6.777J/2.372J Design and Fabrication of Microelectromechanical Devices
Spring Term 2007

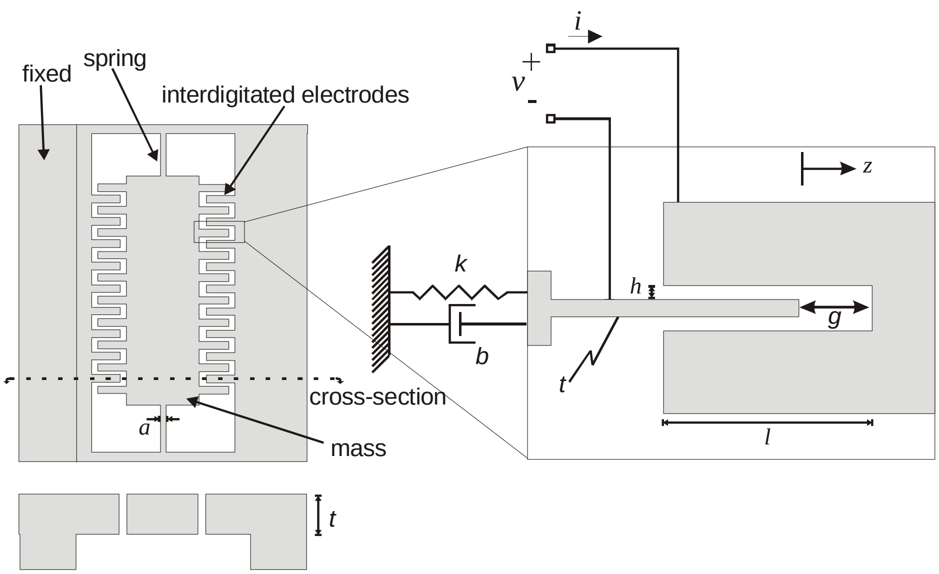
Massachusetts Institute of Technology

PROBLEM SET 4 (21 pts)

Issued: Lecture 9 Due: Lecture 12

Problem 6.5 (5 pts): The in-plane interdigitated electrostatic (or comb drive) transducer

Many MEMS components use interdigitated electrodes as transducers. In this and subsequent problems we will explore this structure. Below on the left is a moveable silicon mass with many interdigitated (comb) fingers that is connected to the fixed substrate by a flexible spring. Using DRIE, it is possible to make such a structure with very thick fingers (>50 μm out of the plane of the paper). On the right is a schematic of one finger of this transducer, with the rest of the system specified by lumped parameters b and k.



For our one-finger model, assume that the device is constrained to only move in the z direction. The thickness of the plate in the plane of the paper (t) is 200 μm and its width w is 5 μm. The spacing h is 5 μm on either side of the finger, the initial gap g_0=10 μm, while the length l=100 μm. The spring constant k=1 kN/m, the damping coefficient b=0.05 mN-s/m, and the device operates in air.

(a) Assume a current drive. Determine analytically the force, the position variable g (the gap), and the voltage on the plates as a function of the charge Q.

(b) State any assumptions that you are making in calculating the force in (a).

(c) Assume a voltage drive. Determine analytically the force, the gap, and the charge on the plates as a function of the voltage V.

(d) Show that the effective spring constant under voltage drive does not change with position (i.e., $\partial F_{net}/\partial g$ is constant with V and g). What does this imply about spring softening/hardening and instabilities for this system?

(e) Using a similar approach for current drive, show that spring softening occurs. Will this transducer experience pull-in? Why?

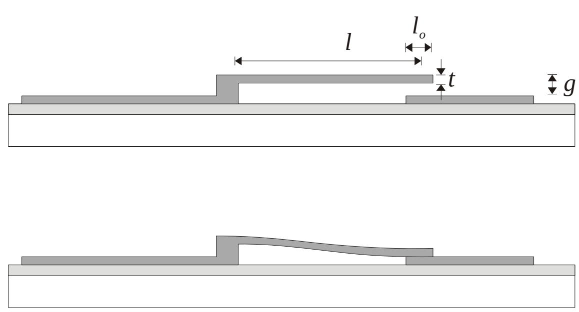
Problem 6.9 (2 pts): Design the spring

The comb-drive transducer in Problem 6.5 is designed to have a spring constant of k=1 kN/m. Given the plate thickness of 200 μm and a spring width a of 10 μm, how long must the spring be to produce this spring constant? You may assume small-deflections. Describe in words and pictures how you might go about fitting such a spring into a minimal wafer area.

Problem 6.10 (7 pts): Design a simple switch

Your task is to design an inexpensive surface-micromachined MEMS electrical switch. The general form of the switch is shown in the figure below. It is composed of a polySi cantilever electrode above a polySi actuating electrode, and is separated from the substrate by an insulator. The cantilever has length l, thickness t, width w (not shown), and is separated from the actuating electrode by a gap g. You are given the following set of constraints:

- Voltage to close the device is V_c=10 V.
- When closed, the cantilever is used to conduct current, and therefore must have no more than 10 Ω of resistance along its length.
- Max cantilever thickness t of 2 μm.
    - You can increase t, but the extra fabrication cost is \$2/μm for each micron over 2 μm.
- Minimize cost by minimizing cantilever beam area, which is priced at \$200/mm^2.
- Cantilever length l must be at least 5× the width to ensure beam-like behavior ($l \geq 5w$).
- Minimum initial gap g is 0.5 μm.
- The overlap length l_o between the actuating electrode and the cantilever is 10 μm.
- The target cost of the part is ≤\$1.
- The width w must be at least 10× the thickness t of the cantilever.
- Assume E=150 GPa, mass density ρ=2300 kg/m^3, and electrical conductivity σ=10^5 S/m for polysilicon.
- You can ignore any deflection of the cantilever due to its own weight, and assume an ideal cantilever support.



