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High Speed Communication Circuits

Lecture 7

High Frequency, Broadband Amplifiers

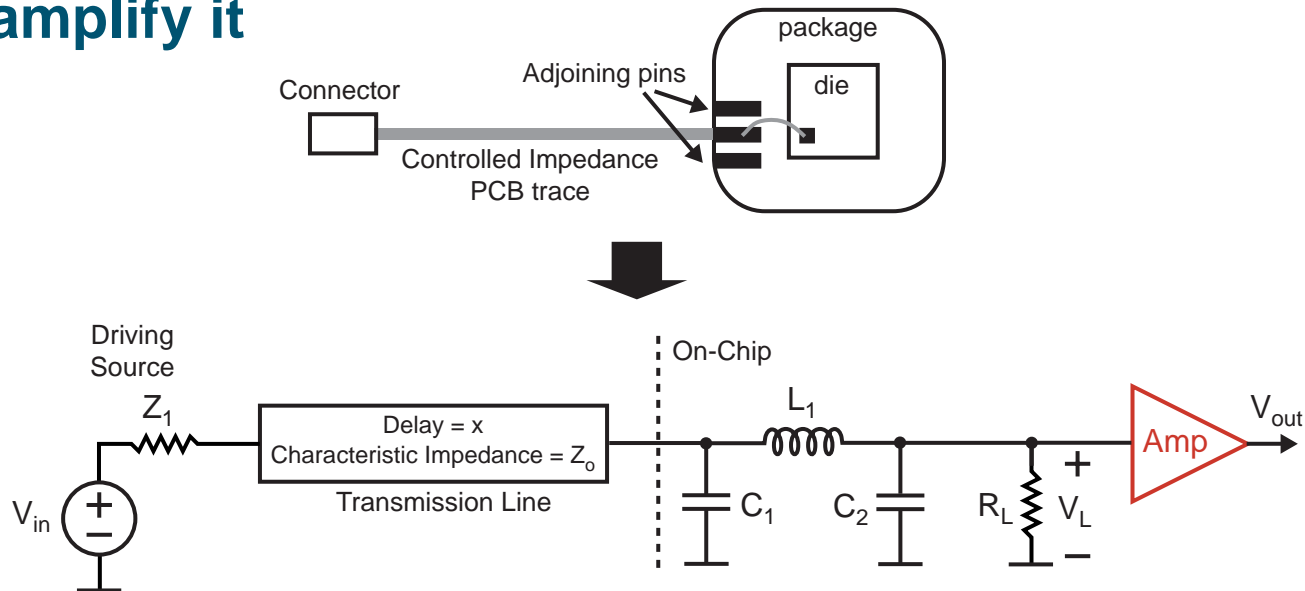
Massachusetts Institute of Technology

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Perrott**

High Frequency, Broadband Amplifiers

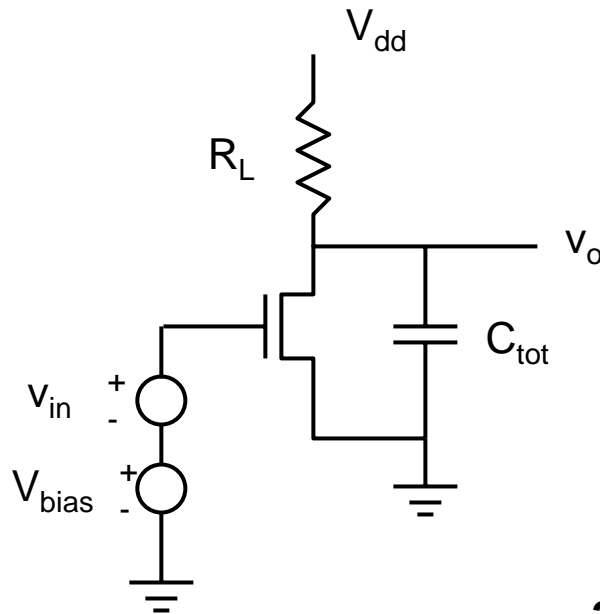
- The first thing that you typically do to the input signal is amplify it



- **Function**
 - Boosts signal levels to acceptable values
 - Provides reverse isolation
- **Key performance parameters**
 - Gain, bandwidth, noise, linearity

Gain-bandwidth Trade-off

- Common-source amplifier example



C_{tot} : total capacitance at output node

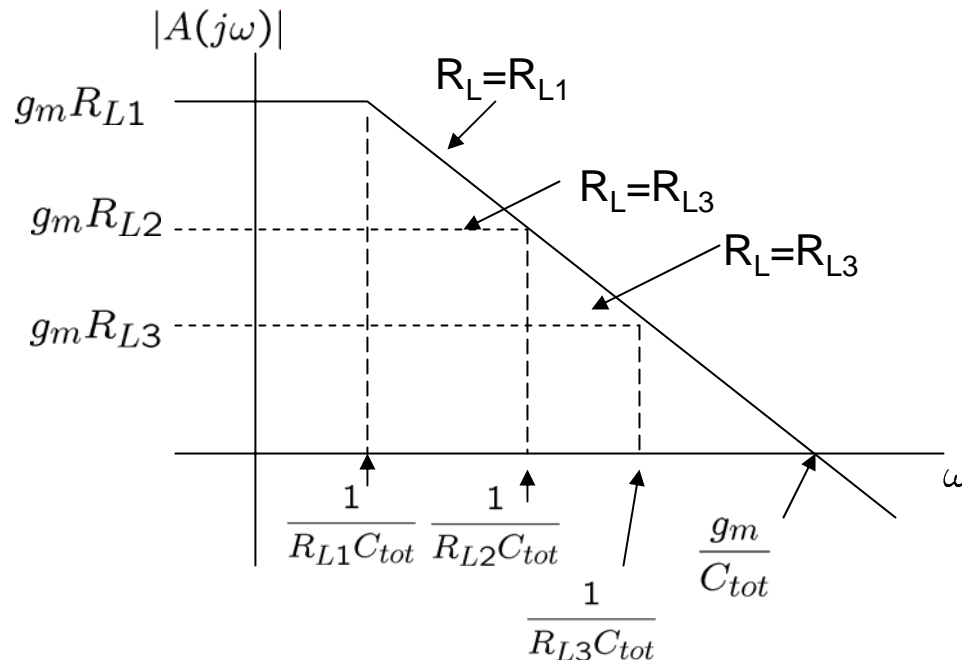
DC gain $A = g_m R_L$

3 dB bandwidth $\omega_h = \frac{1}{R_L C_{tot}}$

Gain-bandwidth $GB = \frac{g_m}{C_{tot}}$

Gain-bandwidth Trade-off

- Common-source amplifier example



- Given the 'origin pole' g_m/C_{tot} , higher bandwidth is achieved only at the expense of gain
- The origin pole g_m/C_{tot} must be improved for better GB

