Lecture 16 - p-n Junction *(cont.)*

March 12, 2007

Contents:

1. Non-ideal and second-order effects

Reading assignment:

del Alamo, Ch. 6, §6.4

Announcements:

Quiz 1: **March 13**, 7:30-9:30 PM; lectures #1-12 (up to SCR-type transport). Open book. *Calculator required.*

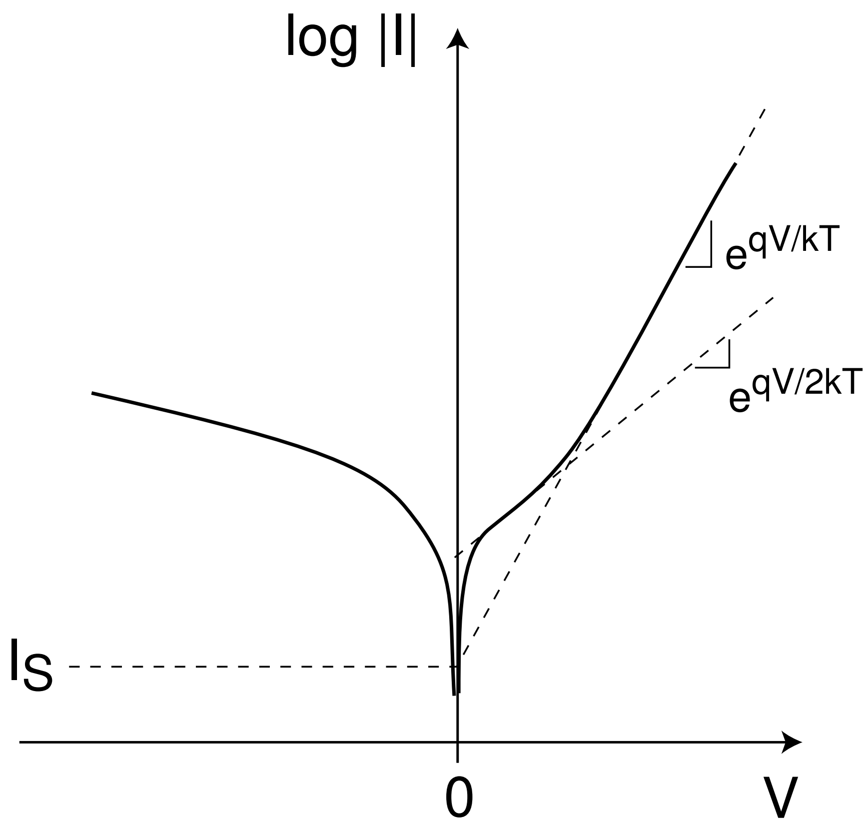
Key questions

- What happens if there is SCR generation and recombination in a pn diode?
- If the doping distribution in a p-n junction is non-uniform, is the basic operation of the diode changed in a fundamental way?
- What happens to the C-V characteristics, I-V characteristics, and the dynamics of a p-n diode with non-uniform doping distributions?

1. Non-ideal and second-order effects

□ Space-charge generation and recombination

In real devices, non-ideal $I - V$ characteristics often seen:



Anomalies often due to:

- recombination through traps in SCR (in forward bias)
- generation through traps in SCR (in reverse bias)

⋆ Simple model for SCR generation and recombination.

Starting point: trap-assisted G/R rate equation:

$$U_{tr} = \frac{np - n_o p_o}{\tau_{ho}(n + n_i) + \tau_{eo}(p + n_i)}$$

In SCR:

$$np = n_i^2 \exp \frac{qV}{kT}$$

Then:

$$U_{tr}|_{SCR} = \frac{n_i^2(\exp \frac{qV}{kT} - 1)}{\tau_{ho} n + \tau_{eo} p + (\tau_{ho} + \tau_{eo}) n_i}$$

SCR G/R current:

$$J_{SCR} = q \int_{-x_p}^{x_n} U_{tr} dx$$

Since n and p changing quickly with x in SCR, no analytical solution.

