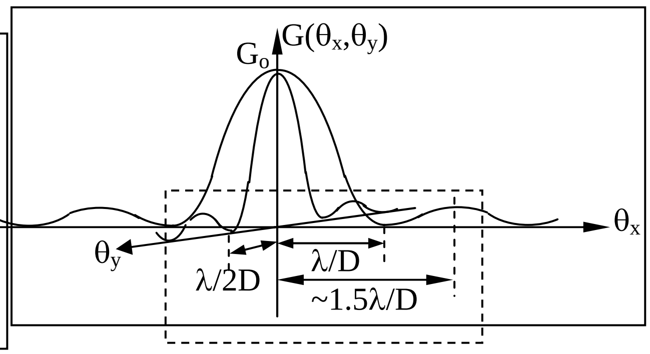
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science
Receivers, Antennas, and Signals – 6.661

Solutions -- Problem Set No. 7 December 22, 2003

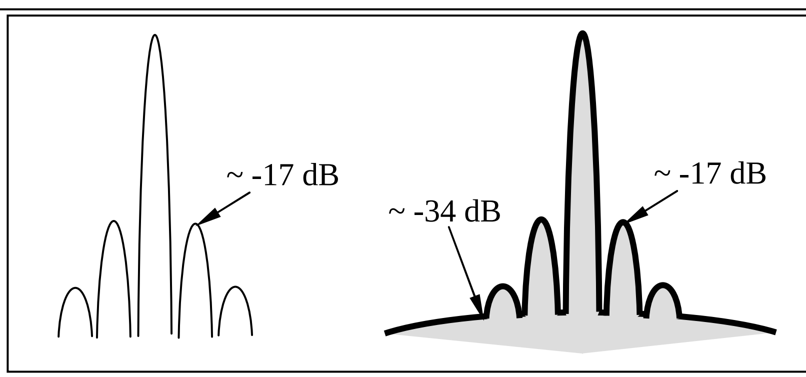
Problem 7.1

a) The on-axis gain G_o is $4\pi A/\lambda^2$ where A is the effective area. But for a uniformly illuminated aperture, A is the physical area, $2D\times D$ here. So $\boxed{G_o = 8\pi D^2/\lambda^2}$. The antenna pattern is proportional to the square of the 2-D Fourier transform of the aperture excitation function, or to a 2-D sinc^2 function, i.e.,

The gains at the two first sidelobes (on x and y axes) are approximately equal because the integrals are the same. See (3.3.3) which shows that the $(1 + \cos\theta)$ factor would be slightly different for the two sidelobes because they are at slightly different angles θ.



b) The on-axis electrical far field is proportional to the integral (see Eqn. 3.3.3) of the electrical field over the illuminated portion of the aperture, which is equivalent to the integral over the whole aperture (yielding the original $\overline{E}$ pattern), minus the integral over the blocked portion (yielding the original $\overline{E}$ pattern, but 5 and 10 times wider and with 0.02^2 times the on-axis gain). The magnitude of the far-field $\overline{E}$ is thus reduced by a fraction of $(0.2)^2/(1\times 2) = 0.02$, leaving $0.98|\overline{E}|$. The gain is proportional to the square of the field, so it is reduced about 4 percent ($0.98^2 \cong 0.96$). This 4 percent power is half (2%) distributed into the new wide diffraction sidelobes shown in the figure, and half into side and back lobes due to scattering from the structure itself. Since a linear scale would not reveal the new sidelobes because they are too small, a log scale is more useful. The new sidelobe pateau added has gain relative to G_o of $0.02^2 = 0.0004$, which is 34 dB less. The minima positions are changed only negligibly until we get to sidelobes below about 30 dB, so the blocked-portion integral becomes comparable to the sidelobe contribution from the unblocked portion.



c) $G_o \cong 0.96 \times 8\pi D^2/\lambda^2$ (see (a) and (b) solutions).

d) For a reasonable aperture taper, the beamwidth $\theta_B \cong 1.3\lambda/D = 1/57.3\times60$ radians, where $\lambda = 0.003$. It follows that $D \cong 1.3\times0.003\times57.3\times60 =$ 13.4 meters.