Gain-bandwidth Improvement

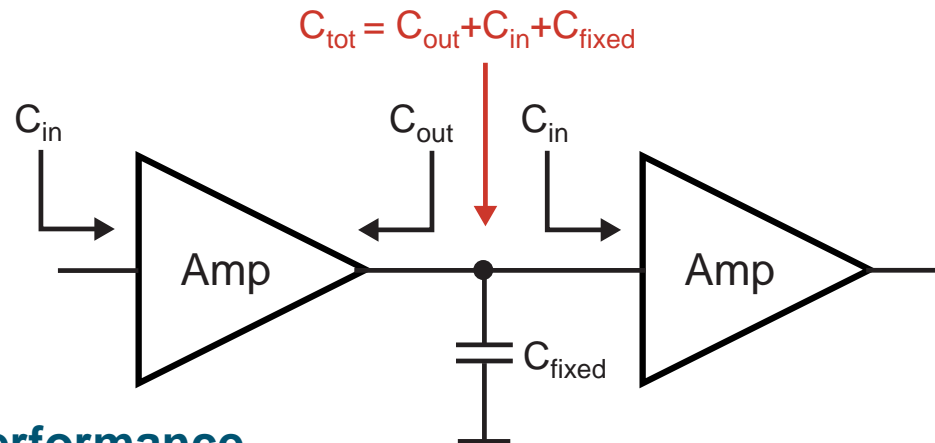
- How do we improve g_m/C_{tot} ?
- Assume that amplifier is loaded by an identical amplifier and fixed wiring capacitance is negligible
- Since $g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)$ and $C_{tot} \propto W$
$$\frac{g_m}{C_{tot}} \propto \frac{V_{GS} - V_T}{L}$$
- To achieve maximum GB in a given technology, use minimum gate length, bias the transistor at maximum $V_{GS} - V_T$
- When velocity saturation is reached, higher $V_{GS} - V_T$ does not give higher g_m
- In case fixed wiring capacitance is large, power consumption must be also considered

Gain-bandwidth Observations

- **Constant gain-bandwidth is simply the result of single-pole roll off – it's *not* fundamental!**
- **It implies a single-pole frequency response may not be the best for obtaining gain and bandwidth simultaneously**
- **Single-pole roll off is necessary for some circuits, e.g. for stability, but not for broad-band amplifiers**

Assumptions (for now) for Bandwidth Analysis

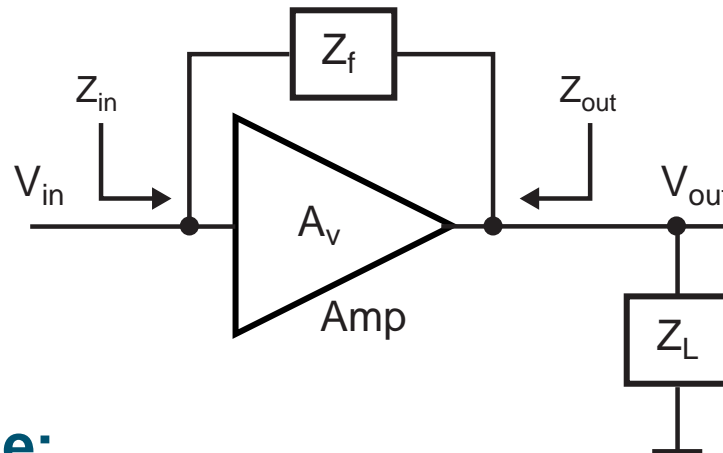
- Assume for now that amplifier is loaded by an identical amplifier and by fixed wiring capacitance
- Assume amplifier is driven by an ideal voltage source for now



- Intrinsic performance
 - Defined as the bandwidth achieved for a given gain when C_{fixed} is negligible
 - Amplifier approaches intrinsic performance as its device sizes (and current) are increased
- In practice, point of diminishing return for bandwidth vs. size (and power) of amplifier is roughly where $C_{in} + C_{out} = C_{fixed}$

The Miller Effect

- Concerns impedances that connect from input to output of an amplifier



- Input impedance:**

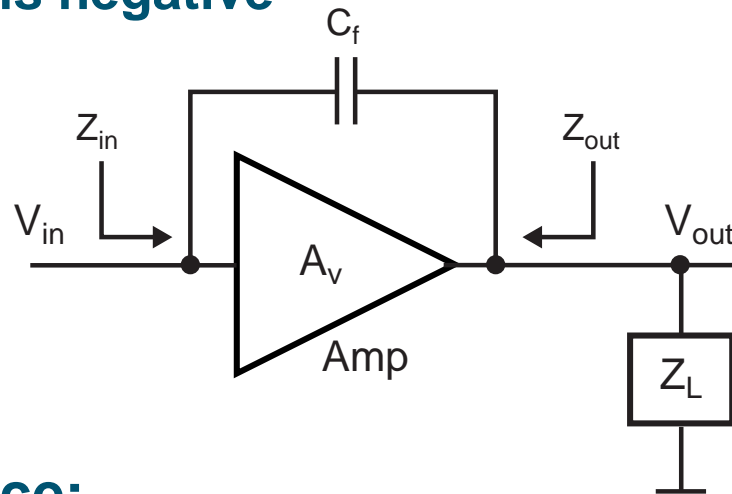
$$Z_{in} = \frac{V_{in}}{(V_{in} - V_{out})/Z_f} = \frac{Z_f}{1 - A_v}$$

- Output impedance:**

$$Z_{out} = \frac{V_{out}}{(V_{out} - V_{in})/Z_f} = \frac{Z_f}{1 - 1/A_v} \approx Z_f \text{ for } |A_v| \gg 1$$

Example: Miller Capacitance

- Consider C_{gd} in the MOS device as C_f
 - Assume gain is negative

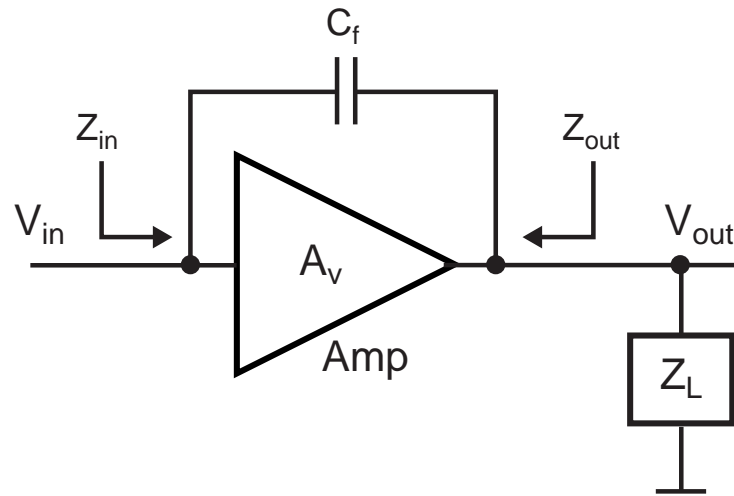


- Input capacitance:

