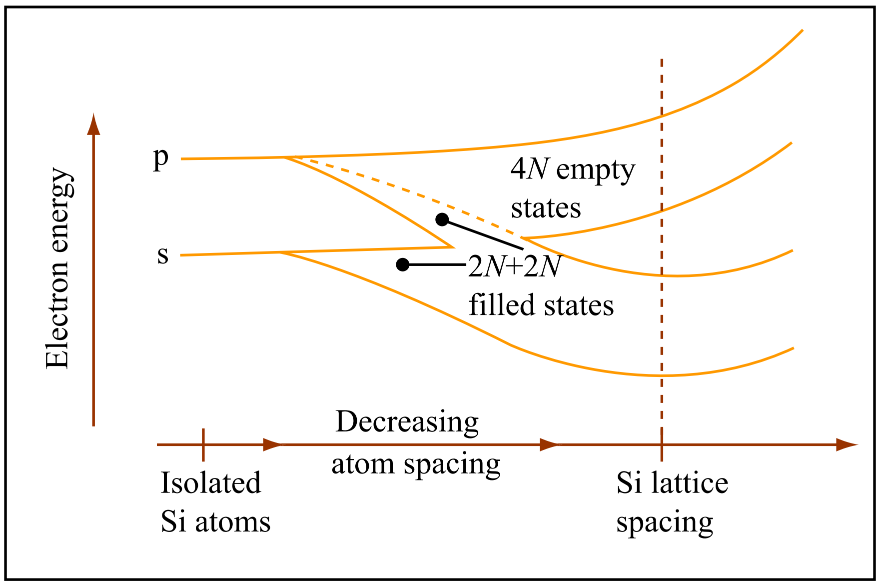


Image by MIT OpenCourseWare.

Adapted from Figure 2.5 in: Pierret, Robert F. *Semiconductor DeviceFundamentals*. Reading, MA: Addison-Wesley, 1996, p. 28. ISBN: 9780131784598.

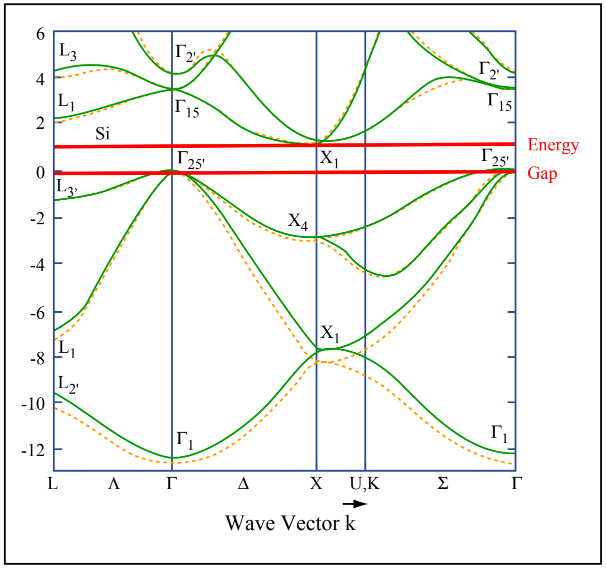


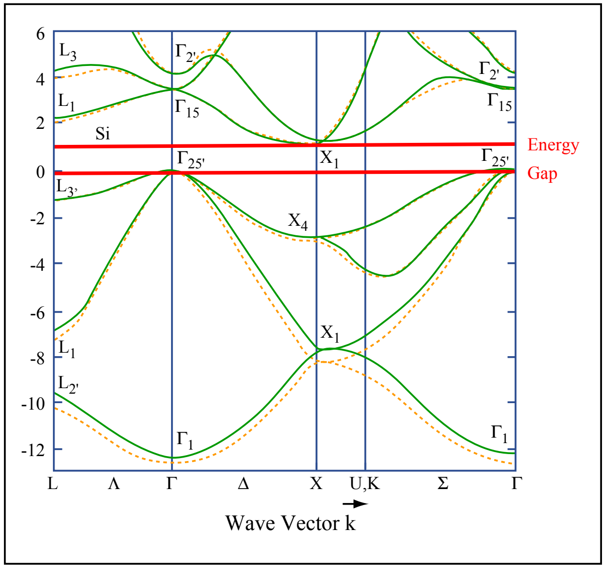
Image by MIT OpenCourseWare.

Figure 1 on p. 559 in: Chelikowsky, J. R., and M. L. Cohen. "Nonlocal Pseudopotential Calculations for the Electronic Structure of Eleven Diamond and Zinc-blende Semiconductors." *Physical Review* B 14, no. 2 (July 1976): 556-582.

Razeghi, *Fundamentals of Solid State Engineering*

Elements of semiconductor physics

- The highest normally filled set of electronic states is the valence band
- The lowest normally empty set of electronic states is the conduction band
- An energy gap separates these states
- At T=0 K, all the valence band states are filled
- A filled band cannot conduct electricity ➔ This is an insulator



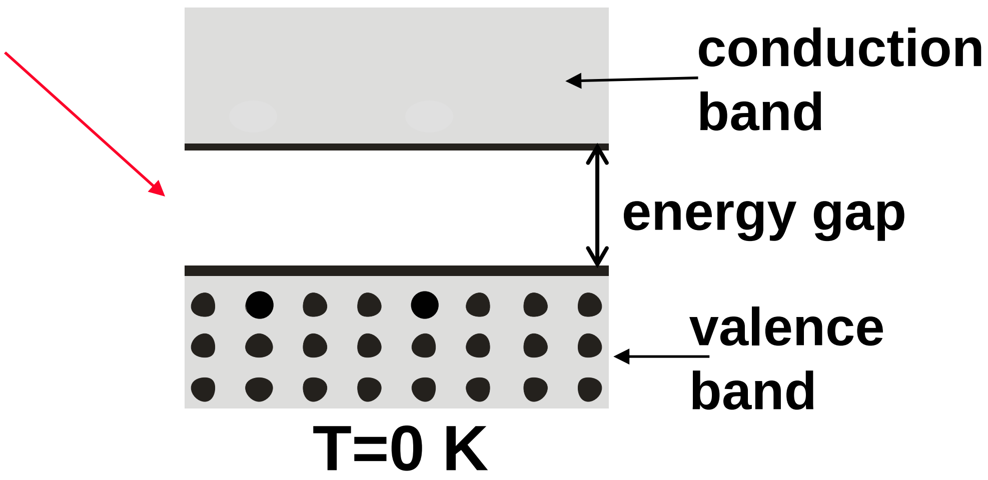


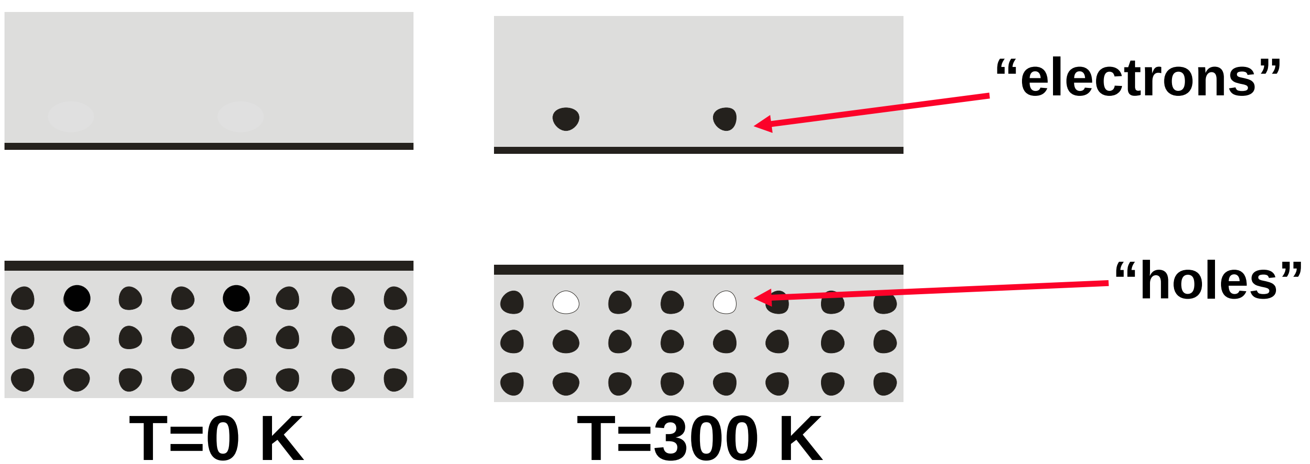
Figure by MIT OpenCourseWare.

Figure 1 on p. 559 in: Chelikowsky, J. R., and M. L. Cohen. "Nonlocal Pseudopotential Calculations for the Electronic Structure of Eleven Diamond and Zinc-blende Semiconductors." *Physical Review* B 14, no. 2 (July 1976): 556-582.

