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High Speed Communication Circuits
Lecture 18
ABC's of Power Amplifiers

Massachusetts Institute of Technology

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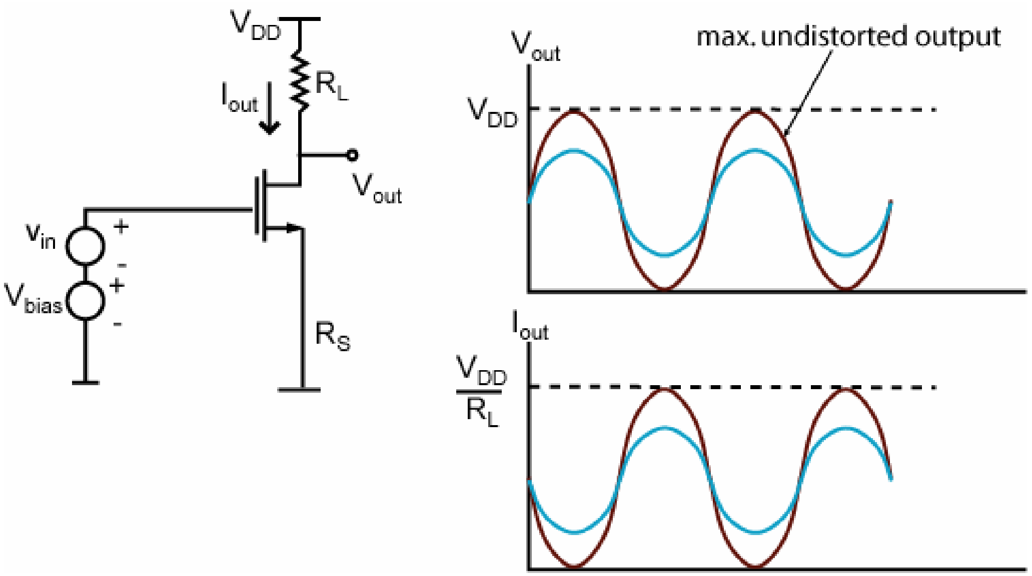
Resistor Loaded Class A Amplifier

- **In a Class A amplifier, the active device conducts current 100% of time**
- **For maximum output swing (and thus output power), the quiescent output voltage is set at $V_{dd}/2$, and bias current at $V_{dd}/2R_L$**



- **Max. power delivered to load (R_L)**

$$P_{L,max} = \frac{\left(\frac{V_{DD}}{2}\right)^2}{2R_L} = \frac{V_{DD}^2}{8R_L}$$

Power Efficiency

- **Power Efficiency is defined as the ratio between power delivered to the load and the DC biasing power**

Max. power delivered to load (R_L)

$$P_{L,max} = \frac{\left(\frac{V_{DD}}{2}\right)^2}{2R_L} = \frac{V_{DD}^2}{8R_L}$$

DC Biasing Power

$$P_{DC} = V_{DD} \cdot \frac{V_{DD}}{2R_L} = \frac{V_{DD}^2}{2R_L}$$

Max. Power Efficiency

$$\eta = \frac{P_L}{P_{DC}} = \frac{1}{4}$$

The efficiency is lower at smaller amplitudes (η is proportional to output power since DC power is constant)

Other Power Efficiency Parameters

- Normalized Power Output Capability P_N
  - Ratio between power delivered to load and peak current times peak voltage on the output device
  - Measure related to output device power handling

$$P_N = \frac{P_{L,max}}{v_{DS,max} \cdot i_{D,max}} = \frac{\frac{V_{DD}^2}{8R_L}}{V_{DD} \cdot \frac{V_{DD}}{R_L}} = \frac{1}{8}$$

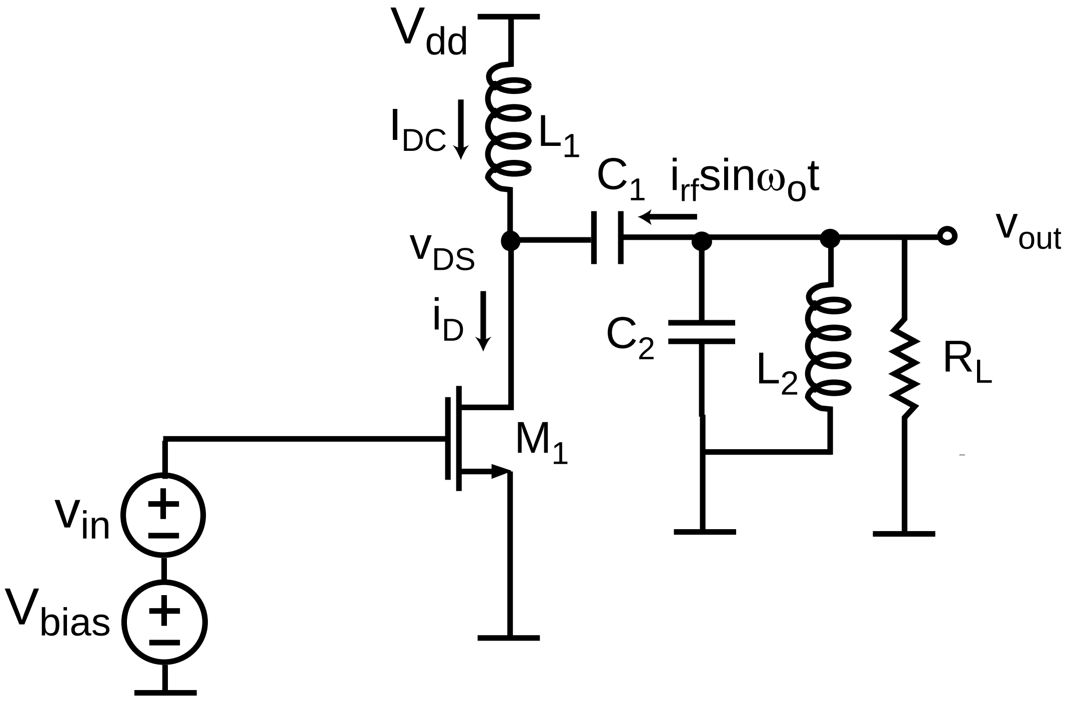
- Power Added Efficiency (PAE)
  - Added signal power by the amplifier divided by DC biasing power

$$PAE = \frac{P_L - P_{in}}{P_{DC}}$$

At low frequencies, PAE=η for the previous amplifier

Class A RF Power Amplifier

- **Inductor improves peak amplitude, thus power efficiency**



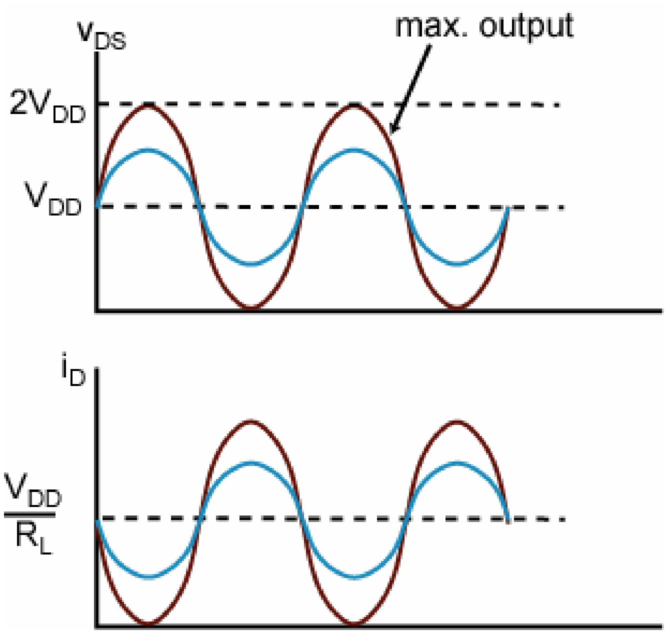
L_1: Large inductor: acts as current source