Analytical model:

$$U_{tr}|_{SCR} = \frac{n_i^2(\exp\frac{qV}{kT} - 1)}{\tau_{ho}n + \tau_{eo}p + (\tau_{ho} + \tau_{eo})n_i}$$

Since np constant, point of SCR with highest U_{tr} where:

$$\tau_{ho}n = \tau_{eo}p$$

At that point:

$$U_{tr}|_{SCR,max} = \frac{n_i}{2\sqrt{\tau_{eo}\tau_{ho}}}(\exp\frac{qV}{2kT} - 1)$$

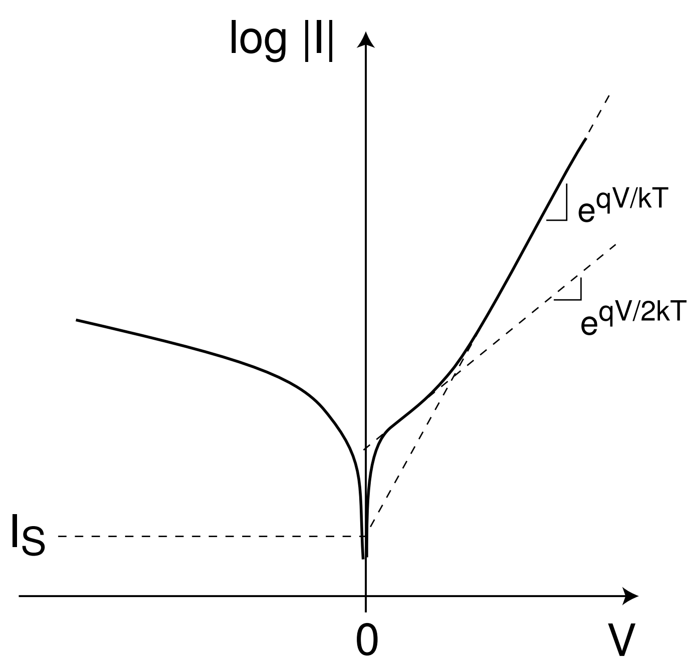
Use this across entire SCR → upper limit to current:

$$J_{SCR,max} = \frac{qn_i x_{SCR}}{2\sqrt{\tau_{eo}\tau_{ho}}}(\exp\frac{qV}{2kT} - 1)$$

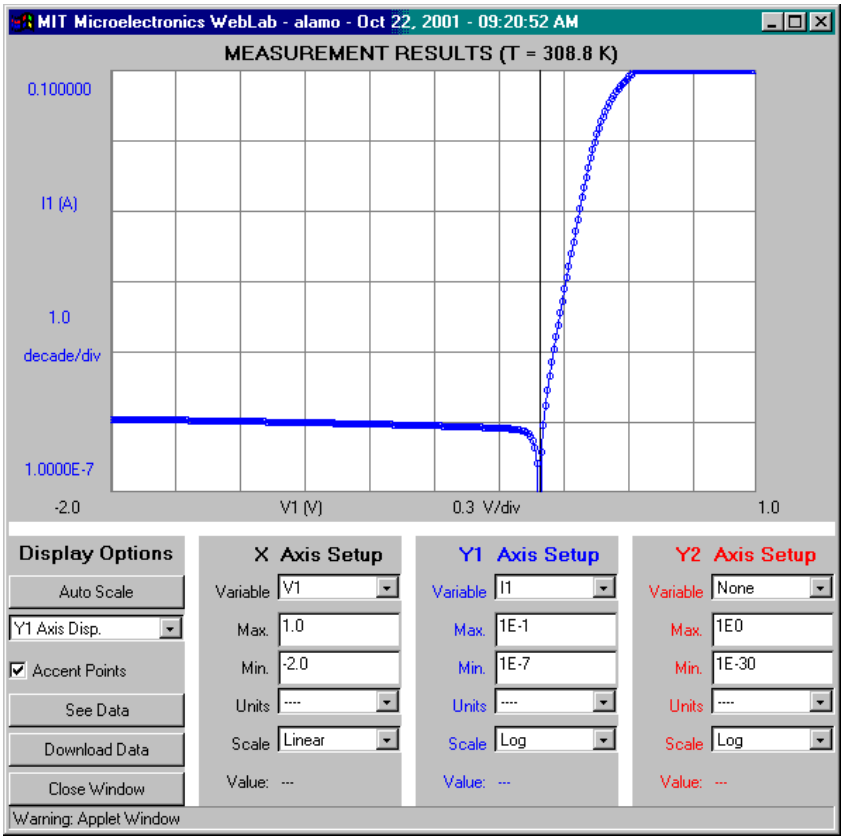
$$J_{SCR,max} = \frac{qn_i x_{SCR}}{2\sqrt{\tau_{eo}\tau_{ho}}}(\exp\frac{qV}{2kT} - 1)$$