$$Z_{in} = \frac{1/(sC_f)}{1 + |A_v|} = \frac{1}{sC_f(1 + |A_v|)}$$

Looks like much larger capacitance by $|A_v|$

Example: Miller Capacitance



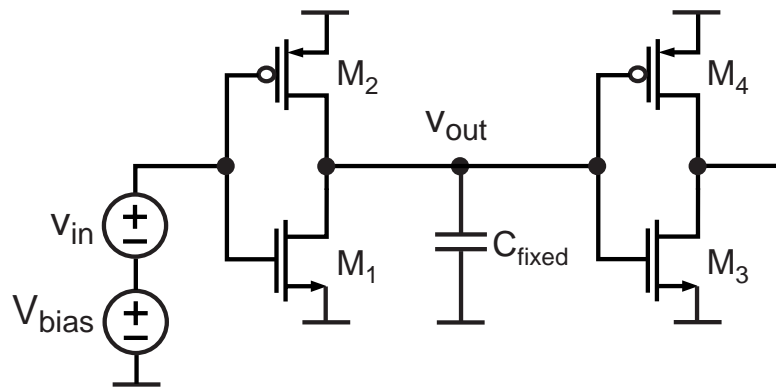
- **Output impedance:**

$$Z_{out} = \frac{1/(sC_f)}{1 + 1/|A_v|} = \frac{1}{sC_f(1 + 1/|A_v|)} \approx \frac{1}{sC_f}$$

This makes sense because the input of the amplifier is 'virtual ground' if gain is large

Amplifier Example – CMOS Inverter

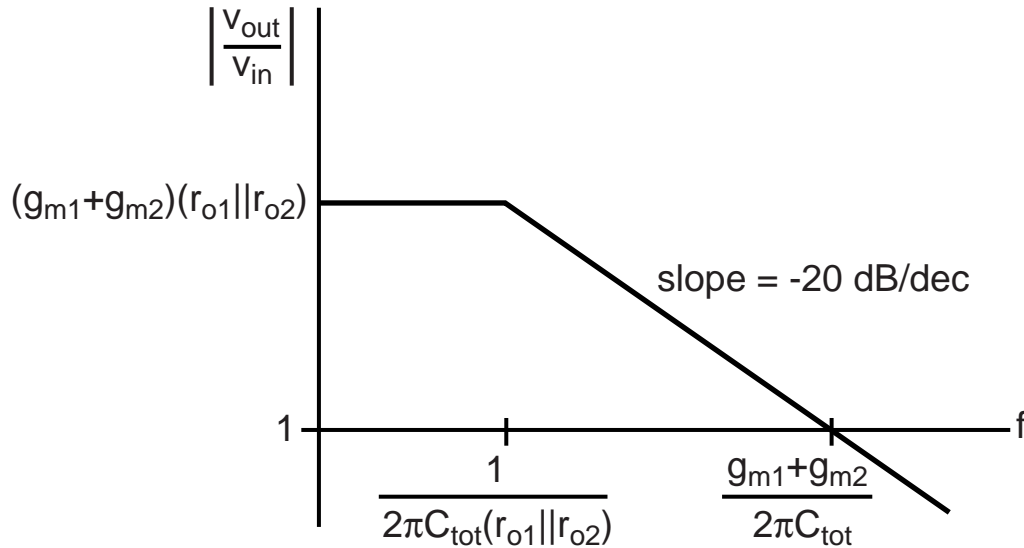
- The Miller effect gives a quick way to estimate the bandwidth of an amplifier without solving node equations: intuition!
- Assume that we set V_{bias} the amplifier nominal output is such that NMOS and PMOS transistors are all in saturation
 - Note: this topology VERY sensitive to V_{bias} : some feedback biasing would be required (6.301)



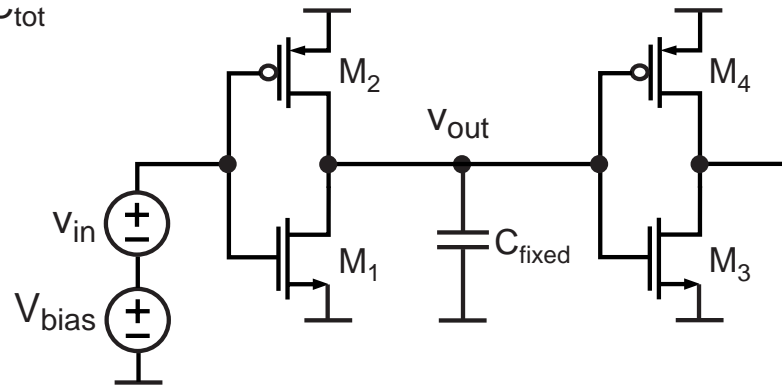
$$C_{tot} = C_{db1} + C_{db2} + C_{gs3} + C_{gs4} + K(C_{ov3} + C_{ov4}) + C_{fixed}$$

(↑) (+C_{ov1}+C_{ov2}) Miller multiplication factor (↑)

Transfer Function of CMOS Inverter



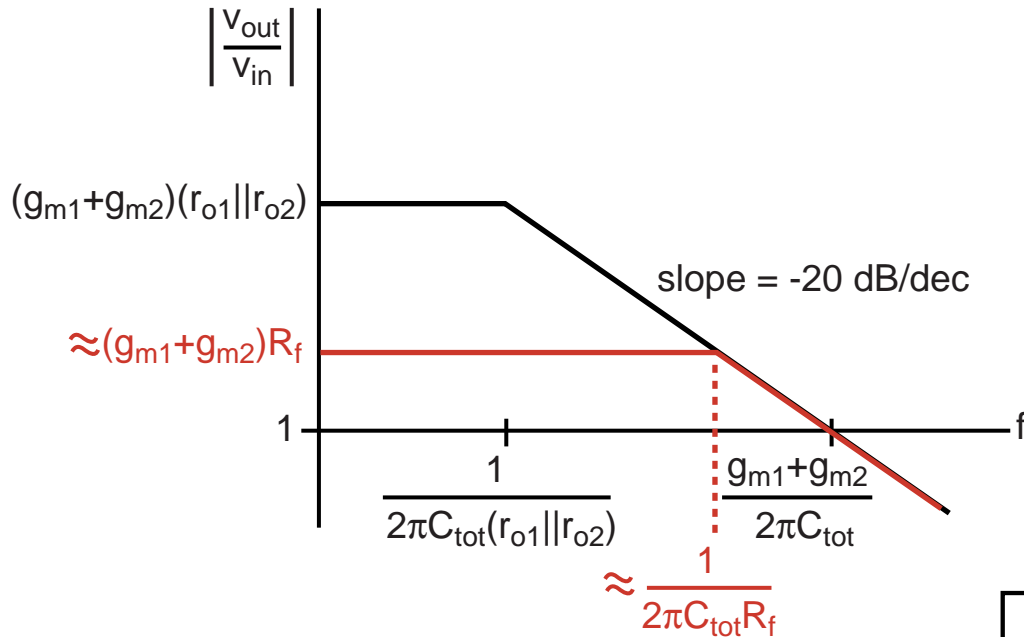
Low Bandwidth!