C_1: DC block (prevents DC power in R_L)

L_2, C_2: Output tank circuit

Drain Voltage and Current Waveforms

- **Since L_1 presents a DC short to V_{DD}, the drain voltage waveform must be symmetric around V_{DD}. the maximum amplitude of sinusoid at the drain is V_{DD}.**



The drain voltage (and the output voltage) swings to twice the power supply! (in practice limited by device breakdown)

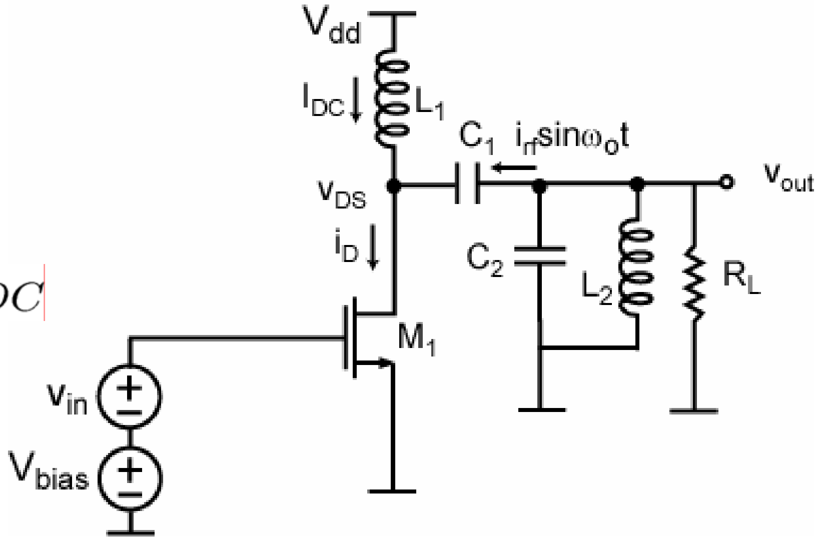
Power Efficiency Calculation

$$i_D = I_{DC} + i_{rf} sin w_o t$$

$$v_{out} = -i_{rf} R_L sin w_o t$$

Since C_1 is effectively a short circuit at ω_o the amplitude of v_{out} is eual to the amplitude of v_{DS}:

$$v_{DS} = V_{DD} - i_{rf} R_L sin w_o t$$



v_{DS} must be positive $$i_{rf} \leq \frac{V_{DD}}{R_L}$$

Also, i_D must be positive $$i_{rf} \leq I_{DC}$$

i_{rf} is maximized if $$I_{DC} = \frac{V_{DD}}{R_L}$$

$$P_{L,max} = \frac{V_{DD}^2}{2R_L}$$

$$P_{DC} = I_{DC} V_{DD} = \frac{V_{DD}^2}{R_L} \qquad \eta_{max} = \frac{1}{2}$$

Normalized Power Output Capability

$$P_N = \frac{P_{L,max}}{v_{DS,max} \cdot i_{D,max}}$$

$$P_{L,max} = \frac{V_{DD}^2}{2R_L}$$

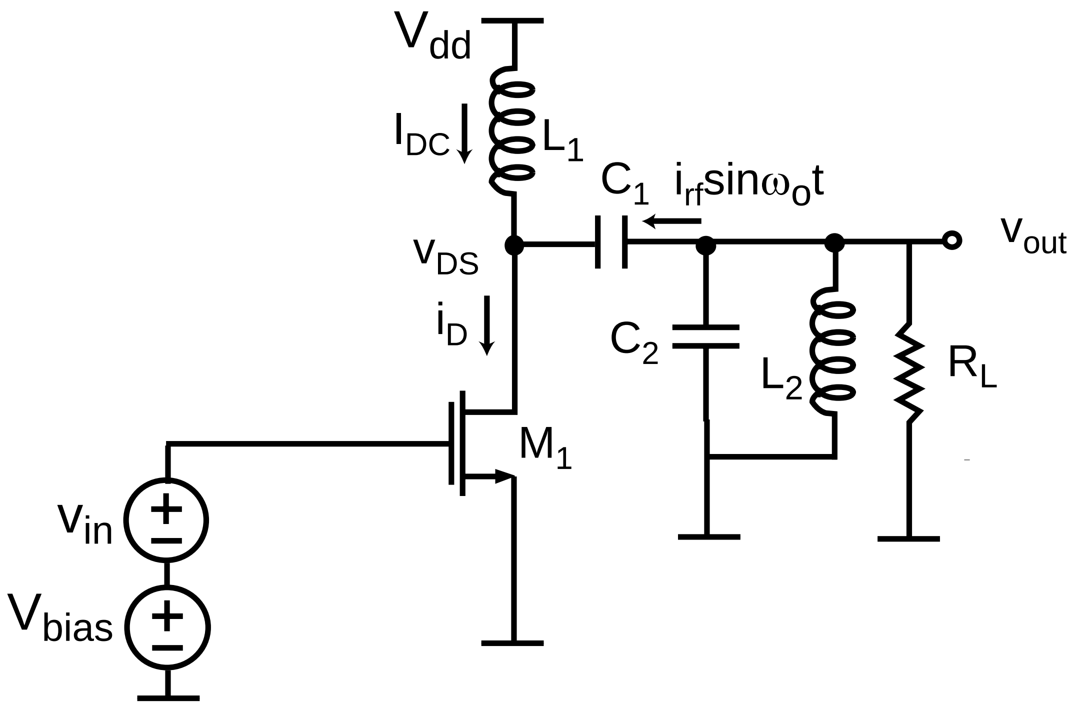
$$v_{DS,max} = 2V_{DD} \qquad i_{D,max} = \frac{2V_{DD}}{R_L}$$

$$P_{N,max} = \frac{1}{8}$$

Class A amplifiers are linear, but have poor efficiency!

Class B Power Amplifier

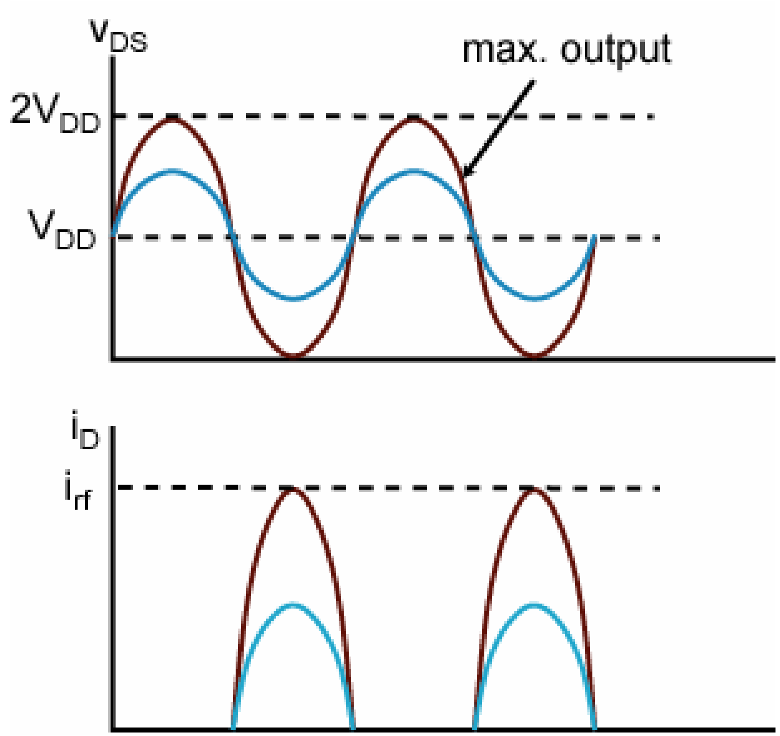
- **Same circuit, but V_{bias} is set so that M1 conducts only 50% of time**



$$V_{bias} \approx V_T$$

Class B Power Amplifier

- V_{bias} is set so that M1 conducts only 50% of time



The harmonics in the output waveform are filtered by output tank circuit

The fundamental component is a linear function of the input

Class B Power Efficiency

- **The fundamental component of i_D half-sine wave is**

$$i_{fund} = \frac{2}{T}\int_0^{T/2} i_{rf}(sinw_o t)(sinw_o t)dt = \frac{i_{rf}}{2}$$