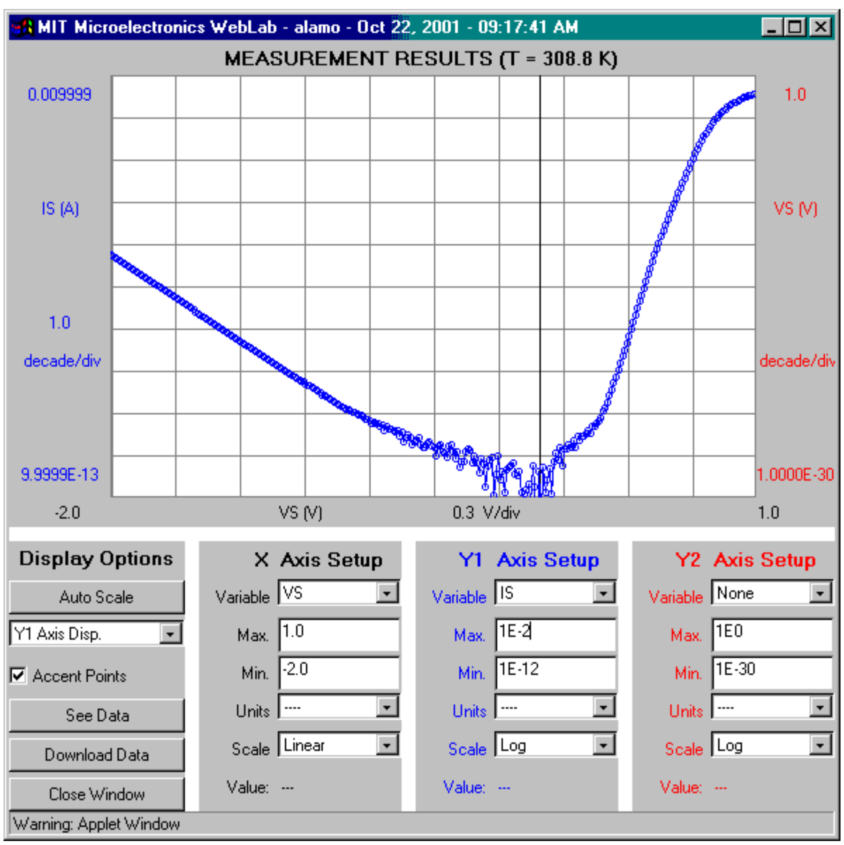
⋆ Key dependencies:

- Forward bias: SCR recombination $\sim \exp\frac{qV}{2kT}$
  in contrast with QNR recombination $\sim \exp\frac{qV}{kT}$
- In practice, $1 < n < 2$ for SCR recombination
- SCR G/R $\sim n_i$, in contrast with QNR G/R $\sim n_i^2$
  $\Rightarrow E_a(SCR) \simeq E_g/2$, in contrast with $E_a(QNR) \simeq E_g$
- SCR G/R highly process sensitive: small SCR G/R current hallmark of "clean" process,
- Reverse bias: SCR generation $\sim x_{SCR} \Rightarrow I \sim \sqrt{|V|}$