Razeghi, *Fundamentals of Solid State Engineering*

Elements of semiconductor physics

- T>0 K, electrons have thermal energy (~kT)
- At equilibrium, the only way for an electron to cross the energy barrier is to be thermally excited
- The number of electrons that can do this is
  - increases with thermal energy (and thus T)
  - decreases with larger bandgap
- The thermally excited electrons give rise to "electrons"
- The resulting empty states in the valence band give rise to "holes"
  - Behave similar to "electrons" but with opposite charge and different mass



Key terminology

- **Energy gap E_G (1.1 eV in silicon at RT)**
- **Si atom concentration = 5×10^{22} /cm^3**
- **The number of carriers in intrinsic Si is related to**
  - **Probability distribution function of energies**
    - **This obeys Fermi-Dirac statistics**
  - **Allowable states**
- **~10^{-9} (prob of being filled) x ~10^{19} (density of states) = ~10^{10} (carriers)**
- **n_i is the intrinsic carrier concentration**
  - **the equilibrium concentration of holes and electrons in the absence of dopants**
  - **~1.5×10^{10} cm^{-3} at RT**
  - **This is a material property**
- **n is the concentration [#/cm^3] of electrons**
- **p is the concentration [#/cm^3] of holes**

$$n_i \propto e^{-\frac{E_G}{2k_BT}}$$

$$n_i = p_i$$

Every thermally generated electron leaves behind a hole

Key terminology

> **Intrinsic carrier concentration of Si, Ge, and GaAs**

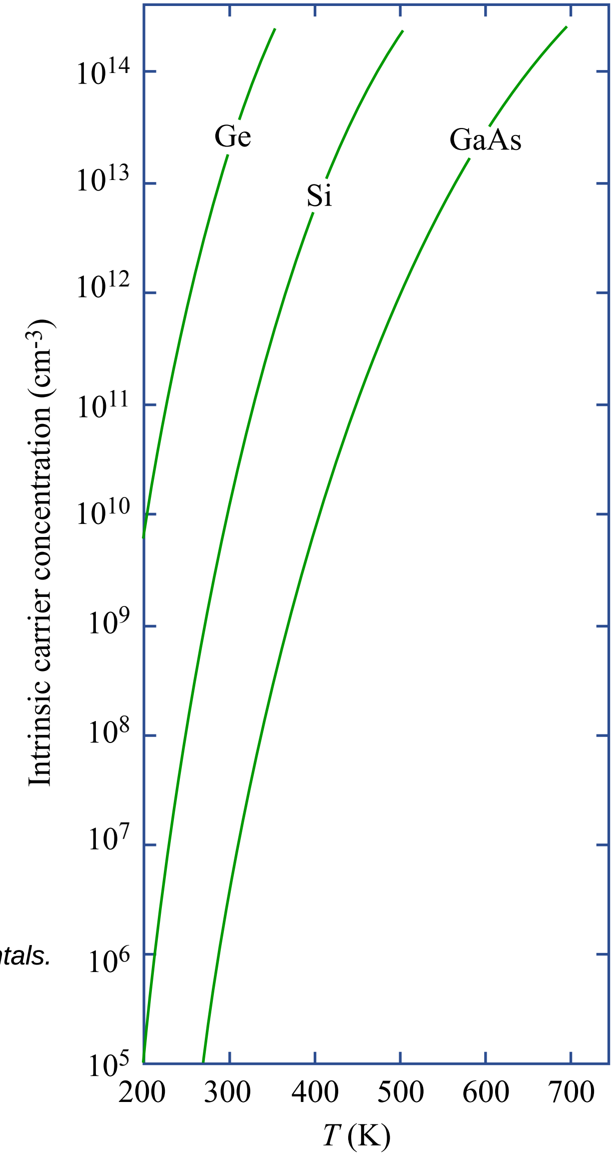


Image by MIT OpenCourseWare.

Adapted from Figure 2.20 in: Pierret, Robert F. *Semiconductor Device Fundamentals.* Reading, MA: Addison-Wesley, 1996, p. 54. ISBN: 9780131784598.

Doped semiconductors

> **Dopants: substitutional impurity atoms introduced having one different valence than the semiconductor**

> **Acceptor dopant concentrations are N_A [#/cm^3] → p-type material**
- **Boron (3 valence electrons)**

> **Donor dopant concentrations are N_D [#/cm^3] → n-type material**
- **Phosphorous (5 valence electrons)**

> **These introduce new energy levels close to valence or conduction bands (~0.05 eV)**

> **Dopant concentrations >> n_i**

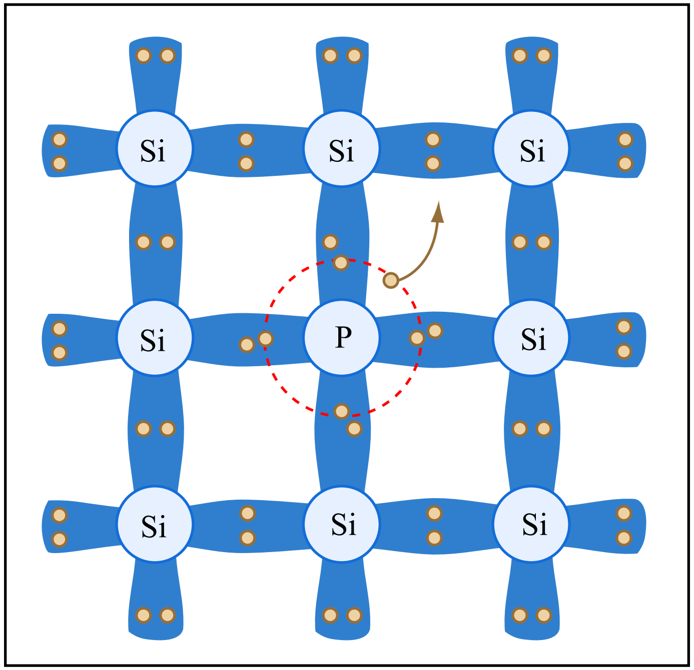
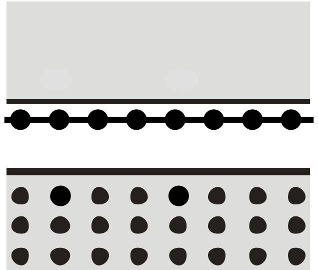


Image by MIT OpenCourseWare.
Adapted from Figure 7.5 in: Razeghi, M. *Fundamentals of Solid State Engineering*. 2nd ed. New York, NY: Springer, 2006, p. 238. ISBN: 9780387281520.



T=0 K

Doped semiconductors

- > **Energy to ionize << kT (at RT)**
- > **We assume all dopants are ionized at RT**

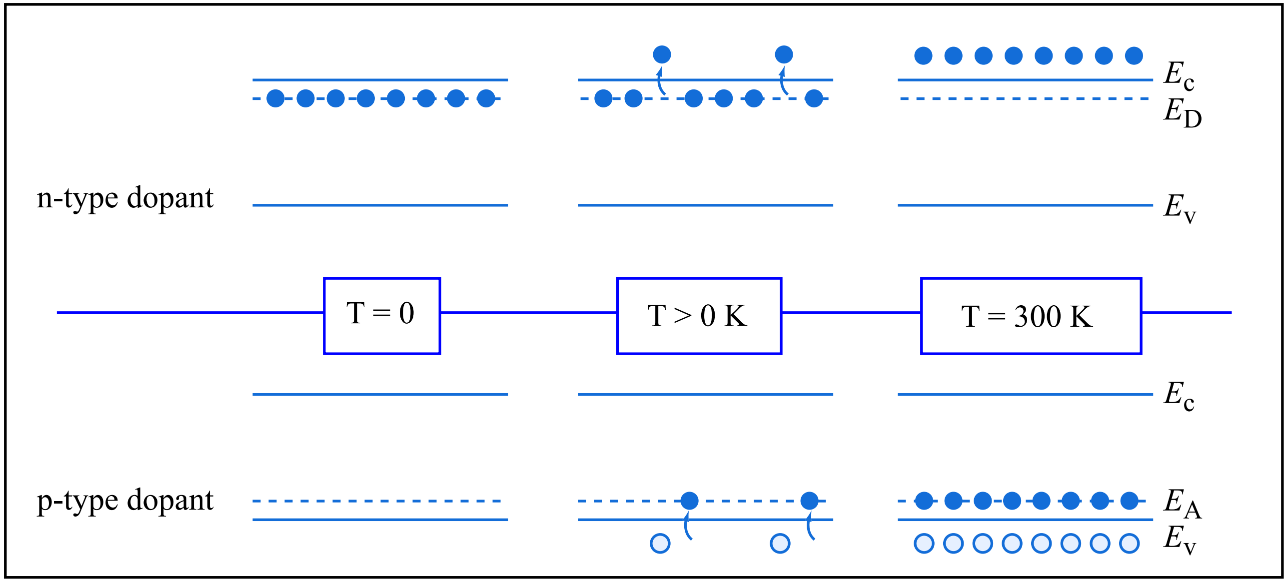


Image by MIT OpenCourseWare.

Adapted from Figure 2.13 in: Pierret, Robert F. *Semiconductor Device Fundamentals*. Reading, MA: Addison-Wesley, 1996, p. 38. ISBN: 9780131784598.