The tank circuit filters harmonics – only the fundamental component is delivered to load:

$$v_{out} = -\frac{i_{rf}}{2} R_L sinw_o t$$

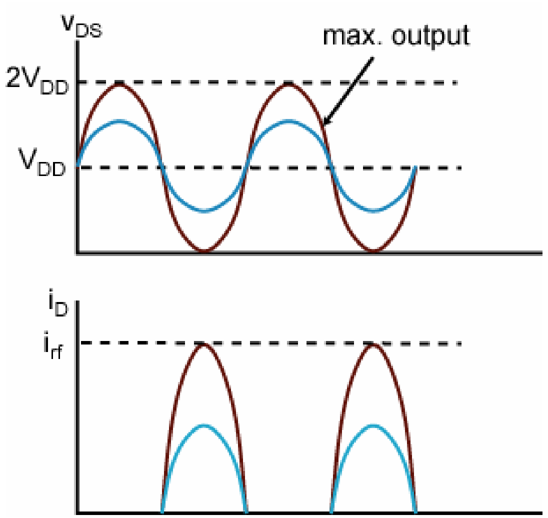
As in Class A amplifier, the maximum amplitude of v_{out} is V_{DD} (since C_1 is an AC short):

$$P_{L,max} = \frac{V_{DD}^2}{2R_L} \quad \textbf{and} \quad i_{rf} \leq \frac{2V_{DD}}{R_L}$$

Class B Power Efficiency, Continued

- **The DC component of max. i_D half-sine wave is**

$$\overline{i_D} = \frac{1}{T}\int_0^{T/2} \frac{2V_{DD}}{R_L} sin w_o t dt = \frac{2V_{DD}}{\pi R_L}$$



The DC power is then $P_{DC} = \frac{2V_{DD}^2}{\pi R_L}$

And the efficiency $\eta = \frac{\pi}{4} = 0.785$

- **Normalized Output Power Capability**

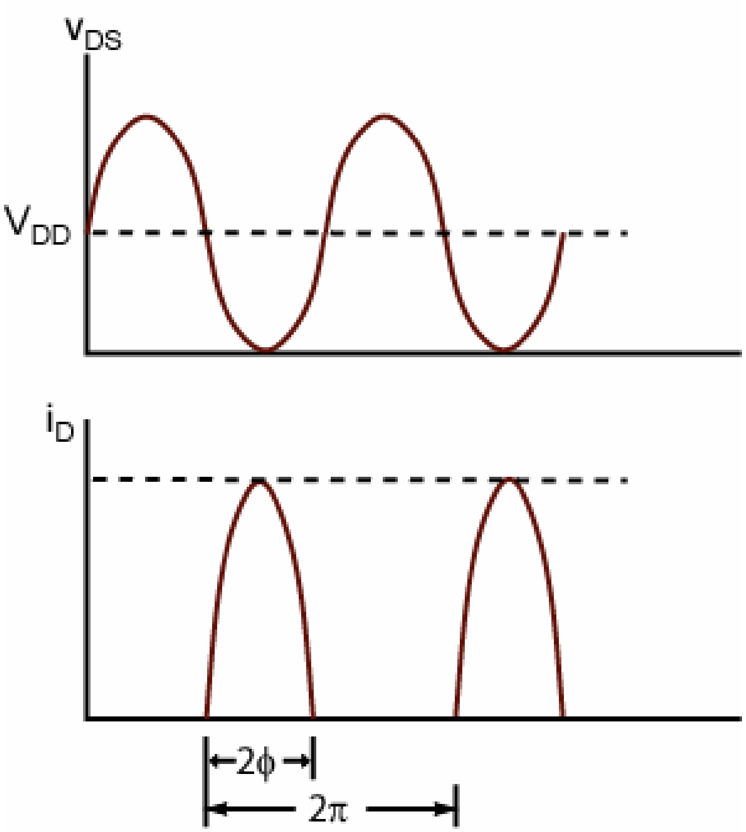
$$v_{DS,max} = 2V_{DD} \quad i_{D,max} = i_{rf} \le \frac{2V_{DD}}{R_L} \quad P_{L,max} = \frac{V_{DD}^2}{2R_L}$$

$$P_N = \frac{P_L}{v_{DS,max} \cdot i_{D,max}} = \frac{1}{8}$$ **: Same as Class A**

Since DC power is proportional to the amplitude, η is proportional to square root of output power: slower degradation at lower power than class A

Conduction Angle vs. Class

- **Conduction Angle ϕ : 2ϕ is the portion of period during which the output transistor M1 conducts**

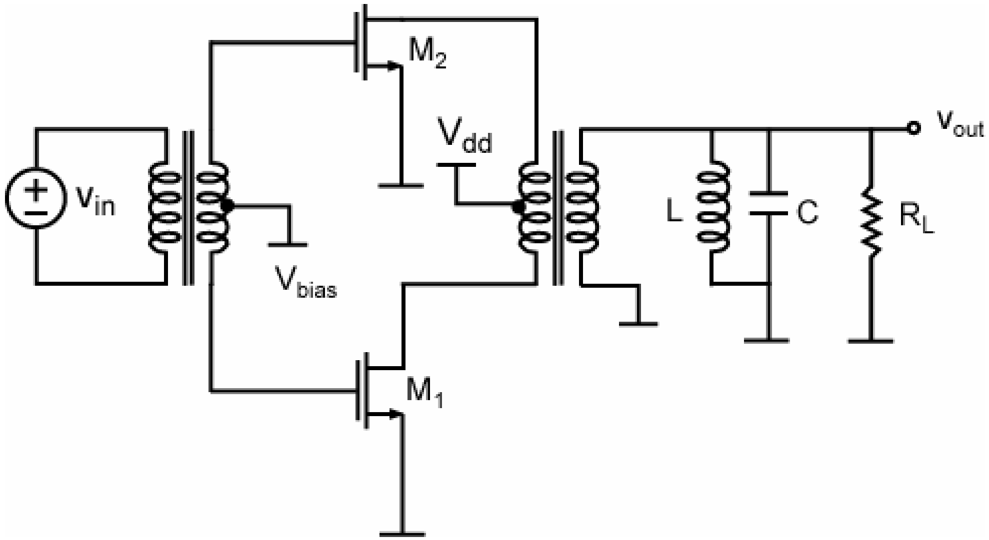


$2\phi=2\pi$: Class A
$\pi<2\phi<2\pi$: Class AB
$2\phi=\pi$: Class B
$0<2\phi<\pi$: Class C

(Class AB or C output cannot be a linear function of input)