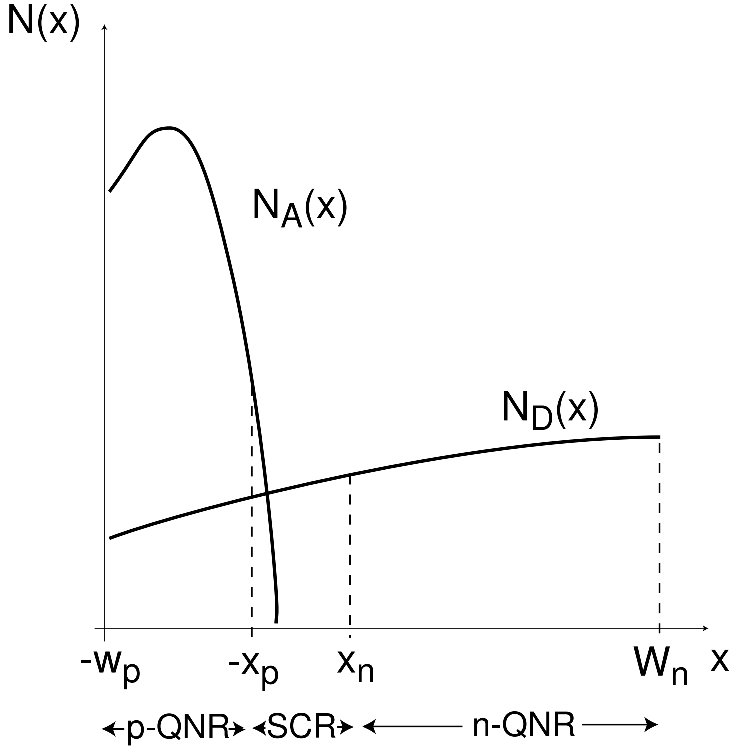
Two diodes in weblab:





□ Non-uniform doping level

"Real" p-n diodes have doping profiles that are highly non-uniform:

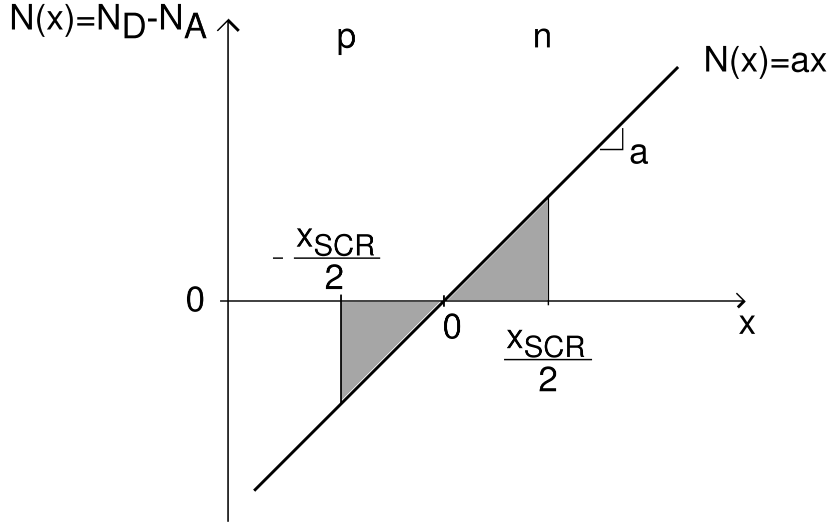


- No major differences in operation of pn diode, *e.g.* rectifying characteristics of p-n diode.
- Some qualitative differences, *e.g.*, the voltage dependence of the C-V characteristics.
- Need to revise computation of I, C, and minority carrier time constants.

⋆ Depletion capacitance

p-n junction electrostatics affected by doping non-uniformity.

Treat simple case: *linearly graded junction*:



$$N(x) = N_D - N_A = ax$$

Do depletion approximation with:

$$x_n = x_p = \frac{x_{SCR}}{2}$$

- Volume charge density:

$$\rho(x) = qax \qquad \text{for} -\frac{x_{SCR}}{2} \leq x \leq \frac{x_{SCR}}{2}$$
$$\rho(x) \simeq 0 \qquad \text{outside}$$

- Electric field:

$$\mathcal{E}(x) = \frac{qa}{2\epsilon}[x^2 - (\frac{x_{SCR}}{2})^2] \qquad \text{for} -\frac{x_{SCR}}{2} \leq x \leq \frac{x_{SCR}}{2}$$
$$\mathcal{E}(x) \simeq 0 \qquad \text{outside}$$

- Electrostatic potential distribution ($\phi(x = 0) = 0$):

$$\phi(x) = \frac{qa}{6\epsilon}[3(\frac{x_{SCR}}{2})^2 x - x^3] \qquad \text{for} -\frac{x_{SCR}}{2} \leq x \leq \frac{x_{SCR}}{2}$$

x_{SCR} determined by demanding that:

$$\phi(\frac{x_{SCR}}{2}) - \phi(-\frac{x_{SCR}}{2}) = \phi_{bi} - V$$

Then:

$$x_{SCR} = [\frac{12\epsilon(\phi_{bi} - V)}{qa}]^{1/3}$$

With:

$$\phi_{bi} = \frac{kT}{q} \ln \frac{n_o(\frac{x_{SCR}}{2})}{n_o(-\frac{x_{SCR}}{2})} = 2\frac{kT}{q} \ln \frac{a x_{SCR}}{2n_i}$$

These two equations need to be solved iteratively.