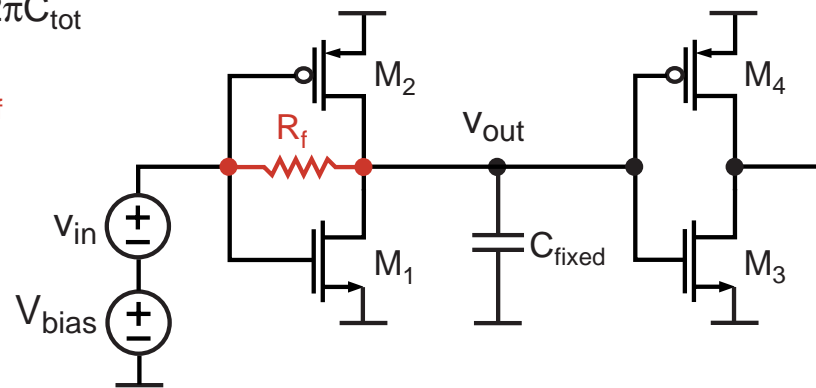
$$C_{tot} = C_{db1} + C_{db2} + C_{gs3} + C_{gs4} + K(C_{ov3} + C_{ov4}) + C_{fixed}$$

$(+C_{ov1} + C_{ov2})$ Miller multiplication factor

Add Resistive Feedback?



Bandwidth extended and less sensitivity to bias offset (does not improve GB, though)

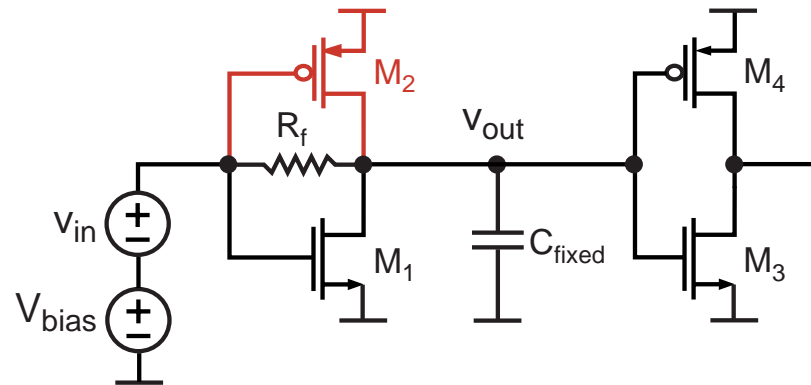


$$C_{tot} = C_{db1} + C_{db2} + C_{gs3} + C_{gs4} + K(C_{ov3} + C_{ov4}) + C_{Rf}/2 + C_{fixed}$$

$(+C_{ov1} + C_{ov2})$ Miller multiplication factor

We Can Still Do Better

- We are fundamentally looking for high g_m to capacitance ratio to get the highest bandwidth
 - PMOS degrades this ratio
 - Gate bias voltage is constrained
- However, when C_{fixed} is dominant and power consumption is important, the PMOS increases g_m without additional power
- On the other hand, below velocity saturation, higher g_m can be achieved by biasing the gate of M_1 close to V_{dd} instead of using PMOS

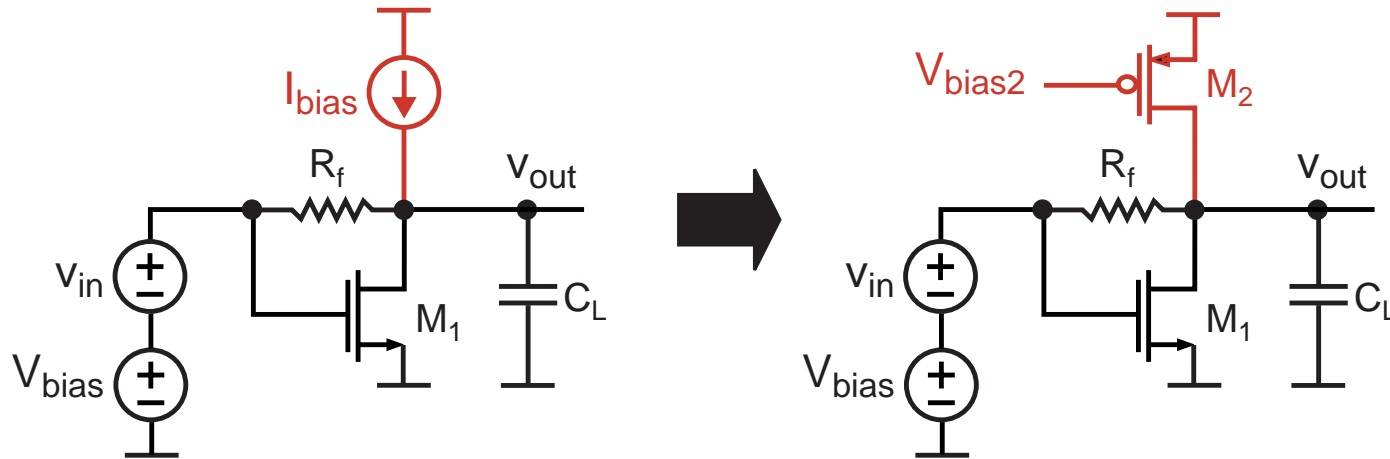


$$C_{\text{tot}} = C_{\text{db1}} + C_{\text{db2}} + C_{\text{gs3}} + C_{\text{gs4}} + K(C_{\text{ov3}} + C_{\text{ov4}}) + C_{\text{Rf}}/2 + C_{\text{fixed}}$$

↑
↑

(+ C_{ov1} + C_{ov2})
Miller multiplication factor

Take PMOS Out of the Signal Path



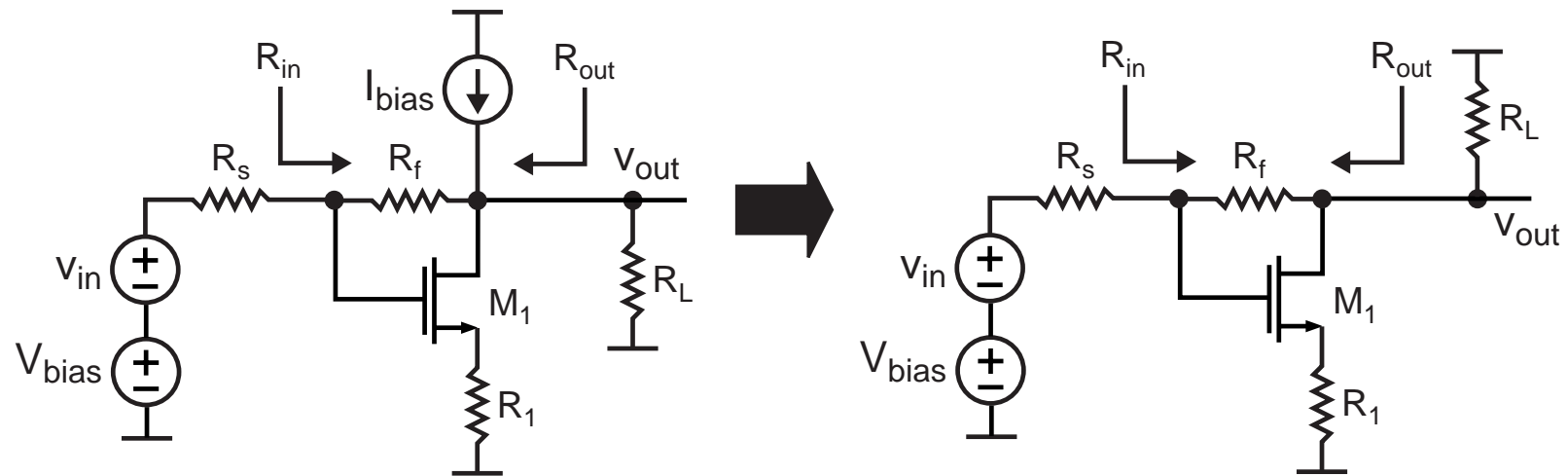
■ Advantages

- PMOS gate no longer loads the signal
- NMOS device can be biased at a higher voltage (higher g_m up to velocity sat. limit)