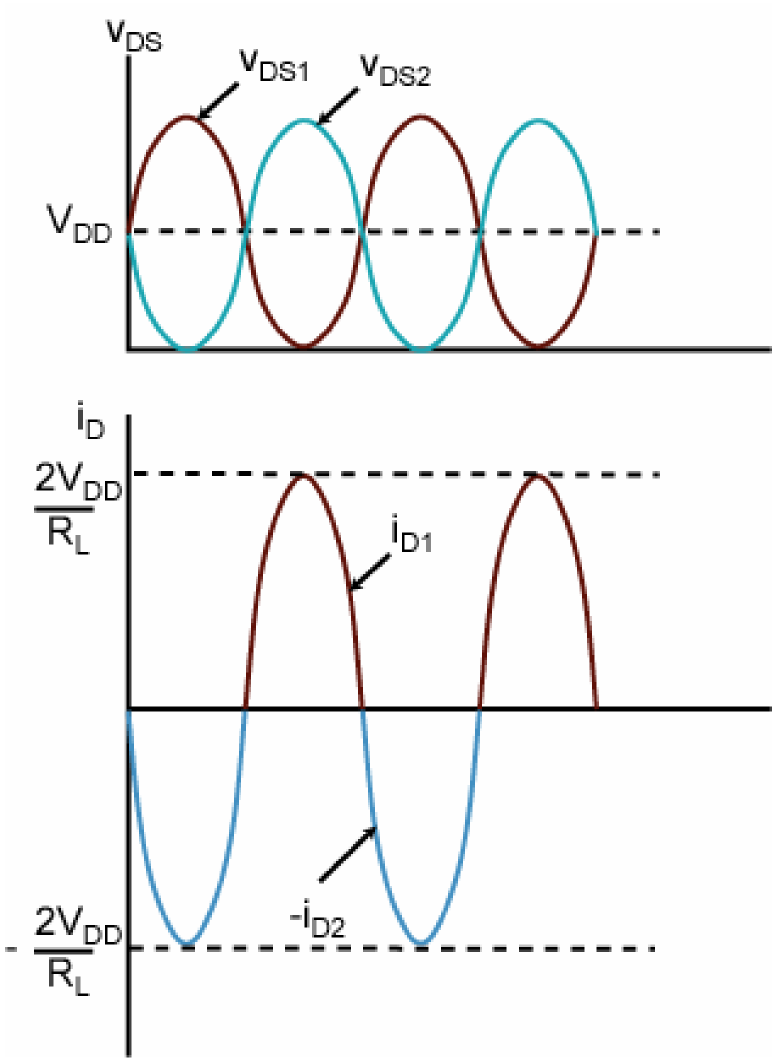
Push-Pull Amplifier

- **Depending on V_{bias} and V_{in} push-pull amplifier can be operated as Class A, B, AB, C, or D amplifier.**



- **Theoretically a Class B push-pull amplifier has low distortion comparable to class A because either half will be conducting at any time.**
- **Real Class B is not possible because devices do not have abrupt turn-on characteristic– most are Class AB**

Voltage and Current Waveforms of Class B Push-Pull



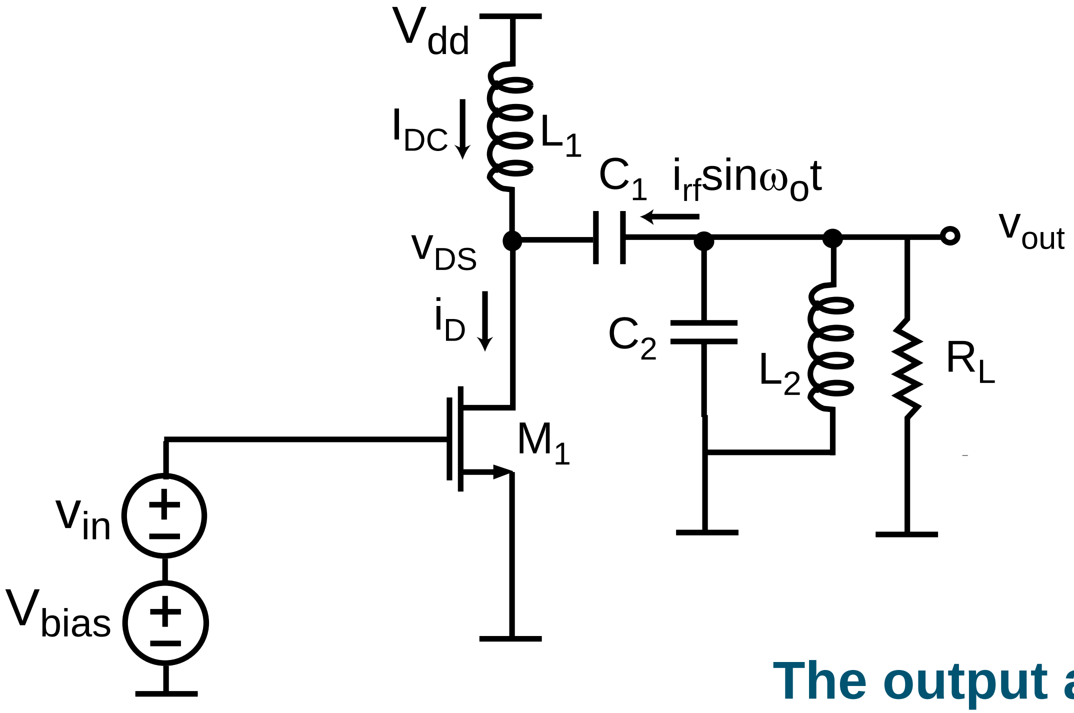
Waveforms shown for maximum amplitude output

typically, 'crossover distortion' arises at the switching point of the two halves due to imprecise turn-on voltages

Crossover distortion is reduced by class AB operation

Class C Amplifier

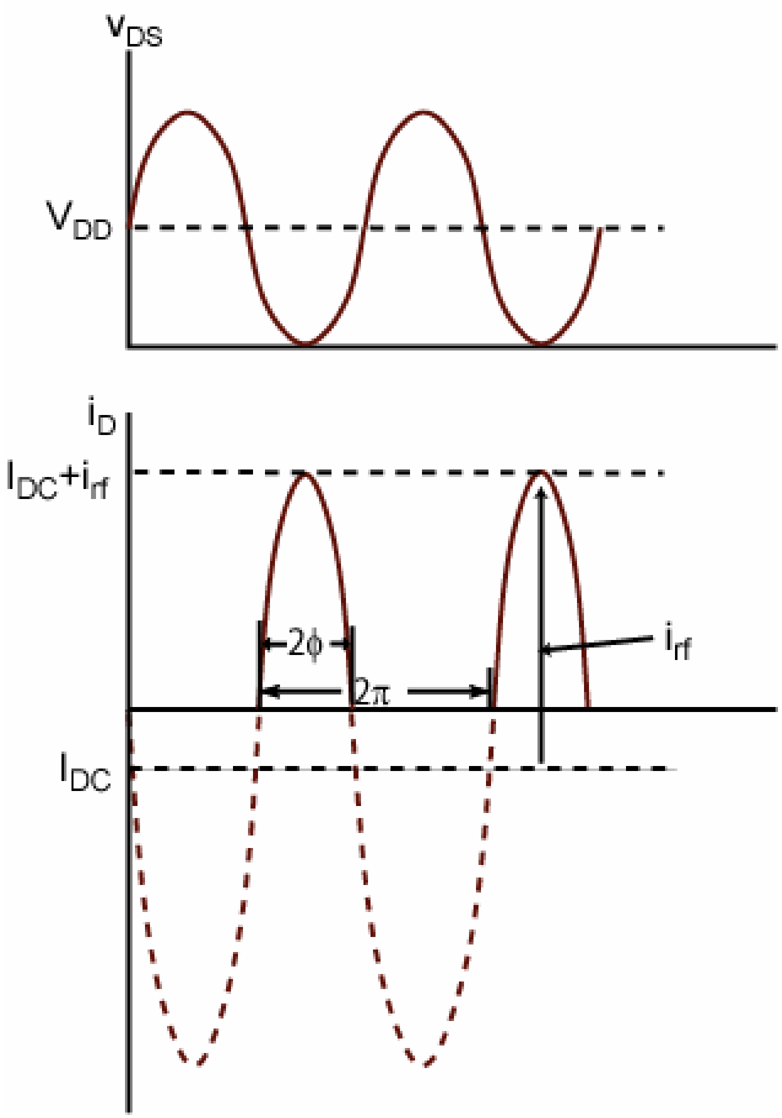
- **Same amplifier, but biased to conduct less than 50% of time**



$$V_{bias} < V_T$$

The output amplitude is not a linear function of input: more suitable for constant-amplitude power amp (such as in PM or FM)