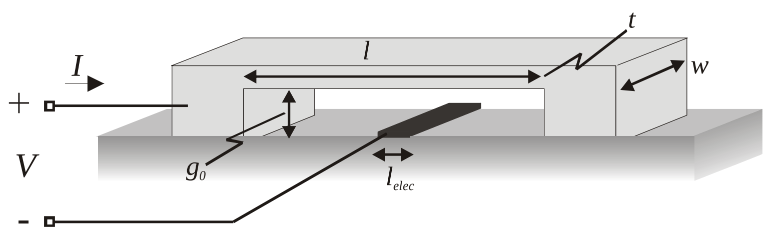
(a) Determine analytically the set of equations governing the design constraints. You will need to make engineering approximations to solve this problem expeditiously. For instance, you can ignore the effects of the non-ideal support and the extra stiffness induced by the cantilever deforming at the contact. You can also model the electrical force as being applied as a point to the tip of the cantilever (if $l_o << l$)..

(b) Using hand calculations, Matlab, or any other suitable approach, determine l, t, w, and g that will meet these specifications and result in the lowest cost. State your V_c, cost, and resistance. If you believe that you cannot meet the cost objective, then give the dimensions that result in the lowest cost.

(c) Using the principles of crayon engineering, design a process flow and mask set to create your device.

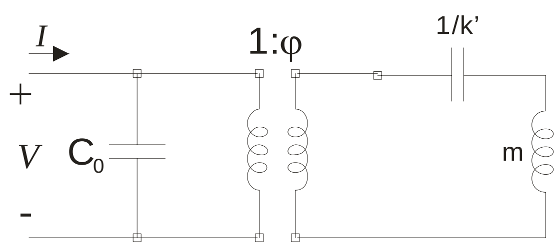
Problem 6.11 (7 pts): A simple MEMS resonator

One of the things that make MEMS resonators attractive to circuit designers is that one can recreate large electrical inductors and capacitors using vibrating mechanical components, and further that one can tune the value of these components by changing an applied DC bias. In this problem we'll examine some of the design issues with a simple clamped-clamped MEMS resonator shown below, which has length l, thickness t, width w, gap g_o, and electrode width l_e. We will first determine the equations describing its static (i.e., fixed point) behavior, and then use a linearized model to examine the tenability of the resonator.

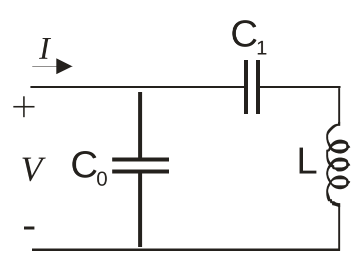


(a) Determine analytically the set of equations governing the static behavior of the beam. Specifically, provide an expression for the mechanical spring constant k and the lumped effective mass, which we will approximate as $m_{eff} = 0.4m_{actual}$. Also find the pull-in voltage V_{PI} for this structure. You may assume the $l_{elec} << l$ and thus that the electrical force acts a point force in the mechanical domain.

(b) Using the linearized model from the figure below, determine C_0, k', and φ**,** in terms of each other, physical dimensions, material constants, the bias voltage V_0, and the static actuated gap $\hat{g}_0$.



(c) Next, remove the transformer to obtain the following equivalent circuit, and determine expressions for C_1 and L.



(d) Determine the input impedance Z_{in} of this circuit as a function of C_o, C_1, and L. This is a 3rd order system that has one pole at ω=0. Find the frequencies (ω) of the other poles and zeros.

(e) The way that we tune this device is to set the fixed point of the nonlinear system by applying a bias voltage V_0 that will deflect the beam and set k' (or C_1), C_0, and φ, and then superpose a small sinusoidal voltage as our signal. As we actuate from V_0=0 to $V_0=V_{PI}$ we will tune the location of the zero. Determine analytically ω_1, the zero of Z_{in} when V_0=0, and ω_2, the zero of Z_{in} when $V_0=\alpha V_{PI}$, in terms of physical dimensions, material constants, the bias voltage V_0, and the static actuated gap $\hat{g}_0$.

(f) As α approaches (but does not quite reach) 1, you can simplify ω_2 into a particularly simple form. Find this form (in terms of ω_1 and α) and use it to estimate the maximum change in ω_2 as V_0 is varied from zero to 95% of V_{PI}. The actual change will be less. Why?

(g) Now that we know how to pick where the zero occurs and how it tunes as we change V_0, we are ready to design. Pick l, w, g_0, and l_e subject to the following constraints:

- $t = 1$ μm
- $V_{PI} = 25$ V
- $\omega_1 = 2\pi 10^6$ rad/s
- $l_e \leq 0.1 \cdot l$
- $g_0 \geq 0.25$ μm
- E= 150 GPa, ρ=2300 kg/m^3 for polySi, and the gap is vacuum

You may use hand calculations or Matlab to determine suitable parameters. You may want to familiarize yourself with the Matlab function fzero to determine the gap. Plot the zero of Z_{in} from $V_0=0.05V_{PI}$ to $V_0=0.95V_{PI}$. Also generate a Bode plot at those two voltages to show how the zero (and pole) move as we change V_{PI}. For the Bode plot you will need to use a very-fine frequency vector and concentrate on the frequency range from 10^6 to 10^7 Hz to easily see the behavior.