■ Issue

- PMOS is not an efficient current provider (I_d /drain cap $C_{gd}+C_{db}$)
 - Drain cap close in value to C_{gs}
- Signal path is loaded by cap of R_f and drain cap of PMOS

Shunt-Series Amplifier



- **Use resistors to control the bias, gain, and input/output impedances**
 - Improves accuracy over process and temp variations
- **Issues**
 - Degeneration of M_1 lowers slew rate for large signal applications (such as limit amps)
 - There are better high speed approaches – the advantage of this one is simply accuracy

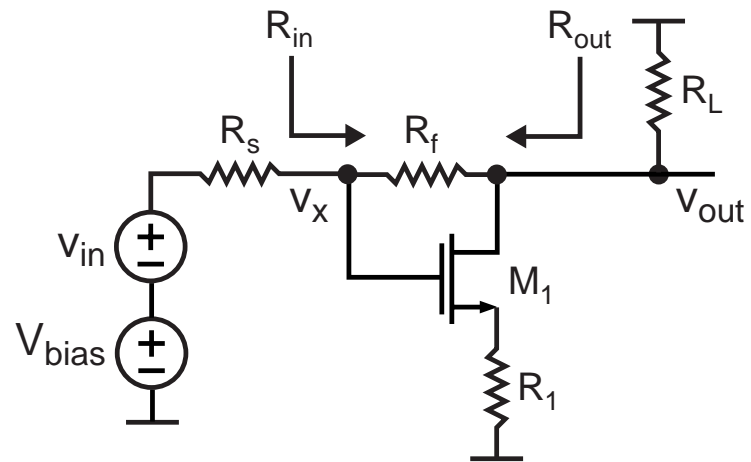
Shunt-Series Amplifier – Analysis Snapshot

- From Chapter 9 (8) of Tom Lee's book:

- Gain

$$A_v = \frac{v_{out}}{v_{in}} = -\frac{R_L}{R_E} \left(\frac{R_f - R_E}{R_f + R_L} \right)$$

where: $R_E = 1/g_m + R_1$



Note: $A_v \approx -\frac{R_L}{R_1}$ for $R_f \gg R_L$, $R_f \gg R_E$, $R_1 \gg 1/g_m$

- Input resistance

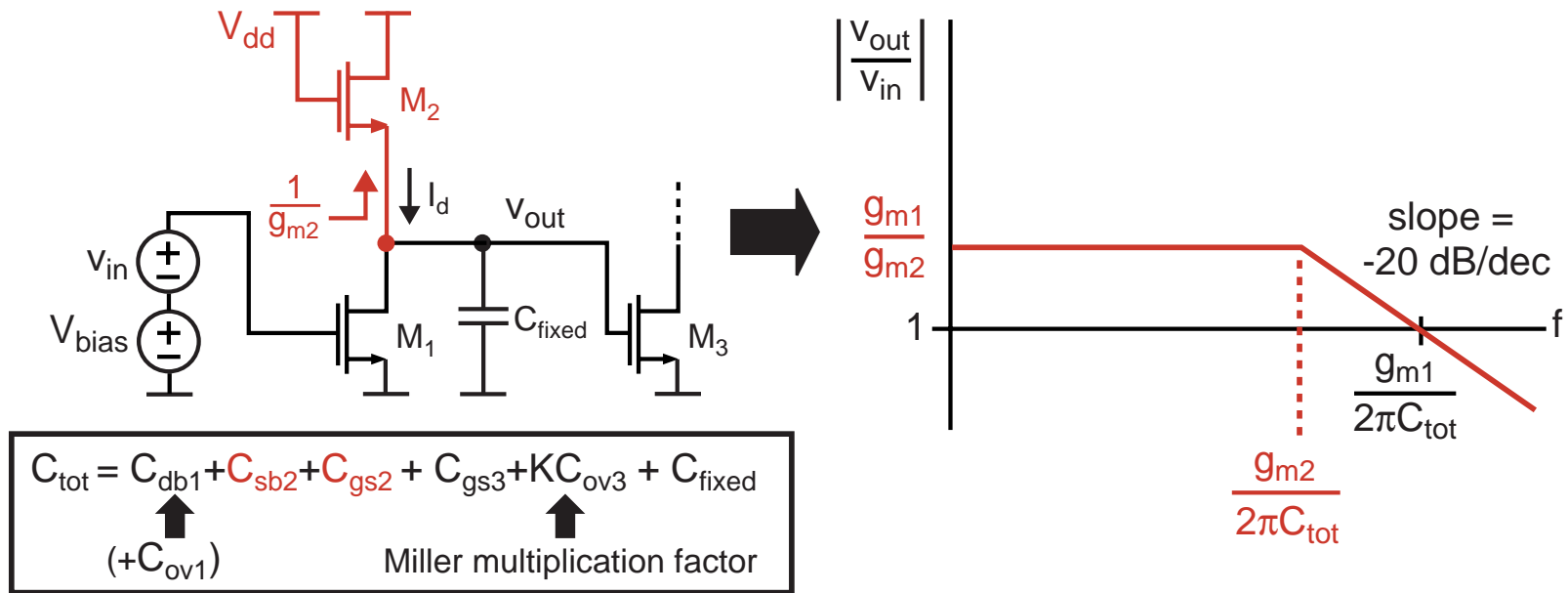
$$R_{in} = \frac{R_f}{1 - A_v} = \frac{R_E(R_f + R_L)}{R_E + R_L} \approx \frac{R_f}{1 + R_L/R_1} \quad \text{for } R_f \gg R_L, R_1 \gg 1/g_m$$

- Output resistance

$$R_{out} = \frac{R_E(R_f + R_s)}{R_E + R_s} \approx \frac{R_f}{1 + R_s/R_1} \quad \text{for } R_f \gg R_s, R_1 \gg 1/g_m$$

Same for $R_s = R_L$!