Class C Amplifier Waveforms



$$i_D = I_{DC} + i_{rf} sin w_o t$$

$$i_D > 0, I_{DC} < 0$$

Conduction angle

$$2\phi = 2 \cdot cos^{-1}\left(-\frac{I_{DC}}{i_{rf}}\right)$$

Solving for the DC bias to achieve conduction angle ϕ

$$I_{DC} = -i_{rf} cos\phi$$

Class C Power Efficiency Calculation

- **The average value of i_D is**

$$\overline{i_D} = \frac{1}{2\pi}\int_{-\phi}^{\phi}\left(I_{DC} + i_{rf}cos\phi\right)d\phi = \frac{1}{2\pi}2\phi I_{DC} + \frac{1}{2\pi}\left[i_{rf}sin\phi\right]_{-\phi}^{\phi}$$

$$I_{DC} = \frac{i_{rf}}{\pi}\left(sin\phi - \phi cos\phi\right)$$

- **The fundamental component of i_D is**

$$i_{fund} = \frac{2}{T}\int_{0}^{T} i_D cos w_o t dt = \frac{1}{2\pi}(4I_{DC}sin\phi + 2i_{rf}\phi + i_{rf}sin2\phi)$$

$$= \frac{i_{rf}}{2\pi}\left(2\phi - sin2\phi\right)$$

- **Maximum output swing is reached when**

$$i_{fund}R_L = V_{DD}$$

Class C Power Efficiency, Continued

- **Solving for i_{rf}:**

$$i_{rf} = \frac{2\pi V_{DD}}{R(2\phi - sin2\phi)}$$

Peak drain current

$$i_{D,max} = I_{DC} + i_{rf}$$

$$= \frac{i_{rf}}{\pi}(sin\phi - \phi cos\phi) + \frac{2\pi V_{DD}}{R(2\phi - sin2\phi)}$$

$$= \frac{2\pi V_{DD}}{R(2\phi - sin2\phi)}\left[1 + \frac{(sin\phi - \phi cos\phi)}{\pi}\right]$$

$$\phi \rightarrow 0 : i_{rf} \rightarrow \infty, i_{D,max} \rightarrow \infty$$

Class C Power Efficiency, Continued

- **Maximum efficiency is then**

$$\eta = \frac{2\phi - sin2\phi}{4\left(sin\phi - \phi cos\phi\right)} \qquad \phi \to 0 : \eta \to 1$$

- **The normalized output capability is poor at small conduction angles**

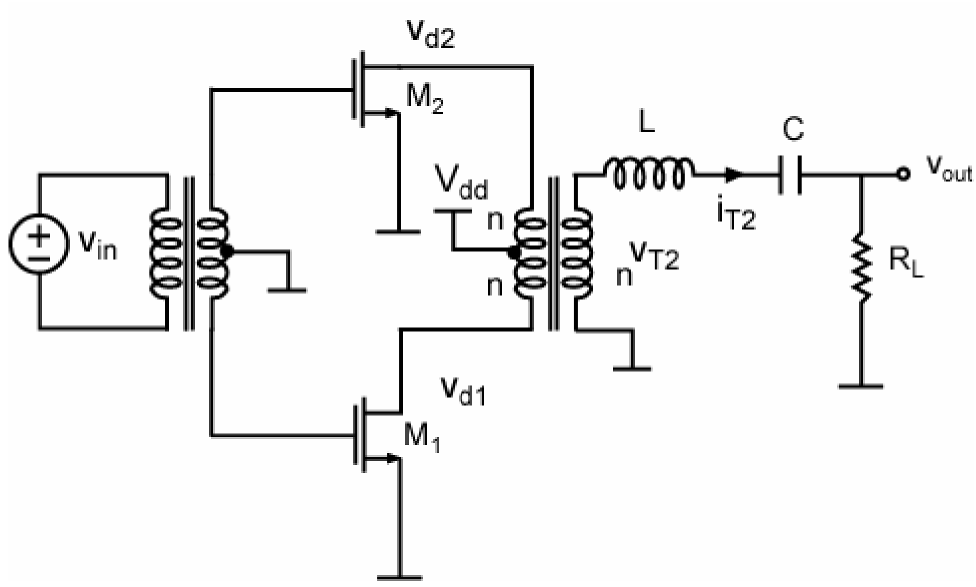
$$v_{DS,max} = 2V_{DD}$$

$$\phi \to 0 : i_{D,max} \to \infty, P_N \to 0$$

Thus, the efficiency must be sacrificed for reasonable P_N

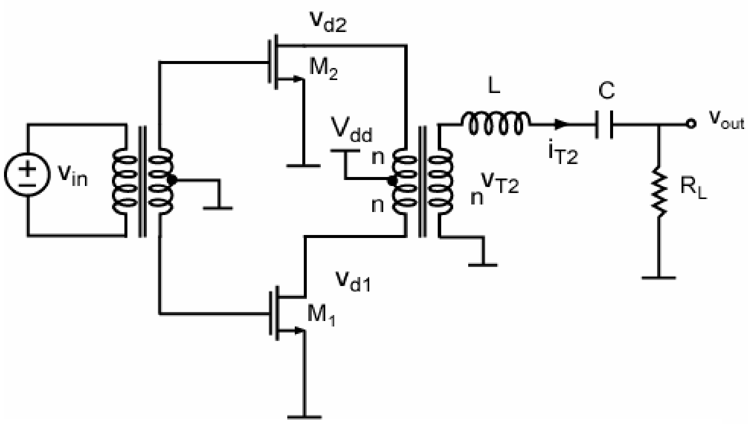
Class D Power Amplifier

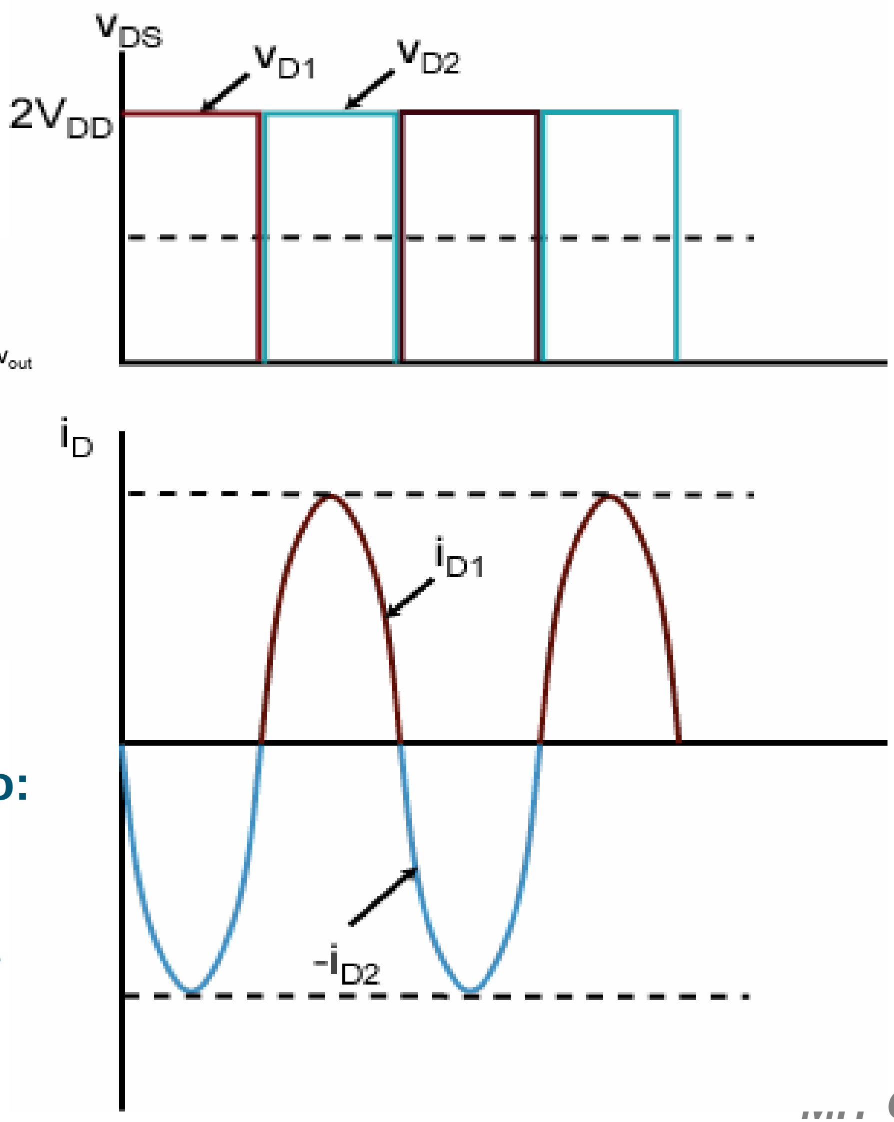
- **Ideal switches dissipate no power (either v or i is zero)**
- **Thus, highest efficiency will be achieved by operating active devices as switches.**
- **Push-pull Class D amplifier example**