Main results

- Donors or acceptors are fully ionized
- Define n_0 and p_0 as the electron and hole concentrations at equilibrium
- n_0 and p_0 follow a mass-action law

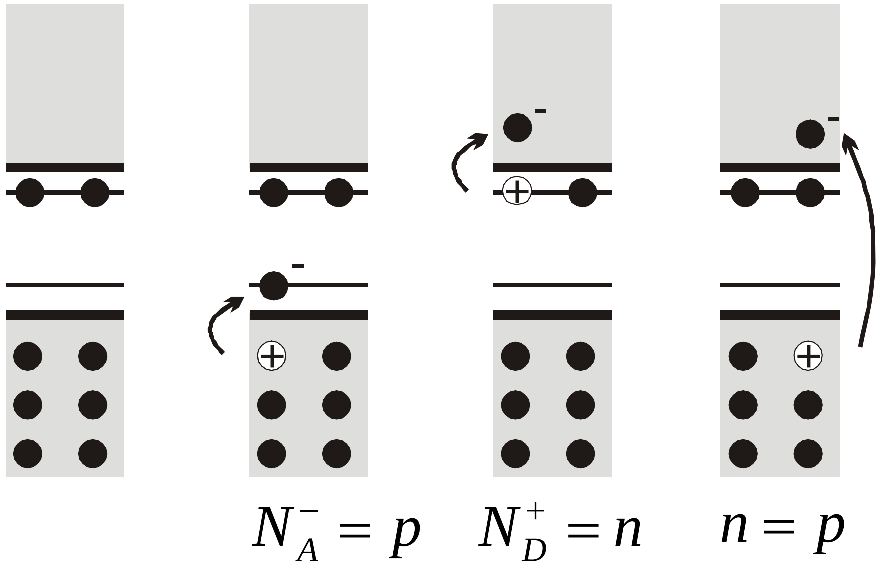
$$N_D \rightarrow N_D^+ + e^-$$

$$N_A \rightarrow N_A^- + h^+$$

$$N_D \approx N_D^+$$

$$N_A \approx N_A^-$$

$$n_0 p_0 = n_i^2$$



Main results

> **Overall, silicon is neutral**
  - **Can use to determine n_0 and p_0**

Charge neutrality requires:

$$N_A^- + n_0 = N_D^+ + p_0$$

$$N_A + n_0 \approx N_D + p_0$$

> **Carrier concentrations typically vary over many orders of magnitude**
  - **Can use this to simplify**

> **Ex:**
  - $n_i \sim 10^{10}$ **cm^{-3}**
  - N_A, $N_D \sim 10^{16}$-10^{19} **cm^{-3}**

> **In a given material at equilibrium**
  - $N_A > 100 N_D$ **(p-type)**
  - $N_D > 100 N_A$ **(n-type)**

For p-type material:

$$p_0 \approx N_A$$

$$n_0 = \frac{n_i^2}{p_0} = \frac{n_i^2}{N_A}$$

Main results

- **Therefore, the equilibrium *majority* carrier concentration is determined by the net doping and the *minority* carrier concentration is determined by the n_0p_0 product.**
- **For typical numbers, minority carrier concentration is *much less* majority carrier concentration**

p-type

$$p_0 = N_A$$

$$n_0 = \frac{n_i^2}{N_A}$$

n-type

$$n_0 = N_D$$

$$p_0 = \frac{n_i^2}{N_D}$$

$$N_A \sim 10^{17}\,cm^{-3}$$

$$p_0 = N_A = 10^{17}\,cm^{-3}$$

$$n_0 = \frac{n_i^2}{N_A} = \frac{\left(10^{10}\,cm^{-3}\right)^2}{10^{17}\,cm^{-3}} = 10^3\,cm^{-3}$$

Excess carriers

> We can do various things to create *excess carriers*
  - Shine light
  - Apply electric fields

> Excess carriers (n', p') represent a departure from equilibrium

$$n' = n - n_0 \qquad p' = p - p_0$$

$$\text{Generally,} \quad n' = p'$$

Electron-hole pair (EHP)

> Excess holes and electrons are created in pairs

Recombination rate

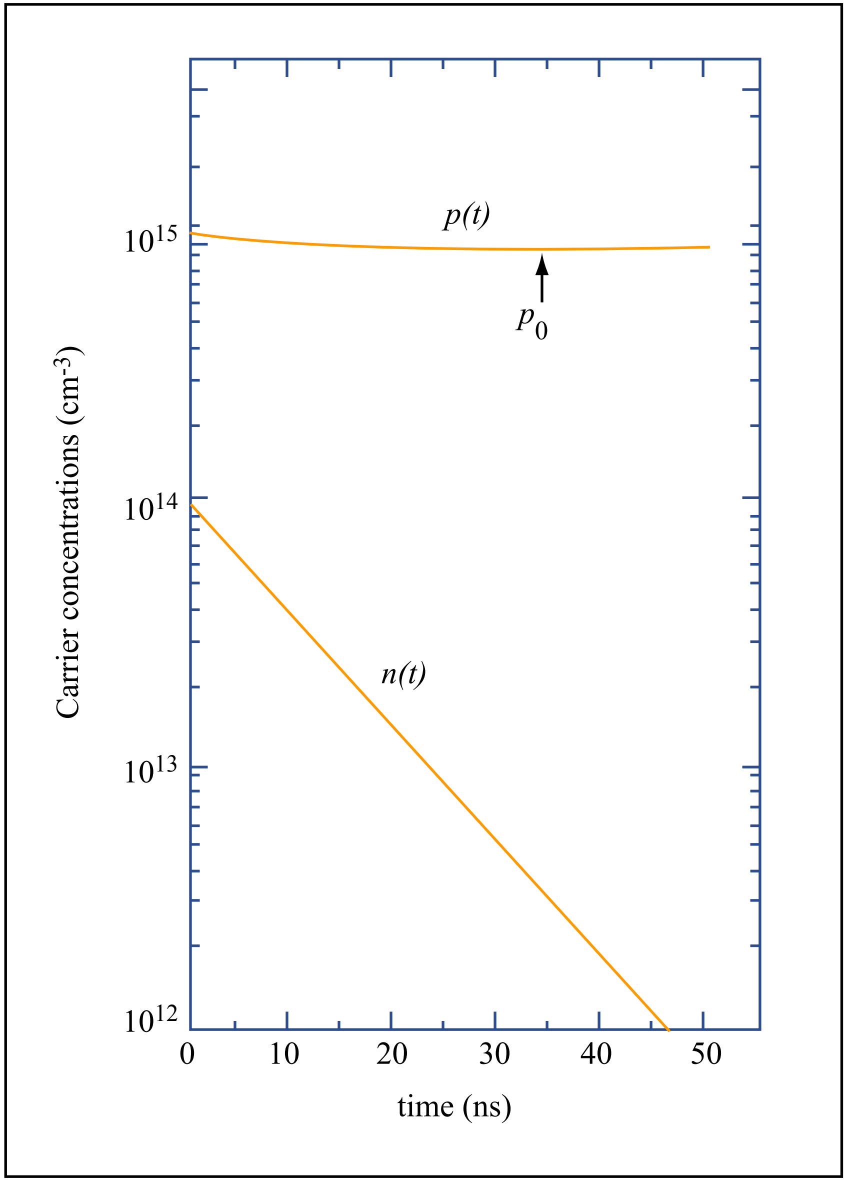
$$\frac{dn'}{dt} \propto -\frac{n'}{\tau_m}$$

> Excess carriers recombine exponentially in pairs, governed by lifetime
  - The *minority* carrier lifetime

$$n'(t) \sim n'(0) e^{-\frac{t}{\tau_m}}$$

Excess carriers

- GaAs w/ $N_A = p_0 = 10^{15}$ cm^{-3}
- GaAs $n_i = 10^6$ cm^{-3}
- Therefore, $n_0 = 10^{-3}$ cm^{-3}
- Create 10^{14} EHP/cm^3 and calculate carrier concentrations over time



Adapted from Figure 4.7 in: Streetman, Ben G., and Sanjay Kumar Banerjee. *Solid State Electronic Devices*. 6th ed. Upper Saddle River, NJ: Pearson Prentice Hall, 2006, p. 127. ISBN: 9780131497269.

Image by MIT OpenCourseWare.

Drift and diffusion of carriers

- Carriers in semiconductors obey a drift/diffusion flux relation
- Drift: carriers move in an electric field
- Diffusion: carriers move in a concentration gradient
- For p-type material
  - n is small ➔ drift current is small
  - ∇n can be big ➔ diffusion current dominates

Electric field [V/cm]

drift diffusion

$$\mathbf{J}_n = q_e\left(\mu_n n \mathrm{E} + D_n \nabla n\right)$$

Mobility [cm^2/V-s]

Diffusivity [cm^2/s]

Carrier concentration [cm^{-3}]

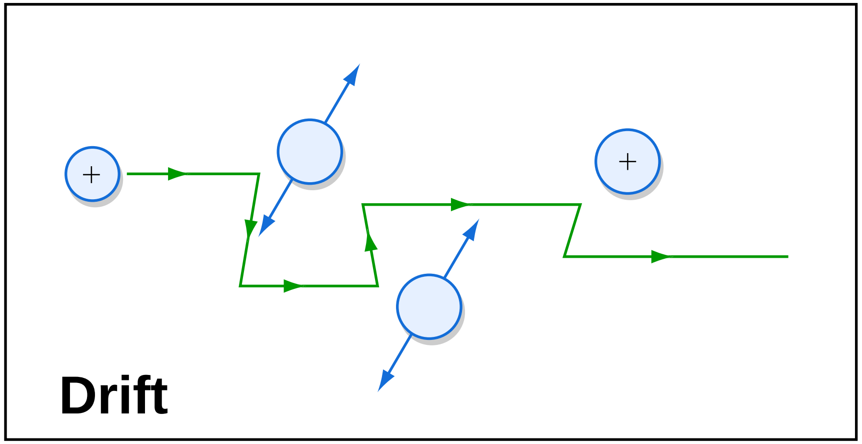


Image by MIT OpenCourseWare.

Figure 3.1b) in: Pierret, Robert F. *Semiconductor Device Fundamentals*. Reading, MA: Addison-Wesley, 1996, p. 76. ISBN: 9780131784598.