NMOS Load Amplifier



- Gain set by the relative sizing of M_1 and M_2

$$M_1 : I_{d1} = (1/2)\mu_n C_{ox} (W_1/L_1) (V_{IN} - V_T)^2$$

$$M_2 : I_{d2} = (1/2)\mu_n C_{ox} (W_2/L_2) (V_{dd} - V_{out} - V_T)^2$$

$\Rightarrow I_{d1} = I_{d2}$

$$\Rightarrow V_{out} = -AV_{IN} + V_{dd} + (A - 1)V_T$$

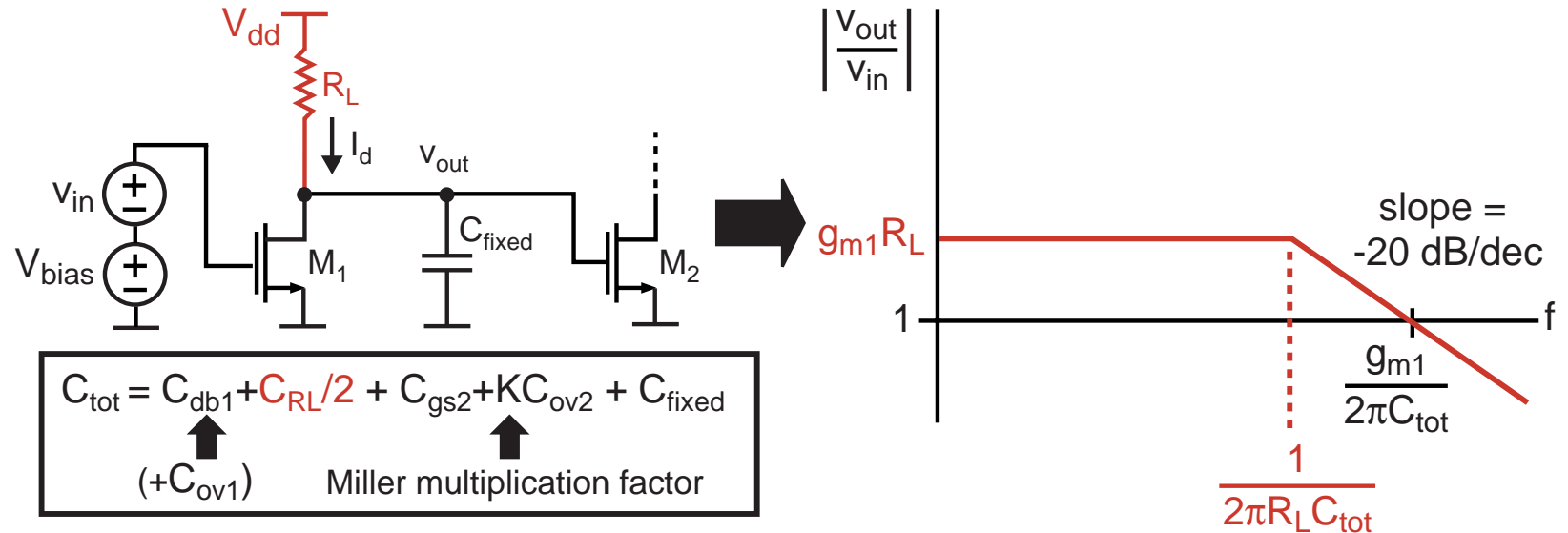
$(V_{IN} = V_{in} + V_{bias})$

$$\text{where } A = \sqrt{\frac{W_1/L_1}{W_2/L_2}}$$

Advantage/Disadvantages of NMOS Load Amplifier

- **Gain is well controlled despite process variations**
- **NMOS is not a low parasitic load**
 - C_{gs} of M2 loads the output
- **Biasing Problem: V_T of M_2 lowers the gate bias voltage of the next stage (thus lowering its achievable f_t)**
 - Severely hampers performance when amplifier is cascaded
 - One paper addressed this issue by increasing V_{dd} of NMOS load (see Sackinger et. al., “A 3-GHz 32-dB CMOS Limiting Amplifier for SONET OC-48 receivers”, JSSC, Dec 2000)

Resistor Loaded Amplifier (Unsilicided Poly)



- **This is the fastest non-enhanced amplifier topology**
 - Unsilicided poly is a low parasitic load (i.e., has a good current to capacitance ratio)
 - Output can go near V_{dd}
 - Allows following stage to achieve high f_t , but at the cost of gain (max gain $\propto V_{\text{RL}}$)
 - Linear settling behavior (in contrast to NMOS load)

Gain Limitations in Resistor Loaded Amplifier

$$g_m = \frac{dI_d}{dV_{gs}} = \frac{2I_d}{V_{GS} - V_T}$$

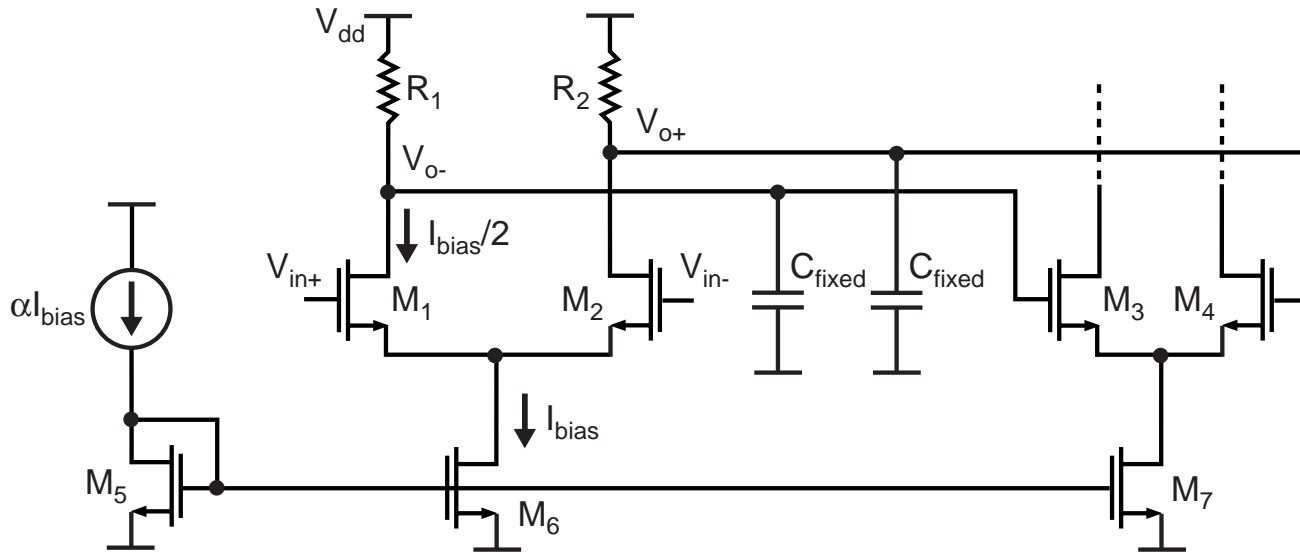
$$A = g_m R_L = \frac{2I_d R_L}{V_{GS} - V_T} = \frac{2V_{R_L}}{V_{GS} - V_T}$$

$$A_{MAX} = \frac{2V_{dd}}{V_{GS} - V_T}$$

- **Want high $V_{GS} - V_T$ for high bandwidth, but this reduces gain. With low V_{dd} gain is very limited.**