- **Load tank circuit is series LC, because the switch has low impedance when on (in contrast to a current-source like behavior in other class amplifiers)**

Class D Push-Pull Power Amplifier Waveforms

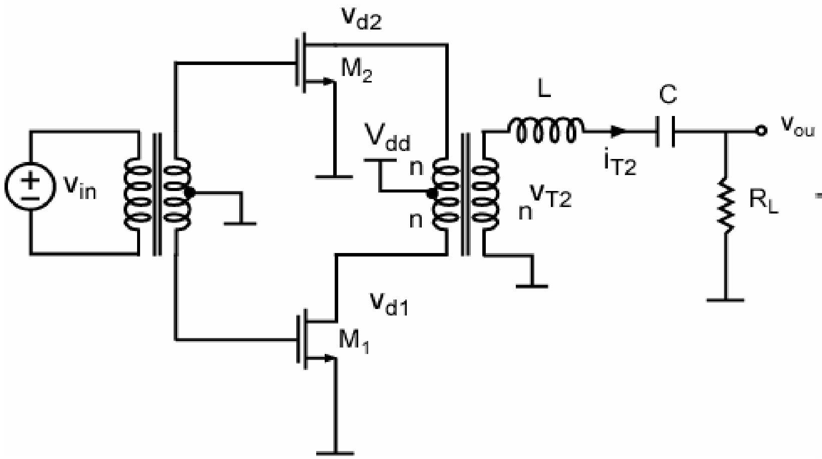


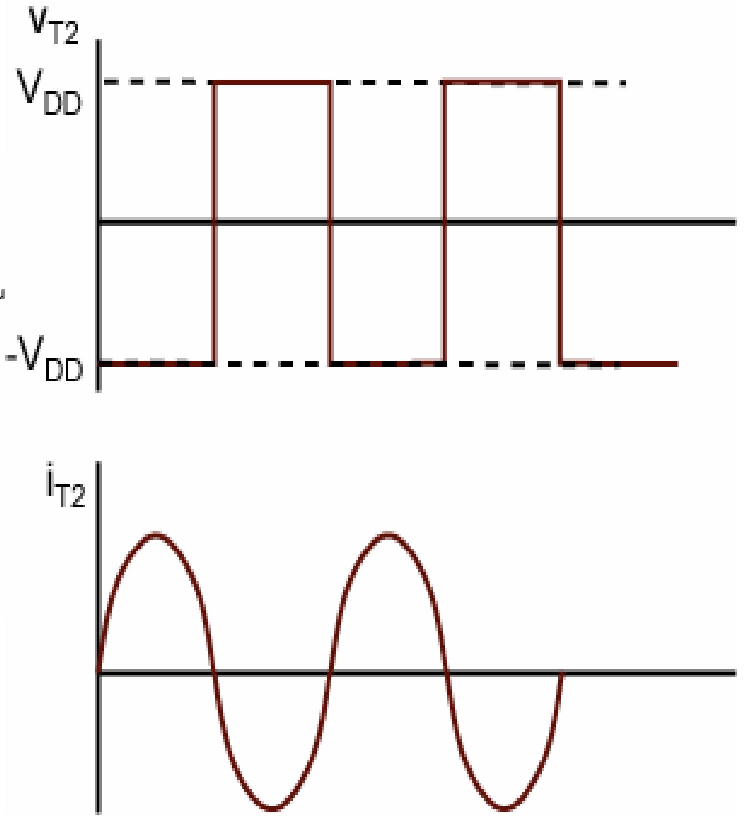


i_d is nonzero only when v_d is zero: No power dissipation

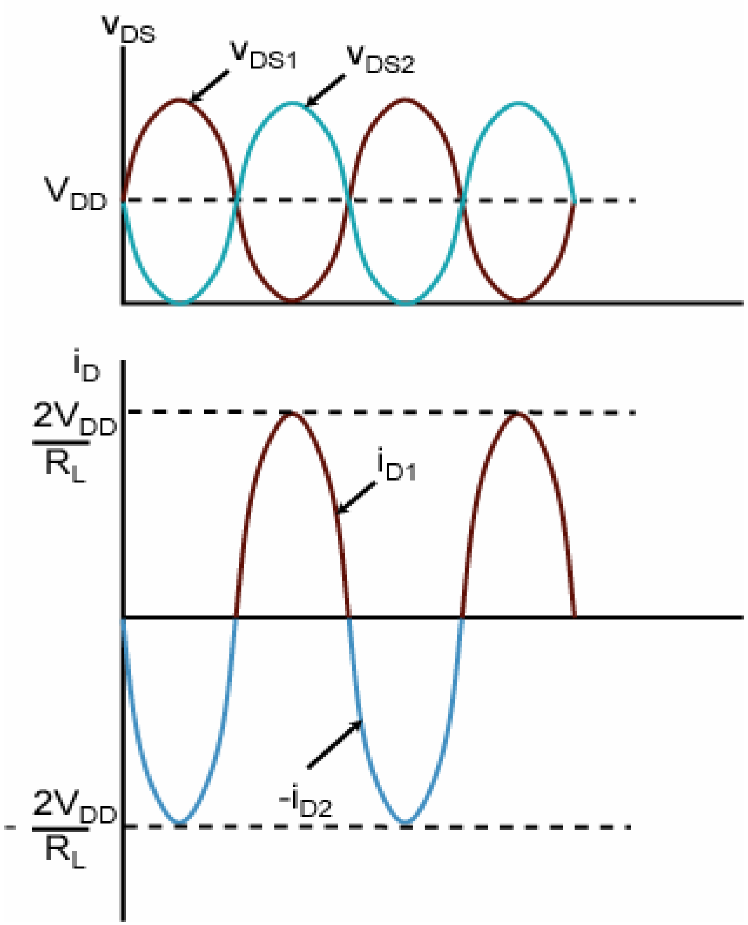
PWM or drain modulation can be used for linear amplification