Outline

- Elements of circuit analysis
- Elements of semiconductor physics
- Semiconductor diodes and resistors
- The MOSFET and the MOSFET inverter/amplifier
- Op-amps

The semiconductor diode

- A p-n junction has very different concentrations of carriers in the two regions, creating a diffusive driving force
- At equilibrium diffusive driving force = electric field in the vicinity of the junction
- In order to set up this electric field, the ionized donors and acceptors are relatively depleted of mobile carriers near the junction – the space-charge layer (SCL) or depletion region

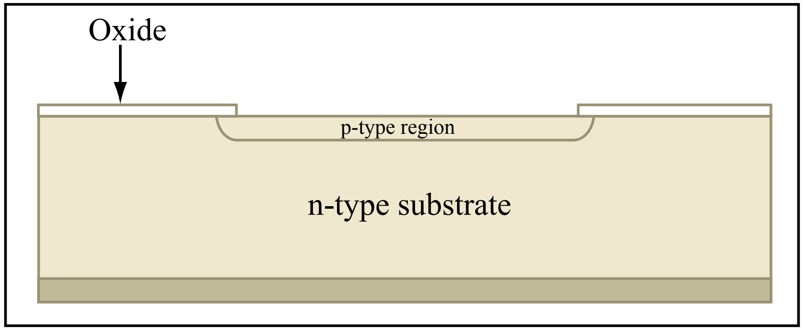
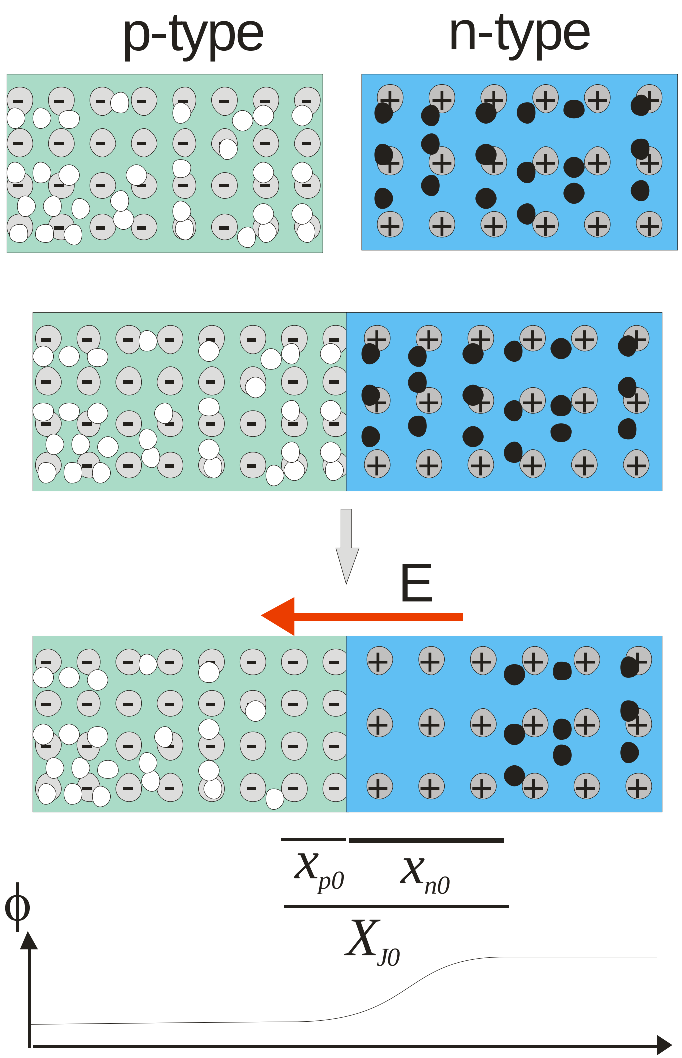


Image by MIT OpenCourseWare.

Adapted from Figure 14.1 in: Senturia Stephen D. *Microsystem Design*. Boston, MA: Kluwer Academic Publishers, 2001, p. 357. ISBN: 9780792372462.



The exponential diode

- An external voltage modifies the net potential drop across the space charge layer
- Forward bias reduces the barrier to diffusion, leading to an increase in current
- Reverse bias increases the barrier, so only current is due to minority carriers generated in or near the space-charge layer

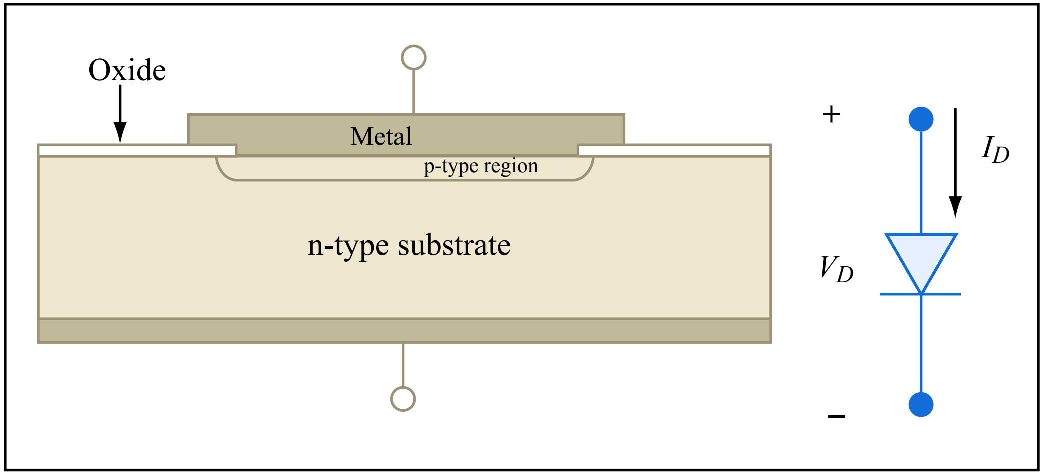


Image by MIT OpenCourseWare.

Adapted from Figure 14.2 in: Senturia, Stephen D. *Microsystem Design*. Boston, MA: Kluwer Academic Publishers, 2001, p. 359. ISBN: 9780792372462.

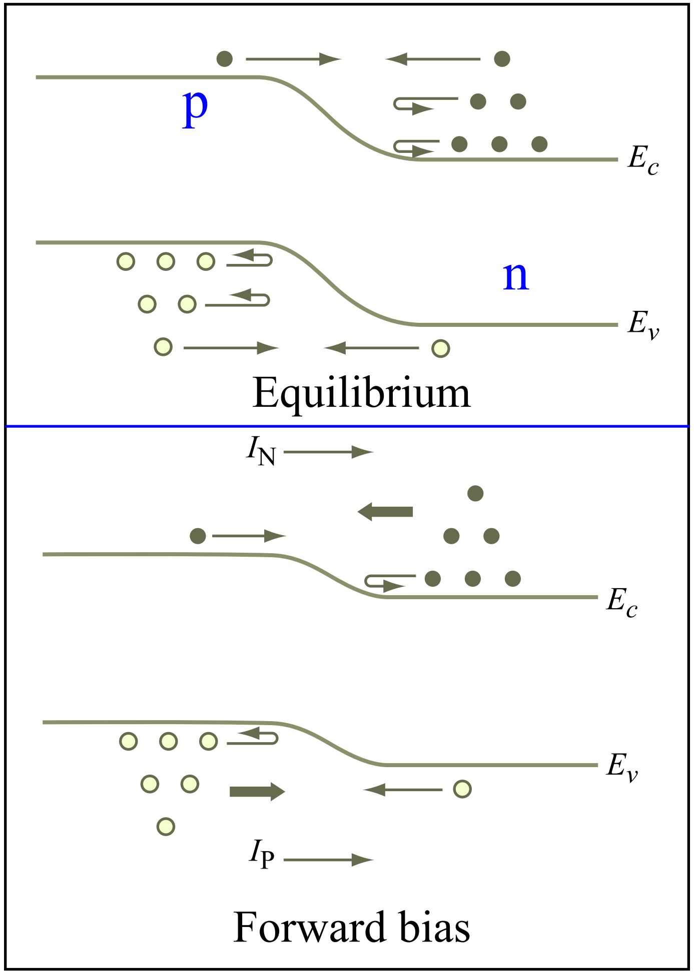


Image by MIT OpenCourseWare.

Adapted from Figure 6.1 in: Pierret, Robert F. *Semiconductor Device Fundamentals*. Reading, MA: Addison-Wesley, 1996, p. 236. ISBN: 9780131784598.

Ideal Exponential Diode Analysis

- ***Linear* changes in voltage lead to *exponential* changes in carrier concentrations**

The total current is

$$I_D = I_0\left(e^{\frac{q_e V_D}{k_B T}} - 1\right)$$

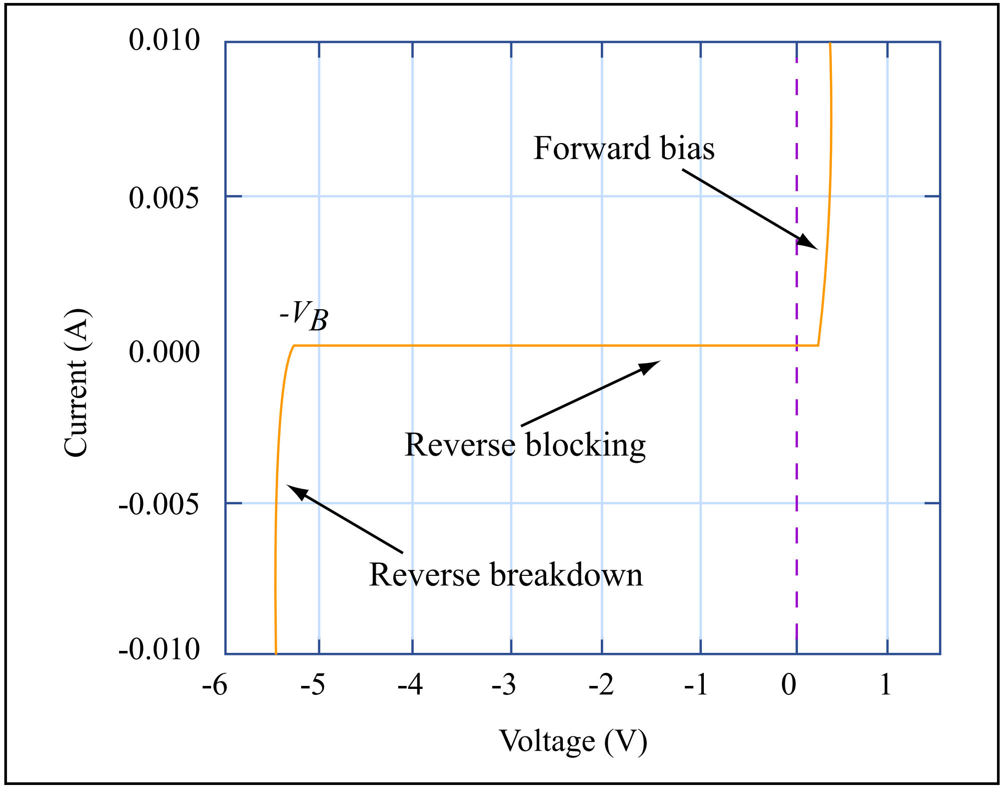


Image by MIT OpenCourseWare.