Capacitance:

$$C = [\frac{qa\epsilon^2}{12(\phi_{bi} - V)}]^{1/3} = \frac{C(V=0)}{(1 - \frac{V}{\phi_{bi}})^{1/3}}$$

With:

$$\frac{1}{C^3} = \frac{12(\phi_{bi} - V)}{qa\epsilon^2}$$

- Experiments:

Image removed due to copyright restrictions

Figure 2 on page 143 in
Lowney, J. R., and W. R. Thurber. "Evidence of Bandgap-narrowing in the Space-charge Layer of Heavily Doped Silicon Diodes." *Electronics Letters* 20, no. 3 (1984): 142-143.

- In general, depletion capacitance well modeled by:

$$C = \frac{C_o}{(1 - \frac{V}{\phi_{bi}})^m}$$

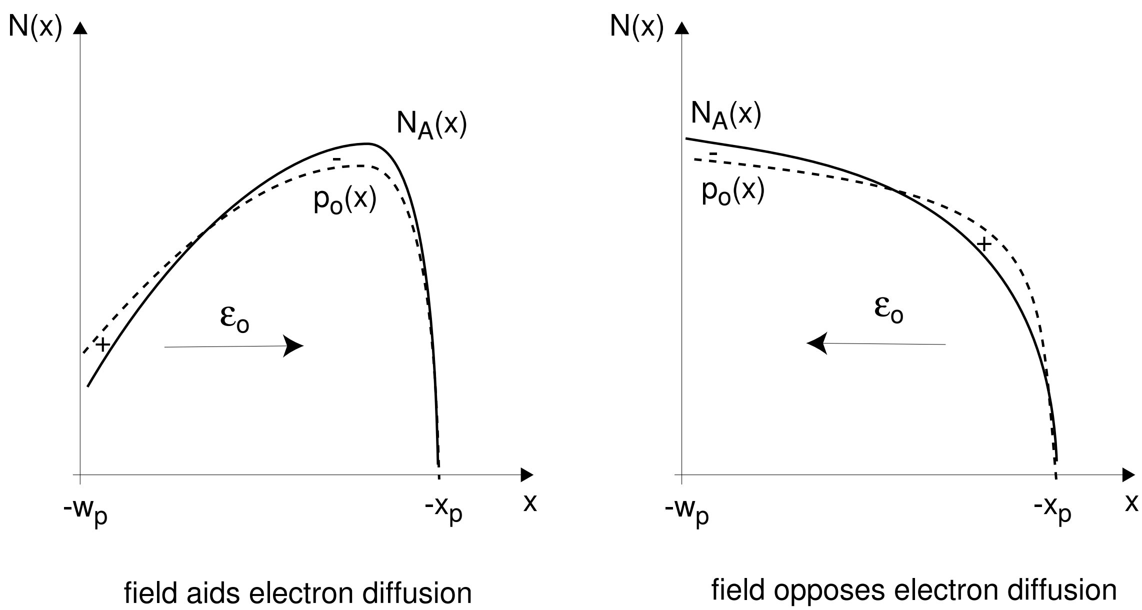
C_o is value of C at $V = 0$.

  - $m = 0.5$ for ideal abrupt junction
  - $m = 0.33$ for ideal gradual junction

⋆ Current

- Non-uniform doping distribution does not affect basic physics of minority carriers.
- Electric field associated with non-uniform doping distribution affects overall carrier velocity: drift added to diffusion.
- Computation of current density and dominant minority carrier time constant more complex.

Electric field affects current and transit time by aiding or opposing minority carrier diffusion.