Problem 7.2

Referring to (3.3.40) we need to determine the determinant of the matrix:

1	-j	j	1	right circular
1	j	-j	1	left circular
0	0	0	1	y polarized
1	1	1	1	45^o polarized

The 3×3 determinant = (-1)j - 1 + j + j + j + 1 = -4j ≠ 0
Therefore the set of measurements is complete.

Problem 7.3

The antenna gain G is proportional to the Fourier transform of $\phi_E(\tau)$, so $G(\theta,\phi)$ will have a beamwidth of $\sim\lambda/D \cong 0.5\times10^{-6}/100 =$ 0.5×10^{-8} radians or ~1 milliarcsec, or roughly the angular diameter of many nearby larger stars. The impulse in $\phi(\tau)$ near the origin corresponds to a sidelobe plateau that is sufficiently small relative to the gain on axis that the telescope would function acceptably well. The difficulty is in phasing each mirror to better than $\sim\lambda/25$, or ~20 nm. The variable refractive index of the atmosphere alone makes this difficult. Without knowing the optical pathlength for each mirror, it is difficult to know which way each mirror should be moved in order to bring an image into coherence. At present it is cheaper to use a small number of apertures with larger light-gathering powers to determine critical dimensions of unknown objects than it is to use many smaller apertures. The cost of each mirror control system is now sufficiently large that only small numbers of mirrors are traditionally used.

Problem 7.4

Referring to (3.3.58), the gain reduction of one dB corresponds to the factor $e^{-(b4\pi/\lambda)^2}$ and also to the factor $10^{-(one/10)} = 0.79$, where a factor expressed as dB = 10 $\log_{10}$(factor). Therefore $-0.23 = -(b4\pi/\lambda)^2$ and
$\sigma_o \equiv b = (\lambda/4\pi)(0.23)^{0.5} = (6\times10^{-7}/4\pi)0.48 =$ 23 nm = $\lambda/26$.

b) If the main diffraction lobe is diminished by 1 dB, or 21 percent, then this power must enter the sidelobes which, in a statistical sense, correspond to the convolution of the diffraction pattern of the original pattern with that of the perturbation correlation function. The perturbation scale length of 1 cm corresponds to a beamwidth for the new sidelobes on the order of $\lambda/0.02$, where we use 2 cm because the autocorrelation function $\phi_\sigma(\tau)$ is even, and the correlation length is 1 cm in one direction and therefore 1 cm in the other. If these new sidelobes had the same width as the unperturbed main beam, then their amplitude would be 21 percent of the original, or down ~6.8 dB. But the beam is wider by a factor of ~20cm/1cm, and its solid angle is 400 times (26 dB) greater. Since the power is still 21 percent of the total, the new sidelobe plateau is down ~6.8 + 26 = 32.8 dB. If we ignore possible interference with the original sidelobes, the level of which was not provided in the problem but which presumably must be below 30 dB to render the problem interesting, then we can afford somewhat poorer surface tolerances here. That is, the 6.8 dB loss can then be reduced by 2.8 dB to equal 3 dB, implying the factor $e^{-(b4\pi/\lambda)^2}$ can now become 0.5. Solving (a) for this assumption yields

$$
\begin{aligned}
\ln 0.5 &= -\left(b4\pi/\lambda\right)^2 \\
b &= \sqrt{\ln 0.5}\ \lambda/4\pi \\
&\cong 39.8 \text{ nm}
\end{aligned}
$$

Obviously other assumptions about the original sidelobe level could be made, and then the interference between the two contributions might be noted. If two equal field contributions coherently add (the old sidelobe plus the tolerance-induced sidelobe), then the power in that direction (sidelobe) is quadurupled, or increased by 6 dB. In other directions these two contributions might cancel, but when we speak of sidelobe levels we usually intend either their peak levels or their average levels, which might approximate half the peak values (say ~3-dB down). Thus the peak levels concern us more.

c) Now the factor $e^{-(b4\pi/\lambda)2}$ = 0.25, where b = 23 nm and λ is unknown. Therefore

$\lambda = 4\pi b(\ln 0.25)^{-0.5}$ = $\boxed{0.34 \text{ microns,}}$ which is in the ultraviolet and beyond the atmospheric cutoff wavelength so that atmospheric absorption precludes long pathlengths for communications or sensing purposes.