Adapted from Figure 14.3 in: Senturia, Stephen D. *Microsystem Design*. Boston, MA: Kluwer Academic Publishers, 2001, p. 360. ISBN: 9780792372462.

The Junction-Isolated Diffused Resistor

- The structure is a diode, but with two contacts
- The diode action prevents currents into the substrate provided the diode is always reverse biased
- The conductivity is controlled by doping
- The resistor value is determined by geometry

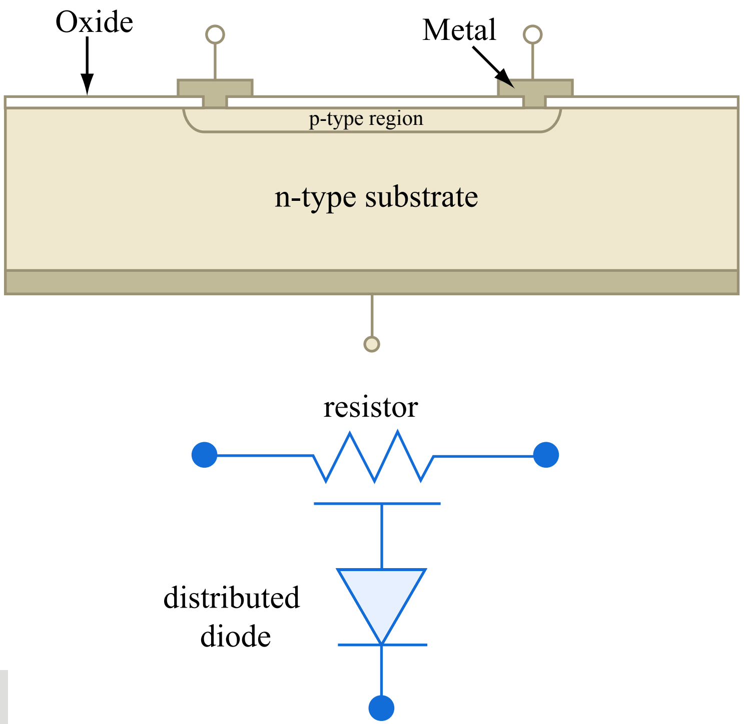
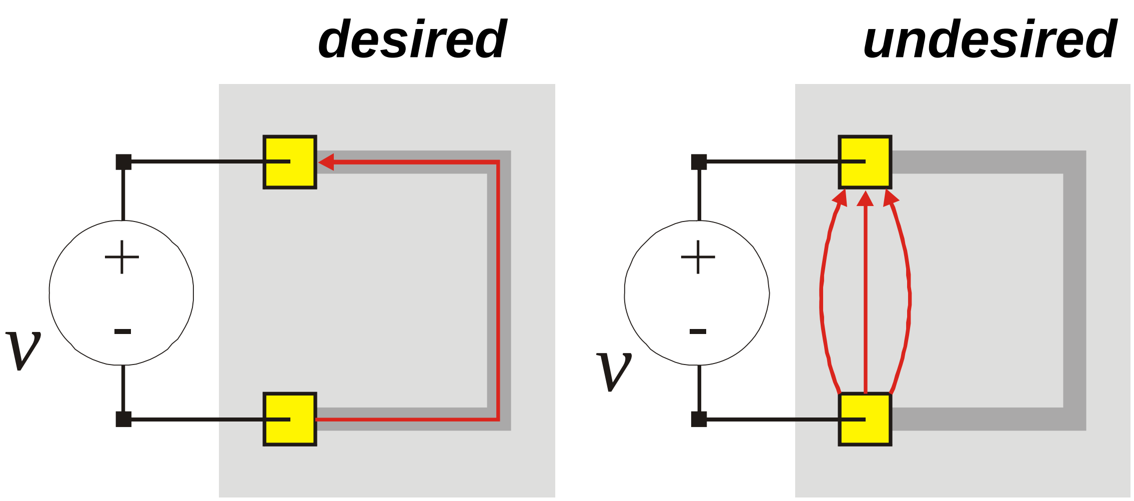


Image by MIT OpenCourseWare.
Adapted from Figure 14.7 in: Senturia, Stephen D. *Microsystem Design*. Boston, MA: Kluwer Academic Publishers, 2001, p. 363. ISBN: 9780792372462.



Outline

- Elements of circuit analysis
- Elements of semiconductor physics
- Semiconductor diodes and resistors
- The MOSFET and the MOSFET inverter/amplifier
- Op-amps

MOSFET Structure

- The MOSFET exploits the concept of a field-induced junction
- The electric field between the gate and the channel region of the substrate can either increase the surface concentration (accumulation) or deplete the surface and eventually invert the surface

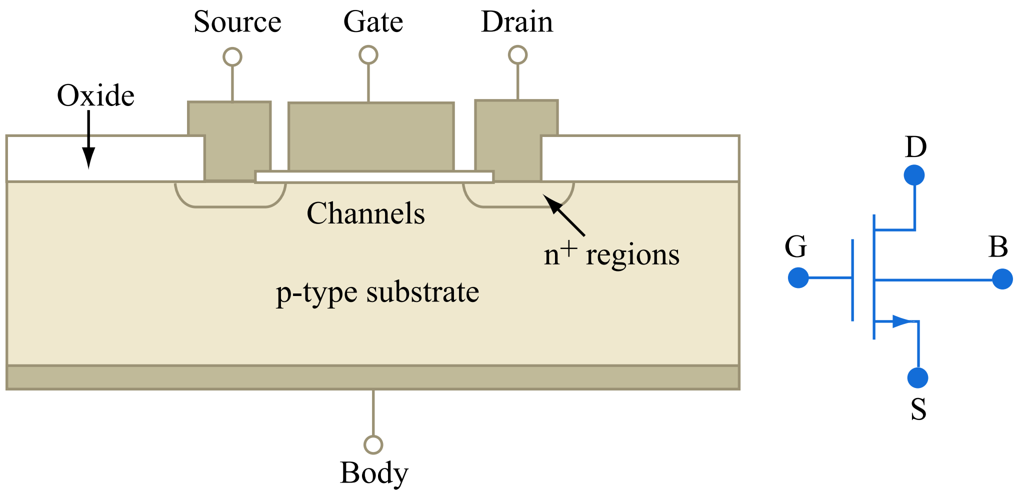
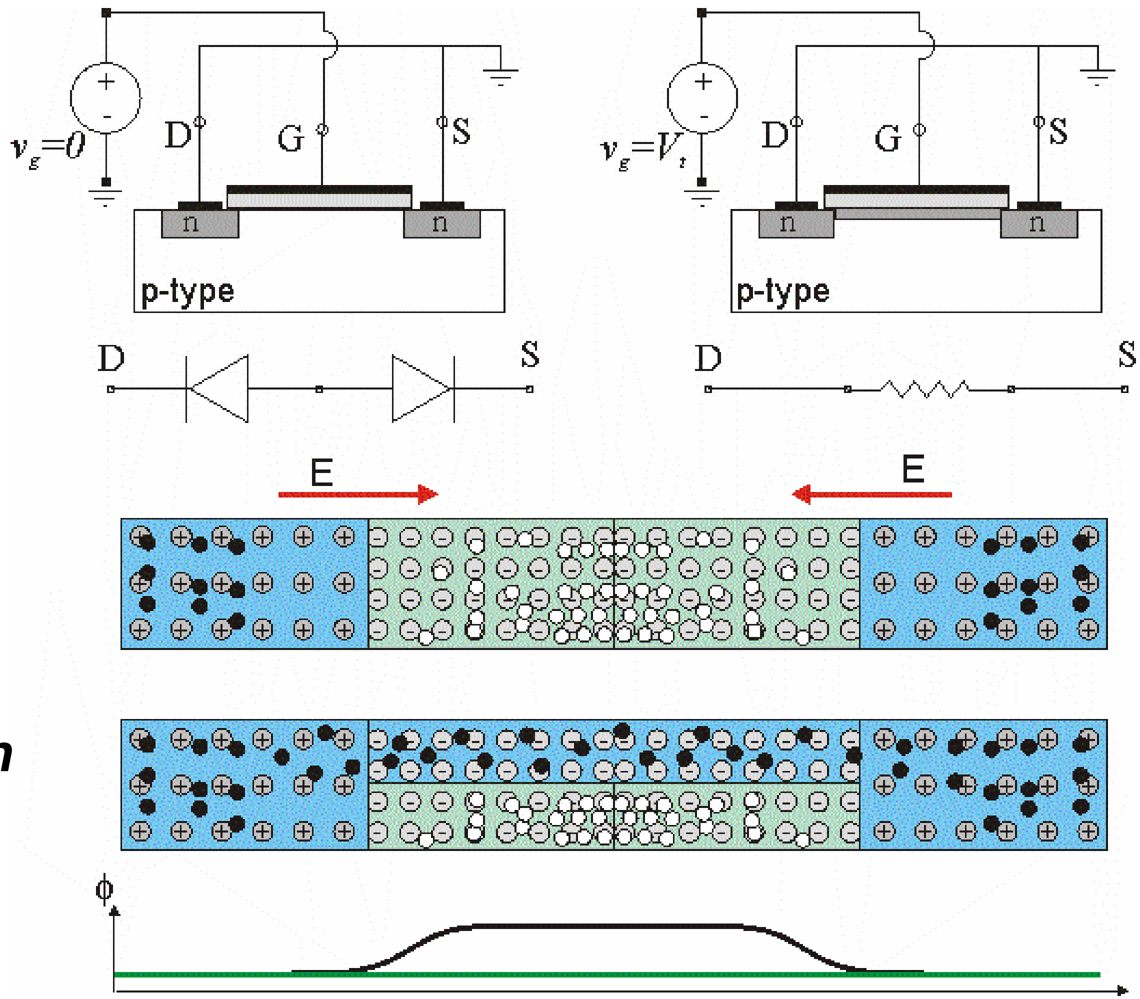