- "Downgoing" doping profile aids diffusion $\rightarrow I \uparrow \;\; \tau_t \downarrow$
- "Upgoing" doping profile opposes diffusion $\rightarrow I \downarrow \;\; \tau_t \uparrow$

Analytical solution obtainable only in case of *transparent* or short region: minority carrier recombination takes place at surface.

For p-region:

$$J_e = qn'\mu_e\mathcal{E}_o + qD_e\frac{dn'}{dx}$$

Since:

$$\mathcal{E}_o = \frac{kT}{q}\frac{1}{N_A}\frac{dN_A}{dx}$$

Then:

$$J_e = q\frac{D_e}{N_A}\frac{d(n'N_A)}{dx}$$

Integrate across p-QNR:

$$\int_{-w_p}^{-x_p} J_e\frac{N_A}{qD_e}dx = \int_{-w_p}^{-x_p}\frac{d(n'N_A)}{dx}dx = n'N_A|_{-x_p} - n'N_A|_{-w_p}$$

B.C.'s:

-at junction ($x = -x_p$):

$$n'(-x_p) = \frac{n_i^2}{N_A(-x_p)}(\exp\frac{qV}{kT} - 1)$$

-at surface ($x = -w_p$) with $S = \infty$:

$$n'(-w_p) = 0$$

If recombination mainly takes place at surface, J_e independent of x, and:

$$J_e(-x_p) = \frac{qn_i^2}{\int_{-w_p}^{-x_p} \frac{N_A}{D_e} dx}(\exp\frac{qV}{kT} - 1) \simeq \frac{qn_i^2 < D_e >}{\int_{-w_p}^{-x_p} N_A \, dx}(\exp\frac{qV}{kT} - 1)$$

Since D_e is slow function of N_A, oftentimes

$$\int_{-w_p}^{-x_p} N_A dx$$

referred to as *Gummel number* → to first order, only integrated doping concentration counts to set the current!

Similar equation for n-type side.

⋆ **Dynamics**: calculation of transit time in "transparent" region.

Go back to:

$$J_e = q\frac{D_e}{N_A}\frac{d(n'N_A)}{dx}$$

Integrate up to x:

$$\int_{-w_p}^{-x} J_e \frac{N_A}{qD_e}dx = \int_{-w_p}^{-x} \frac{d(n'N_A)}{dx}dx = n'N_A|_{-x} - n'N_A|_{-w_p}$$

If $S = \infty$, $n'(-w_p) = 0$, and:

$$n'(x) = \frac{J_e(-x_p)}{qN_A(x)}\int_{-w_p}^{-x}\frac{N_A}{D_e}dx$$

Total minority carrier charge:

$$Q_p = q\int_{-w_p}^{-x_p} n'(x)dx = J_e(-x_p)\int_{-w_p}^{-x_p}\frac{1}{N_A}(\int_{-w_p}^{-x}\frac{N_A}{D_e}dx)dx$$

Diffusion capacitance:

$$C_{dp} = \frac{dQ_p}{dV} = \frac{q}{kT}J_e(-x_p)\int_{-w_p}^{-x_p}\frac{1}{N_A}(\int_{-w_p}^{-x}\frac{N_A}{D_e}dx)dx = \frac{q}{kT}\tau_{tp}J_e(-x_p)$$

Then:

$$\tau_{tp} = \int_{-w_p}^{-x_p}\frac{1}{N_A}(\int_{-w_p}^{-x}\frac{N_A}{D_e}dx)dx$$

Key conclusions

- SCR generation and recombination dominates in low forward bias and in reverse bias.
- Key characteristic of SCR recombination in forward bias:

$$J_{SCR} \propto \exp \frac{qV}{nkT} \qquad \text{with } 1 < n < 2$$

- Key characteristic of SCR generation in reverse bias:

$$J_{SCR} \propto \sqrt{|V|}$$

- Non-uniformly doped regions do not affect basic operation of pn junction.
- Exponent of dependence of depletion capacitance with voltage is function of doping distribution:
  - $m = 0.5$ for abrupt junction
  - $m = 0.33$ for linearly graded junction.
- Integrated doping concentration sets minority carrier current in "transparent" or "short" non-uniformly doped QNR.
- Minority carrier transit time through non-uniformly doped QNR depends on details of impurity profile.