Implementation of Resistor Loaded Amplifier

- Typically implement using differential pairs



- Benefits**
 - Bias stability without feedback
 - Common-mode rejection
- Negative**
 - More power than single-ended version

Open-Circuit Time Constants

- The Miller capacitance analysis is a reasonably good method, but is somewhat limited in applicable topologies
- the OCT method is more general and often gives more insights
- Systematic, intuitive method to determine bandwidth of amplifiers
- Often gives fairly accurate estimates of bandwidth if there is a dominant pole
- Points to the bandwidth *bottleneck*: this is the *real* value of the OCT method!
- Limitation: fails in RF circuits with zero enhancements or inductors
- In typical broadband amplifiers, the OCT estimate is *too pessimistic* due to multiple poles at around similar frequencies

Open Circuit Time Constant Method

Assumptions: No zero near or ω_h
Zero well below ω_h is handled by treating corresponding capacitor as short circuit
Negative real poles only (complex conjugate poles with low Q reduces accuracy of estimation only moderately)
No inductors

$$A(s) = \frac{V_o}{V_i} = \frac{a_0}{(1 + \tau_1 s)(1 + \tau_2 s) \cdots (1 + \tau_n s)}$$
$$= \frac{a_0}{1 + (\tau_1 + \tau_2 + \cdots + \tau_n)s + \cdots + \tau_1 \tau_2 \cdots \tau_n s^n}$$

If we ignore the higher order terms

$$\omega_h = 2\pi f_h \approx \frac{1}{\tau_1 + \tau_2 + \cdots + \tau_n} = \frac{1}{\sum_{i=1}^n \tau_i}$$

OCT Method, Continued

It can be shown

$$\sum_{i=1}^n \tau_i = \sum_{j=1}^n \tau_{jo}$$

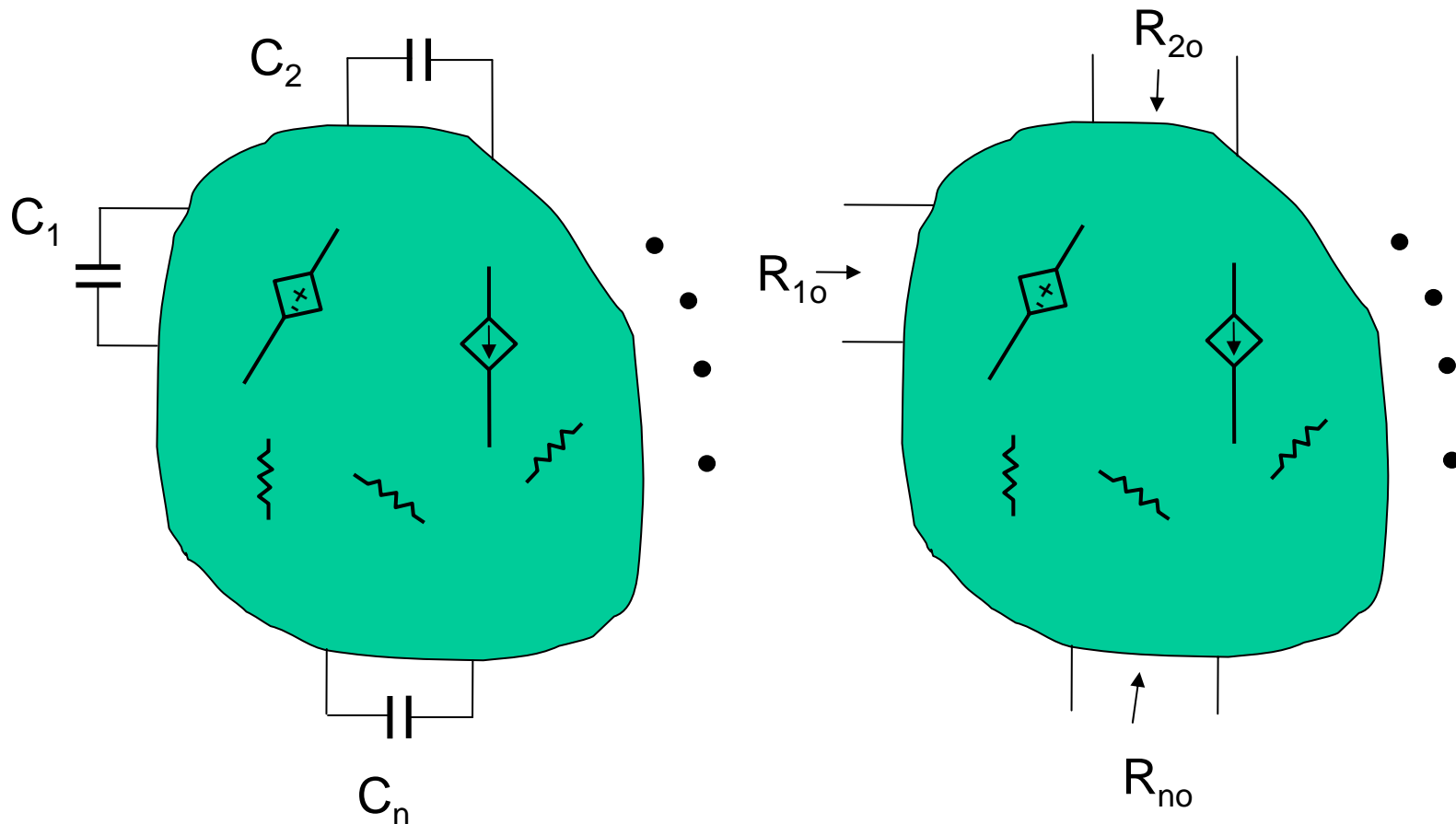
$$\text{Thus } \omega_h \approx \frac{1}{\sum_{j=1}^n \tau_{jo}}$$

where $\tau_{jo} = R_{jo}C_j$: open-circuit time constants

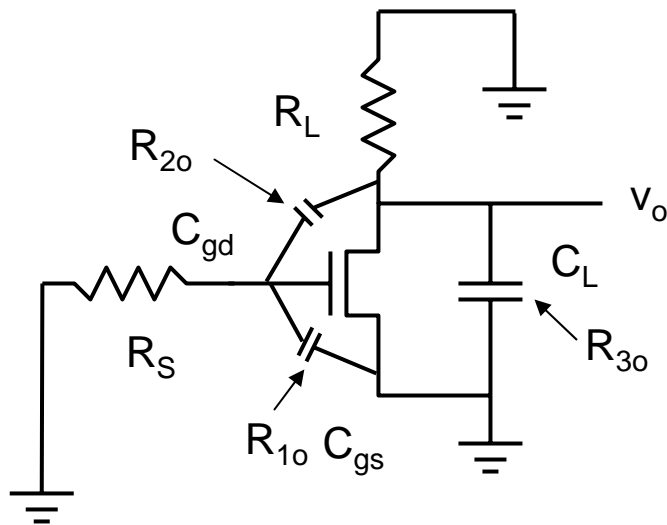
The open-circuit time constants can be found without node equations, often by inspection

OCR Calculation

Let's consider an arbitrary circuit with resistors, dependent sources, and n capacitors (no inductors!). Redraw the circuit to pull capacitors out around the perimeter.