Image by MIT OpenCourseWare.

Adapted from Figure 14.10 in: Senturia, Stephen D. *Microsystem Design*. Boston, MA: Kluwer Academic Publishers, 2001, p. 366. ISBN: 9780792372462.

MOSFET qualitative operation

- With D & S grounded, back-to-back diodes prevent current flow between D & S
- To reduce barrier, apply positive voltage to G (w.r.t. D & S)
- At some *threshold voltage*, this will form an *n-channel inversion layer* that will connect D & S



MOSFET Characteristics

- Need $V_{GS}>V_T$ for device to turn on
- As V_{DS} increases, current will flow between D & S
- As V_{DS} increases, voltage between G and D decreases
- When V_{DS} gets too big, one side of channel *pinches off*, preventing further increases in current

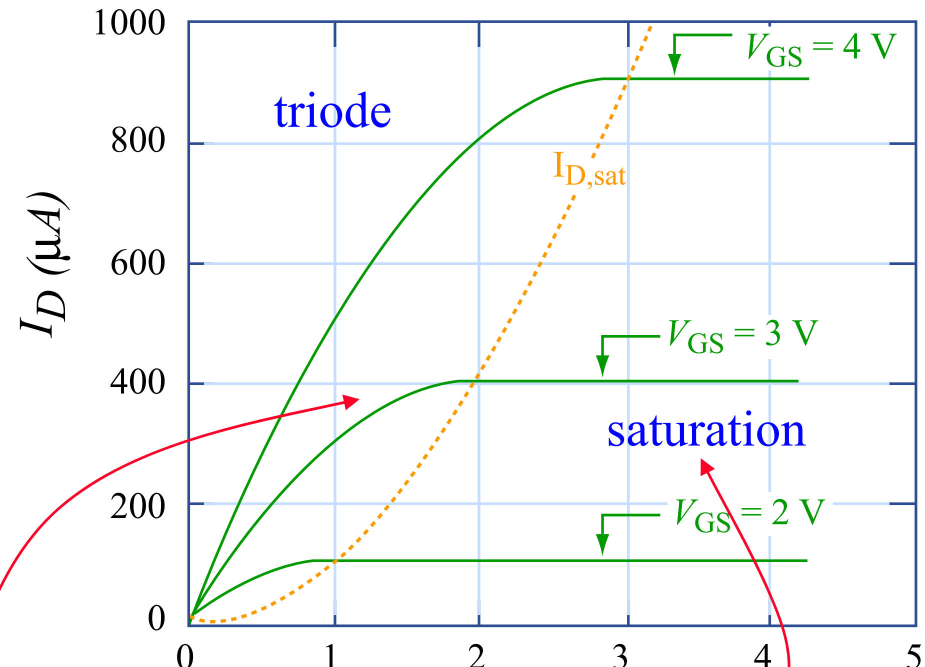
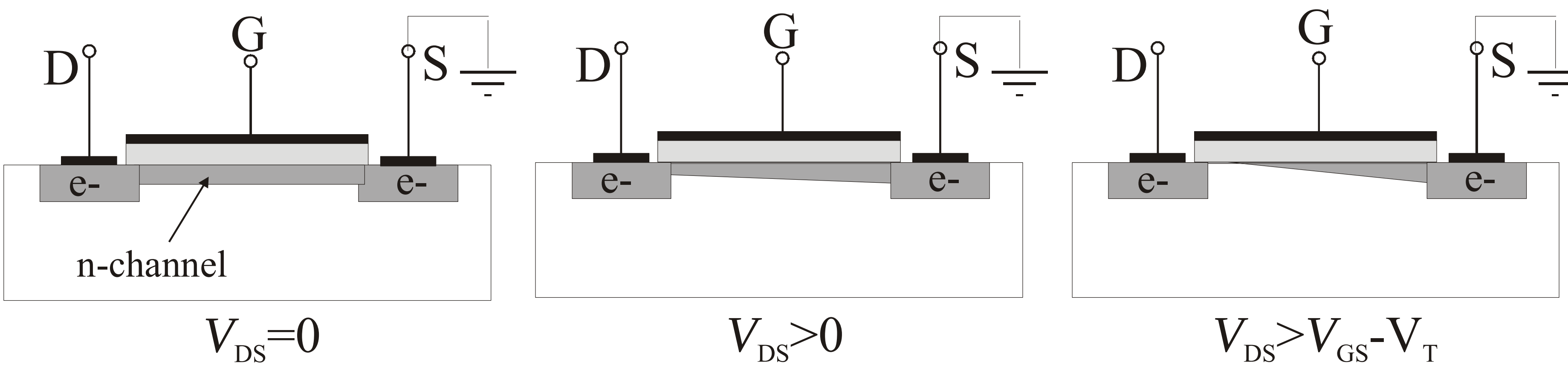


Image by MIT OpenCourseWare.
Adapted from Figure 14.13 in: Senturia, Stephen D. *Microsystem Design*. Boston, MA: Kluwer Academic Publishers, 2001, p. 369. ISBN: 9780792372462.



Different kinds of MOSFETs

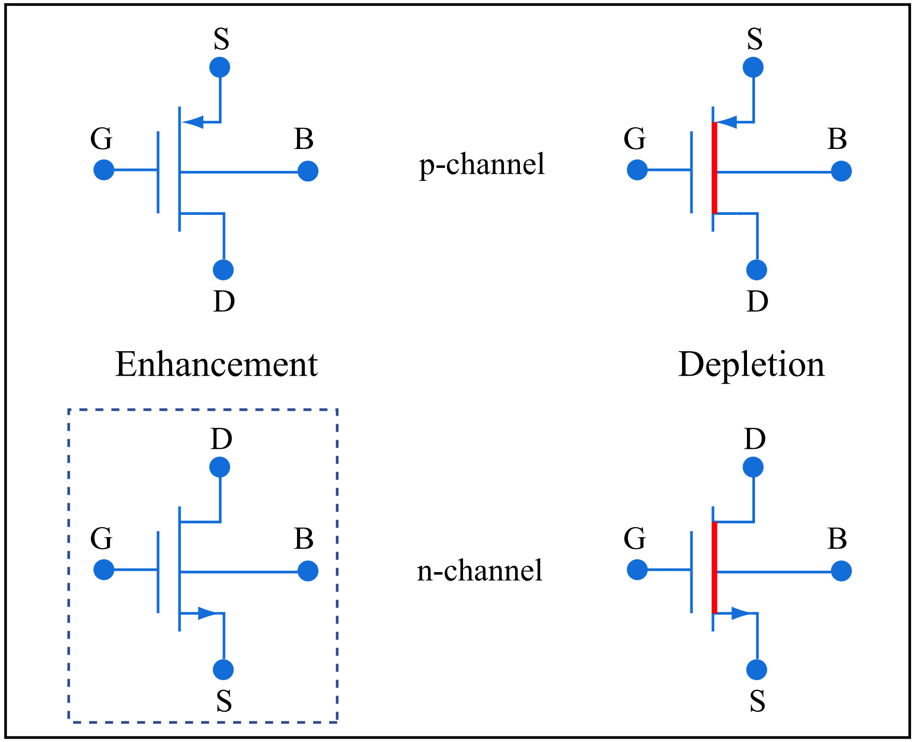


Image by MIT OpenCourseWare.

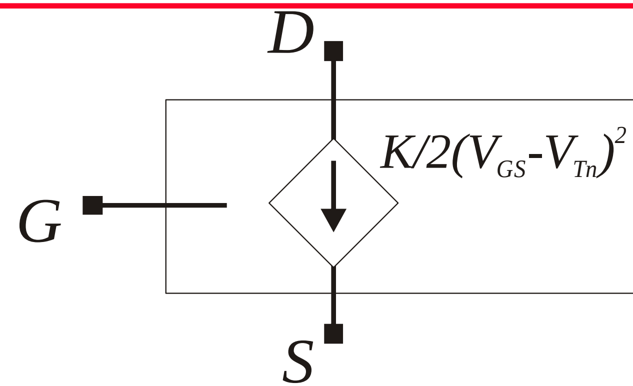
Adapted from Figure 14.11 in: Senturia, Stephen D. *Microsystem Design*. Boston, MA: Kluwer Academic Publishers, 2001, p. 367. ISBN: 9780792372462.