Class D Secondary Winding Waveforms

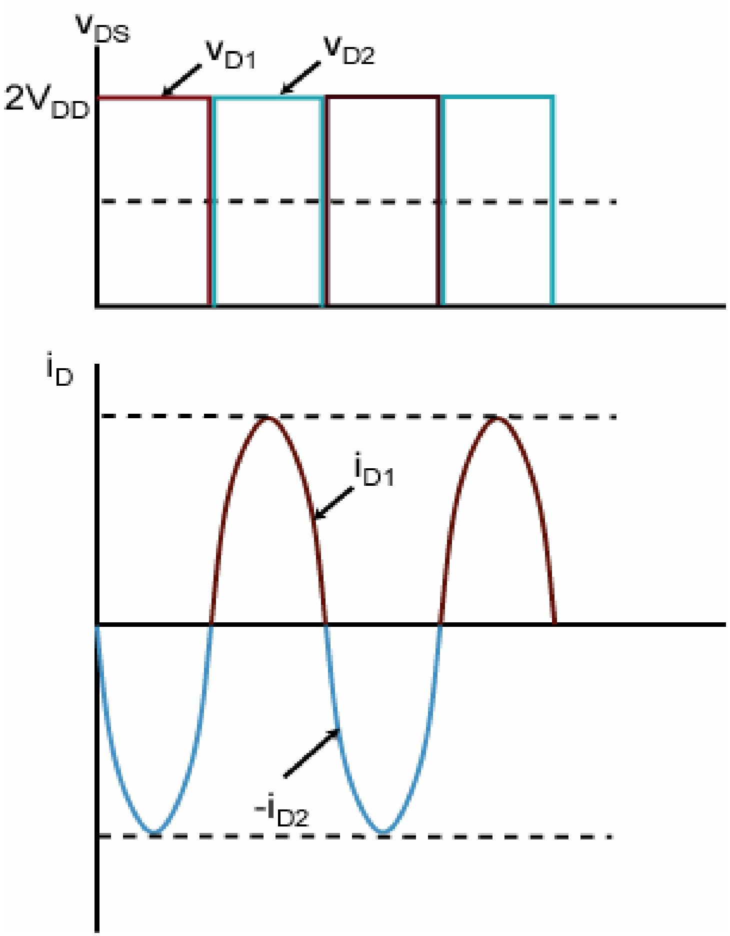




Class B vs. D Push-Pull Amplifier Waveforms



Class B Push-pull



Class D Push-pull

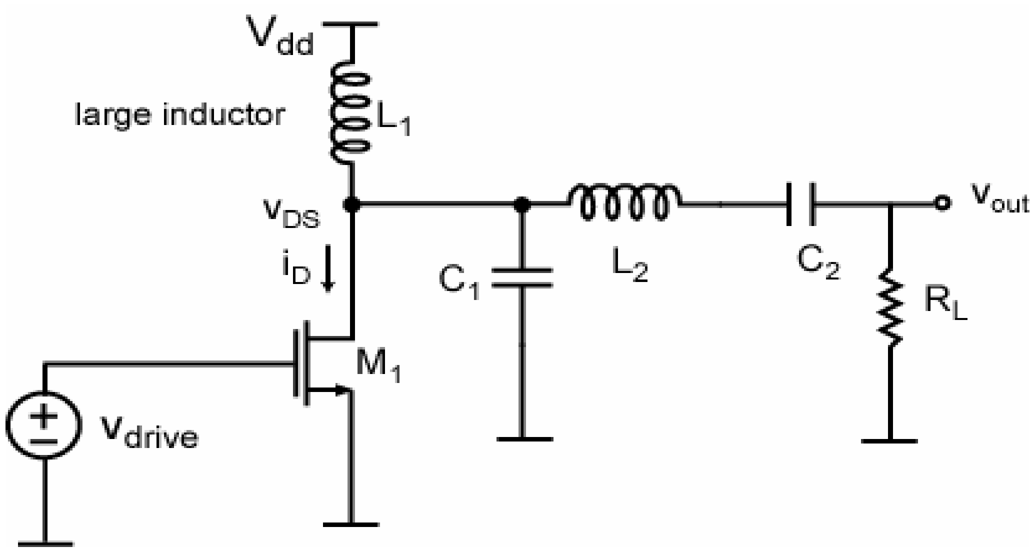
Class D Amplifier Power Efficiency

- **With ideal switches, Class D amplifier efficiency would be 100%**
- **In practice, finite switch ON resistance and nonzero on-off transients limit efficiency (use high f_t device!)**
- **In bipolar Class D amplifier, the efficiency is further compromised due to charge storage in saturation and $V_{CE,SAT}$.**
- **Normalized power capability is shown to be**

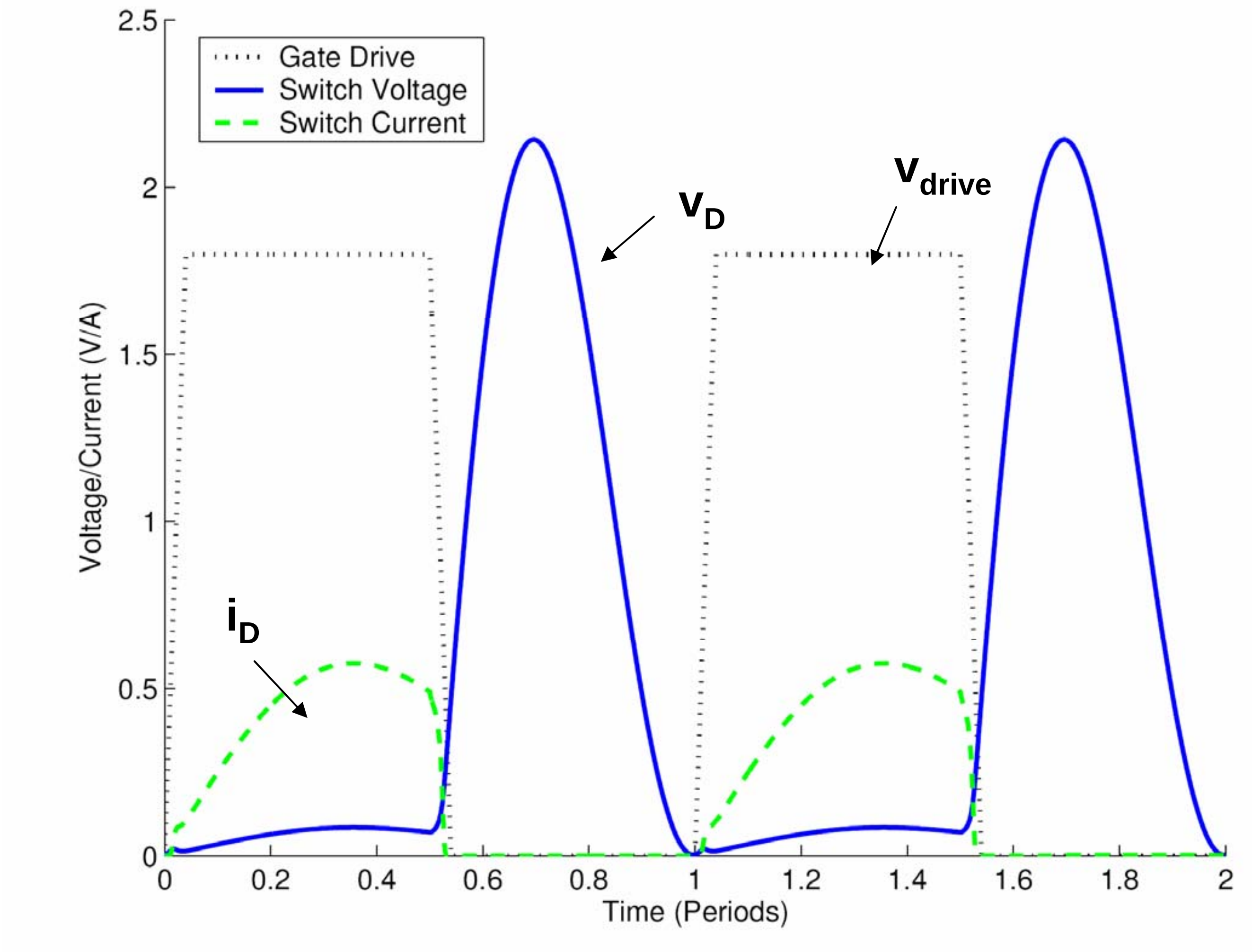
$$P_N = \frac{P_L}{v_{DS,max} \cdot i_{D,max}} = \frac{1}{\pi} = 0.32$$