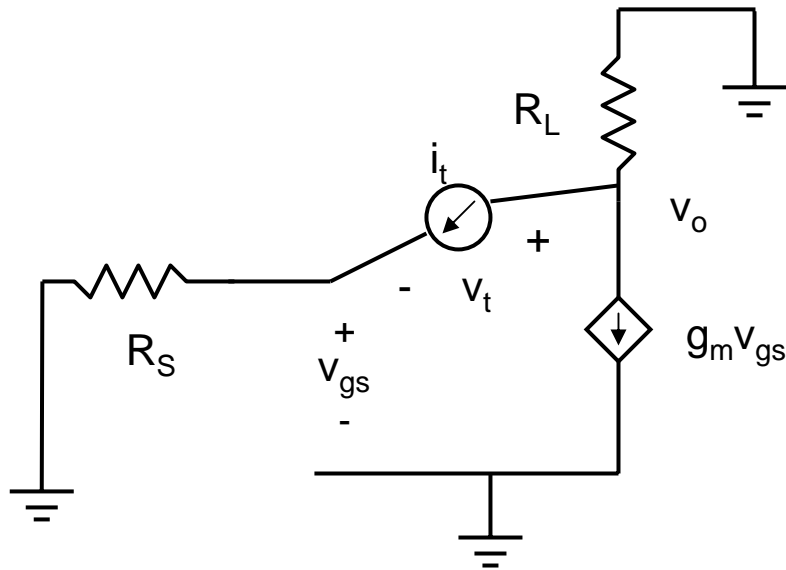
OCT's for CS Amplifier



By inspection: $R_{1o} = R_S, \tau_{1o} = R_S C_{gs}$
 $R_{3o} = R_L, \tau_{3o} = R_L C_L$

It takes some work to figure R_{2o}

OCT's for CS Amplifier



$$v_{gs} = i_t R_S$$

$$-v_o = (i_t + g_m v_{gs}) R_L = i_t (1 + g_m R_S) R_L$$

$$v_t = v_{gs} - v_o = i_t \left(1 + \frac{R_S}{R_L} + g_m R_S \right) R_L$$

$$R_{2o} = \frac{v_t}{i_t} = \left(1 + \frac{R_S}{R_L} + g_m R_S \right) R_L$$

$$\tau_{2o} = C_{gd} \left(1 + \frac{R_S}{R_L} + g_m R_S \right) R_L$$

OCT's for CS Amplifier

$$R_{1o} = R_S, \quad \tau_{1o} = R_S C_{gs}$$

$$R_{3o} = R_L, \quad \tau_{3o} = R_L C_L$$

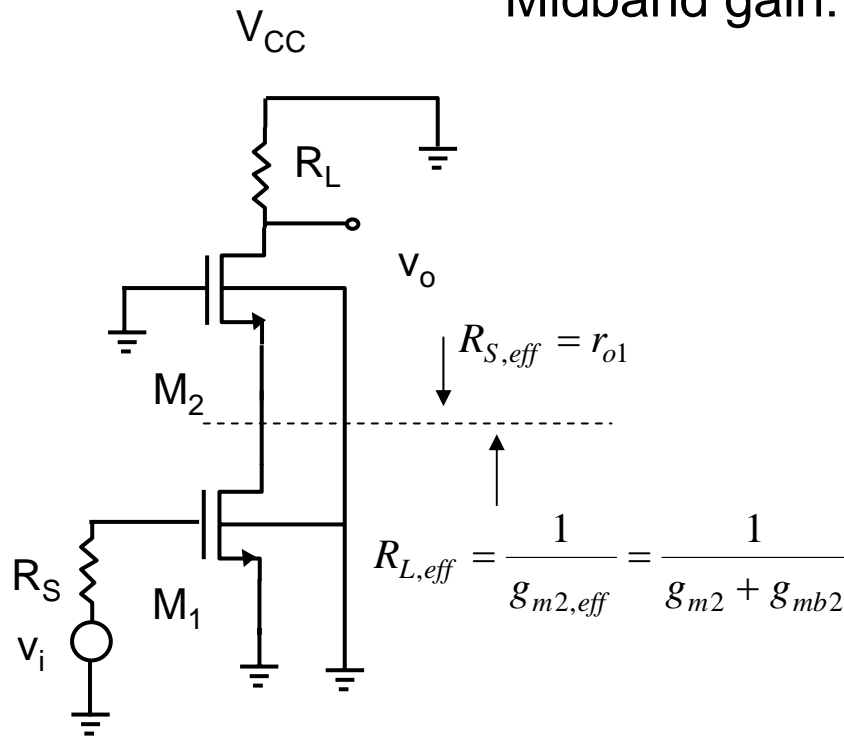
$$R_{2o} = \frac{v_t}{i_t} = \left(1 + \frac{R_S}{R_L} + g_m R_S\right) R_L$$

$$\tau_{2o} = C_{gd} \left(1 + \frac{R_S}{R_L} + g_m R_S\right) R_L = C_{gd} \left(1 + \frac{R_L}{R_S} + g_m R_L\right) R_S$$

- If $R_S=0$, then $\tau_{1o}=0$, $\tau_{2o}=C_{gd}R_L$, and $\tau_{3o}=C_L R_L$ so R_L determines the bandwidth
- Having individual OCT values identifies the bandwidth bottleneck and suggests a game plan
- Note: for cascaded amplifiers, C_L does not include Miller cap of the next stage. Instead, τ_{2o} corresponding to the next stage Miller cap is separately calculated

Cascode to Improve Bandwidth

Midband gain: by inspection $a_v = -g_{m1}R_L$



$$\begin{aligned}
 R_{1o} &= R_S \\
 R_{2o} &= R_{1o} \left(1 + g_{m1}R_{L,eff} + \frac{R_{L,eff}}{R_S} \right) \\
 &= R_S + (1 + g_{m1}R_S)R_{L,eff} \\
 &= R_S + \frac{1 + g_{m1}R_S}{g_{m2} + g_{mb2}} \\
 R_{3o} &\approx R_{S,eff} \left\| \frac{1}{g_{m2,eff}} \right\| \approx \frac{1}{g_{m2} + g_{mb2}} \\
 R_{4o} &= \cancel{r_b} \left(1 + \frac{R_L}{\frac{1}{g_{m,eff}} + R_{S,eff}} \right) + R_L \approx R_L
 \end{aligned}$$

Cascode

- Improves bandwidth in a single-stage amplifier
- Problem in cascading:
 - Bias point: Reduces ω_t of the next stage. PMOS SF is a possibility but low g_m/C ratio is a drawback.