Large-signal and small-signal MOSFET models

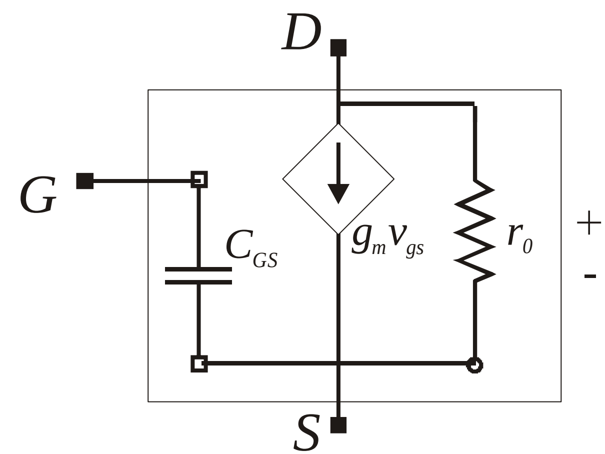
> **Electrical engineers use circuit models to analyze circuits involving MOSFETS**

> **Can use either full nonlinear characteristics**

> **Or linearized small-signal model**
  - **Different models include different components**



Simple <u>large-signal</u> model, in saturation



Simple <u>small-signal</u> model, in saturation

Outline

> **Elements of circuit analysis**

> **Elements of semiconductor physics**

- **Semiconductor diodes and resistors**
- **The MOSFET and the MOSFET inverter/amplifier**

> **Op-amps**

Operational Amplifiers – op-amps

- Let someone else design a high-performance amplifier
- Basic structure and transfer characteristic

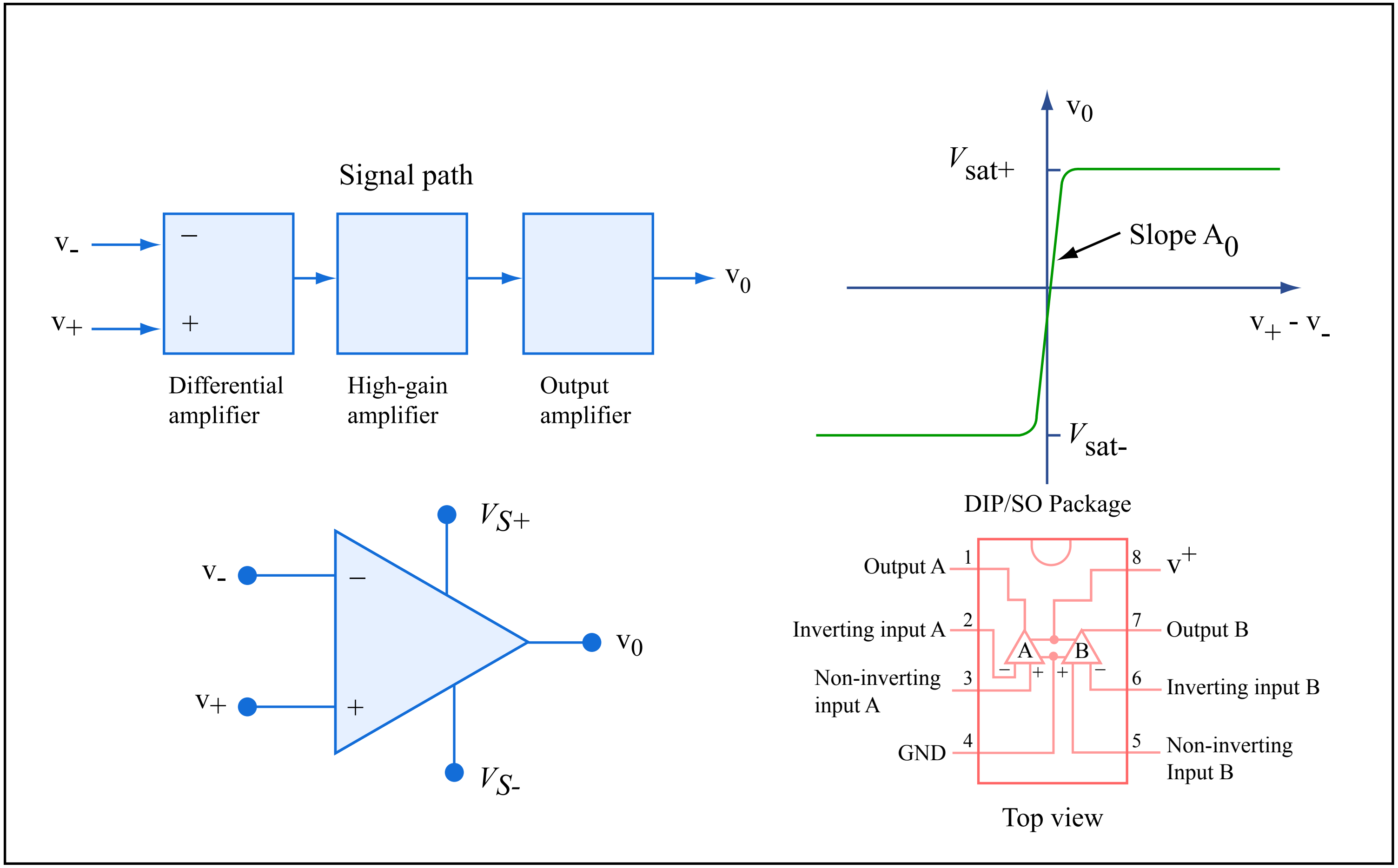


Image by MIT OpenCourseWare.

Adapted from Figures 14.23 and 14.24 in Senturia, Stephen D. *Microsystem Design*. Boston, MA: Kluwer Academic Publishers, 2001, p. 382. ISBN: 9780792372462.

LM158

Ideal and non-ideal behavior

> **We do first analysis using ideal linear model**

> **Op-amp data sheets have pages and pages of limitations**

> **A sampling**

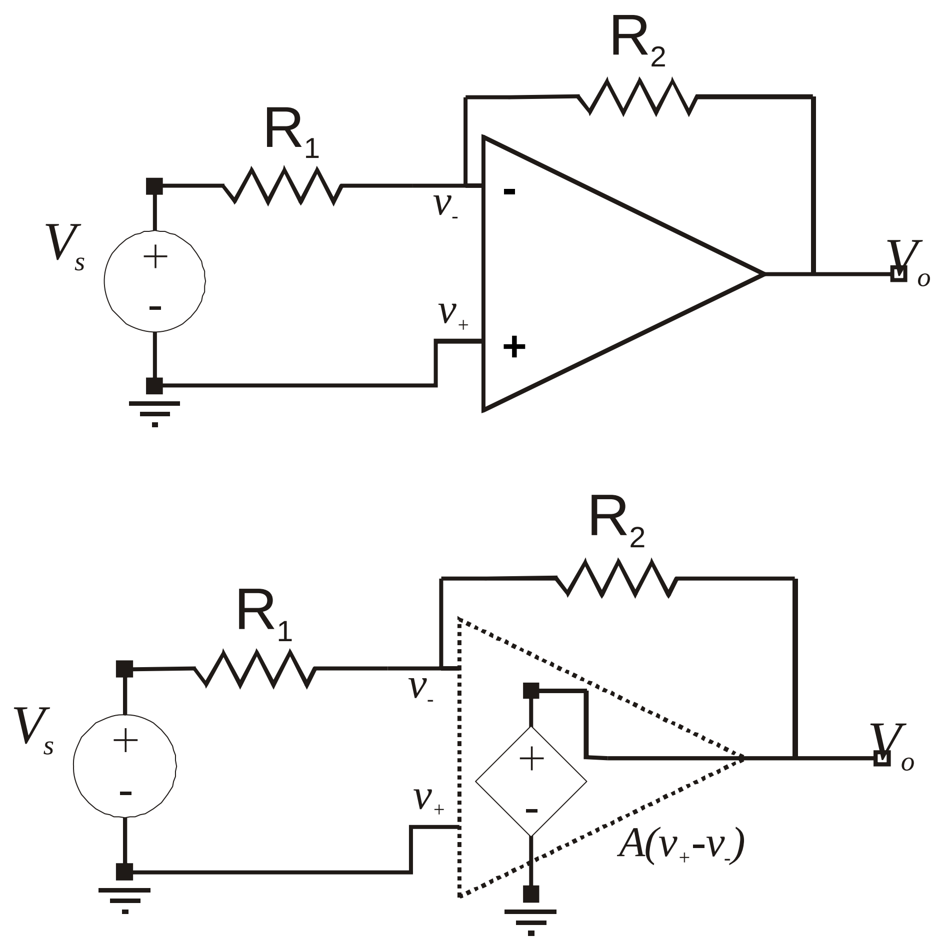
- **Input offset voltage v_{off}: zero volts at input gives non-zero output**
- **Frequency limitations: op-amps can only amplify up to a maximum frequency**

$$v_0(s) = A(s)\left(v_+(s) - v_-(s)\right)$$

where

$$A(s) = \frac{A_0 s_0}{s + s_0}$$

The Inverting Amplifier



Assume op-amp draws no current

Use Nodal analysis:

$$\text{KCL:} \quad \frac{V_s - v_-}{R_1} = \frac{v_- - V_0}{R_2}$$

$$\Downarrow V_0 = A(0 - v_-)$$

$$\Downarrow$$

$$\frac{V_o}{V_s} = -\frac{R_2}{R_1}\left[\frac{1}{1+\frac{1}{A}\left(1+\frac{R_2}{R_1}\right)}\right]$$

$$\Downarrow A \to \infty$$

$$\boxed{\frac{V_o}{V_s} = -\frac{R_2}{R_1}}$$

Short method for analyzing op-amps

> **Assume linear region operation**
>
> - **This implies that the two inputs are essentially at the same voltage (but never exactly equal)**