Class E Power Amplifier

- **Finite switching speed causes v-i product at the switching instant non-zero -> power loss**
- **Class E amplifier tries to make both v and i zero at the off-to-on transient (solves only half of the problem)**



Typical Voltage and Current Wavforms



Souce: Shawn Kuo, “Linearization of a PulseWidth Modulated Power Amplier,” S.B. Thesis, MIT, June 2004

Class E Amplifier Design and Efficiency

- **For detailed design equations and analyses, refer to Tom Lee's book and**

  N.O. Sokal and A.D. Sokal, "Class E, a New Class of High-Efficiency Tuned Single-Ended Power Amplifiers," IEEE J. Solid-State Circuits, v.10, June 1975, pp 168-176

  (Original invention of Class E by G. Ewing in 1964)

- **Ideal Class E amplifier has 100% efficiency, but in practice, finite ON resistance of MOS switches as well as off-transients reduce efficiency**

- **Normalized Output Power Capability**

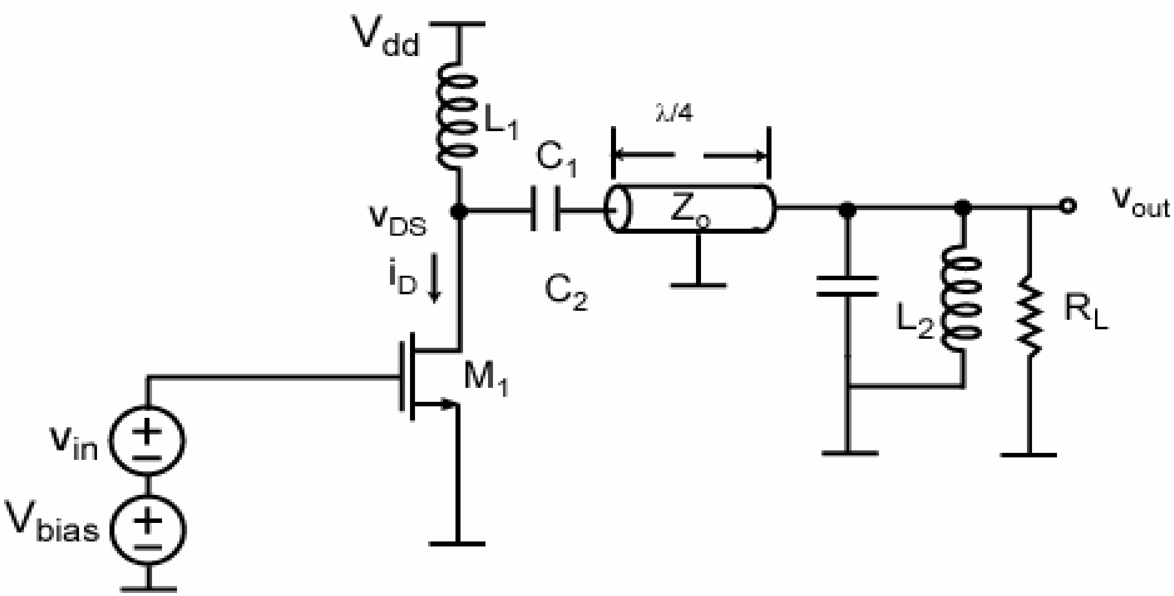
$$v_{DS,max} \approx 3.6V_{DD}, i_{D,max} \approx 1.7\frac{V_{DD}}{R}$$

$$P_{L,max} = \frac{2}{1+\frac{\pi^2}{4}} \approx 0.577 \cdot \frac{V_{DD}^2}{R} \quad P_N = \frac{P_{L,max}}{v_{DS,max} \cdot i_{D,max}} \approx 0.098$$

Due to very low P_N, Class E amplifiers are more suitable for low frequency amplifiers in which large devices can be used

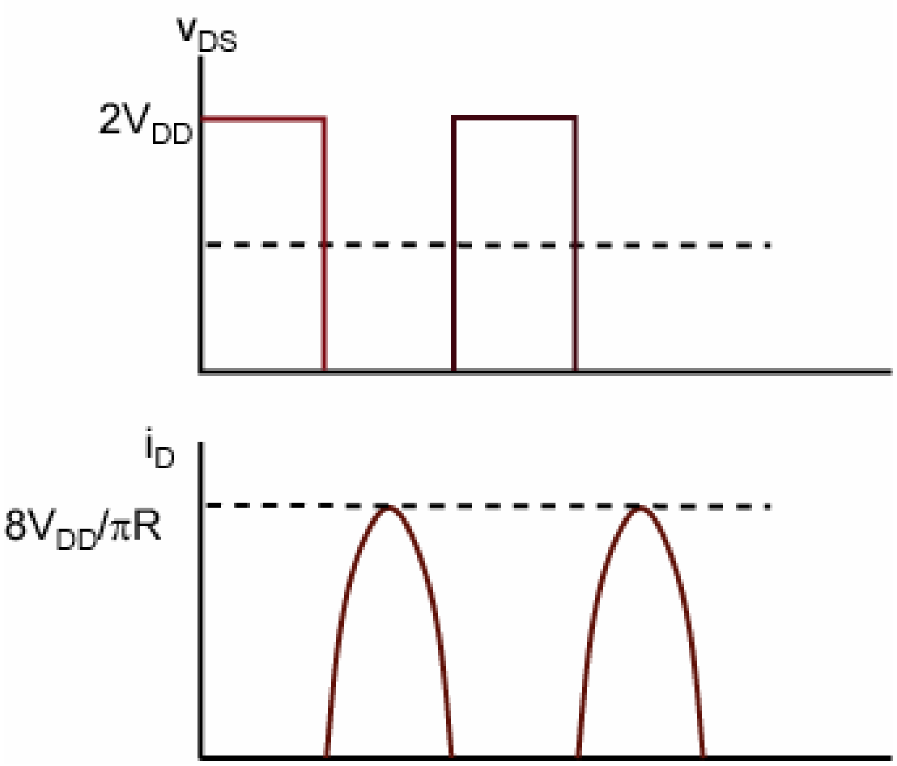
Class F Power Amplifier

- **Allows square wave on drain**
- **Single-ended version of Class D**



Class F Power Amplifier Waveforms

- **The waveforms are similar to half of Class-D Push-Pull**



Class F Power Amplifier Analysis

- **Refer to Tom Lee's book**

Amplitude of fundamental frequency of drain voltage

$$v_{fund} = \frac{4}{\pi} \cdot V_{DD}$$

Power delivered to the load

$$P_L = \frac{v_{fund}^2}{2R} = \frac{8V_{DD}^2}{\pi^2 R}$$

$$i_{D,max} = \frac{4}{\pi} \cdot \frac{2V_{DD}}{R} = \frac{8V_{DD}}{\pi R} \qquad v_{DS,max} = 2V_{DD}$$

$$P_N = \frac{v_{fund}^2}{2R} = \frac{8V_{DD}^2}{\pi^2 R} / \left(2V_{DD} \cdot \frac{8V_{DD}}{\pi R}\right) = \frac{1}{2\pi} \approx 0.16$$