> **Assume zero currents at both inputs**

> **Analyze the external circuit with these constraints**

> **Check to verify that the output is not at either saturation limit ($\pm V_S$)**

More op-amp configurations

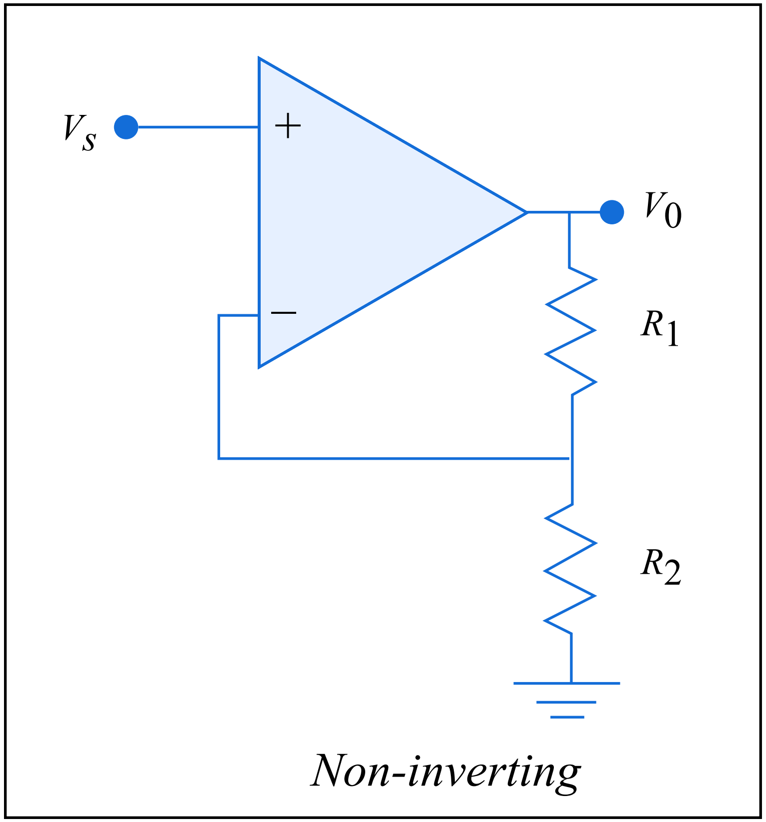


Image by MIT OpenCourseWare.

Adapted from Figure 14.28 in Senturia, Stephen D. *Microsystem Design*.
Boston, MA: Kluwer Academic Publishers, 2001, p. 388. ISBN: 9780792372462.

$$v_- = v_+ = V_S$$

$$V_s = V_0 \frac{R_2}{R_2 + R_1}$$

$$\boxed{\frac{V_0}{V_S} = 1 + \frac{R_1}{R_2}}$$

More op-amp configurations

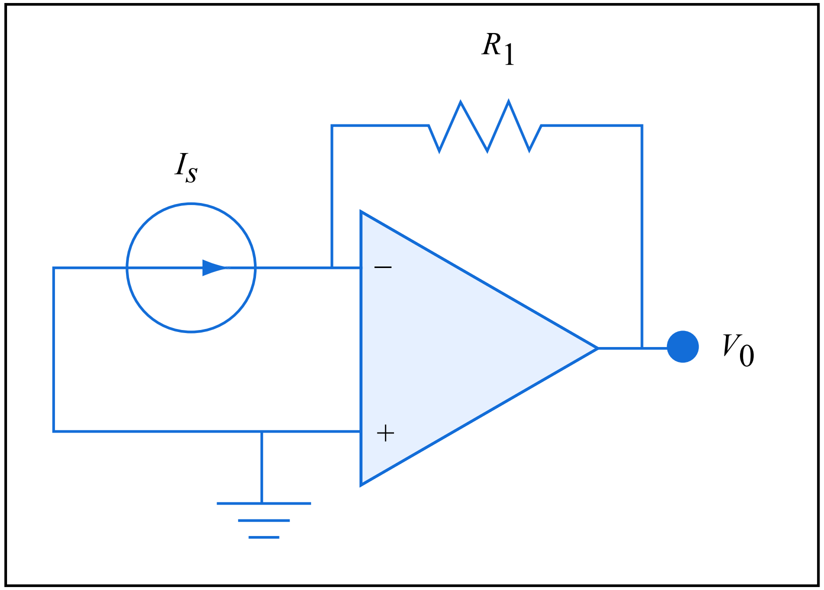


Image by MIT OpenCourseWare.

Adapted from Figure 14.29 in Senturia, Stephen D. *Microsystem Design.* Boston, MA: Kluwer Academic Publishers, 2001, p. 389. ISBN: 9780792372462.

Transimpedance amplifier

$$V_0 = -R_1 I_S$$

Integrator

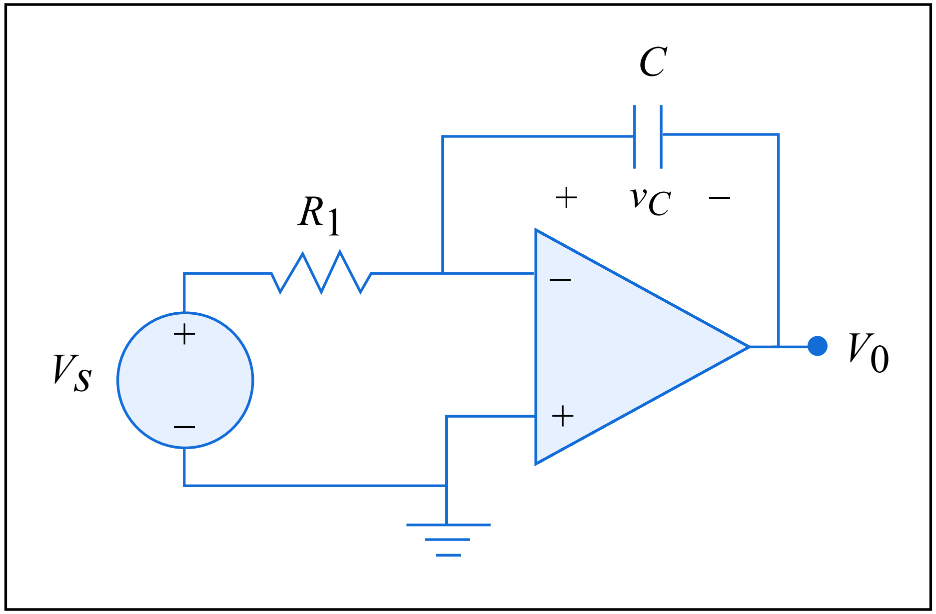


Image by MIT OpenCourseWare.

Adapted from Figure 14.30 in Senturia, Stephen D. *Microsystem Design*. Boston, MA: Kluwer Academic Publishers, 2001, p. 389. ISBN: 9780792372462.

$$V_0 = -\frac{1}{R_1 C}\int V_s(t)dt$$

Differentiator

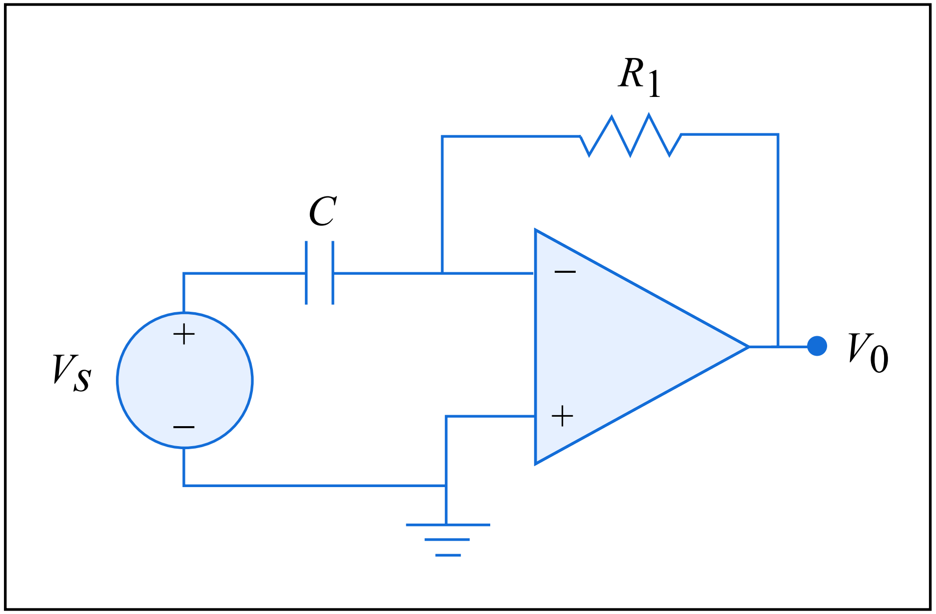


Image by MIT OpenCourseWare.
Adapted from Figure 14.31 in Senturia, Stephen D. *Microsystem Design*. Boston, MA: Kluwer Academic Publishers, 2001, p. 390. ISBN: 9780792372462.

The differentiator is less "ideal"

$$\frac{V_s}{1/sC} = \frac{0 - V_0}{R_1}$$

$$\frac{V_0}{V_s} = -sR_1C$$