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6.641 Electromagnetic Fields, Forces, and Motion, Spring 2005

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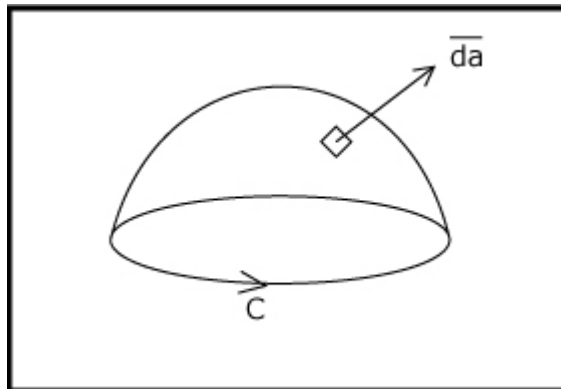Note: Please use the actual date you accessed this material in your citation.

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Lecture 1: Integral Form of Maxwell's Equations

I. <u>Maxwell's Equations in Integral Form in Free Space</u>

1. Faraday's Law

$$\underbrace{\oint_C \bar{E} \cdot \overline{ds}}_{} = -\frac{d}{dt} \underbrace{\int_S \mu_0 \bar{H} \cdot \overline{da}}_{}$$

      Circulation   Magnetic Flux
        of $\bar{E}$

$\mu_0 = 4\pi \times 10^{-7}$ henries/meter
[magnetic permeability of
free space]

EQS form: $\oint_C \bar{E} \cdot \overline{ds} = 0$ (Kirchoff's Voltage Law, conservative electric
field)

MQS circuit form: $v = L\dfrac{di}{dt}$ (Inductor)

2. Ampère's Law (with displacement current)

$$\underbrace{\oint_C \bar{H} \cdot \overline{ds}}_{} = \underbrace{\int_S \bar{J} \cdot \overline{da}}_{} + \underbrace{\frac{d}{dt} \int_S \varepsilon_0 \bar{E} \cdot \overline{da}}_{}$$

  Circulation Conduction   Displacement
   of $\bar{H}$ Current   Current

MQS form: $\oint_C \bar{H} \cdot \overline{ds} = \int_S \bar{J} \cdot \overline{da}$

EQS circuit form: $i = C\dfrac{dv}{dt}$ (capacitor)

3. Gauss' Law for Electric Field

$$\oint_S \varepsilon_0 \bar{E} \cdot \overline{da} = \int_V \rho \, dV$$

$$\varepsilon_0 \approx \frac{10^{-9}}{36\pi} \approx 8.854 \times 10^{-12} \text{ farads/meter}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \approx 3 \times 10^8 \text{ meters/second (Speed of electromagnetic waves in free space)}$$

4. Gauss' Law for Magnetic Field

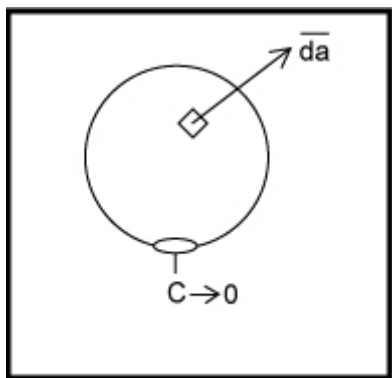$$\oint_S \mu_0 \bar{H} \cdot \overline{da} = 0$$

In free space:

$$\bar{B} = \mu_0 \bar{H}$$

magnetic   magnetic
flux   field
density   intensity
(Teslas)   (amperes/meter)

5. Conservation of Charge

Take Ampère's Law with displacement current and let contour $C \rightarrow 0$

$$\lim_{C \rightarrow 0} \oint_C \bar{H} \cdot \overline{ds} = 0 = \oint_S \bar{J} \cdot \overline{da} + \frac{d}{dt} \underbrace{\oint_S \varepsilon_0 \bar{E} \cdot \overline{da}}_{\int_V \rho \, dV}$$

$$\underbrace{\oint_S \bar{J} \cdot \overline{da}}_{} + \frac{d}{dt} \underbrace{\int_V \rho \, dV}_{} = 0$$

Total current   Total charge
leaving volume   inside volume
through surface

6. Lorentz Force Law

$$\bar{f} = q\left(\bar{E} + \bar{v} \times \mu_0 \bar{H}\right)$$

II. <u>Electric Field from Point Charge</u>

Mathematical Gaussian Spherical Surface of radius r
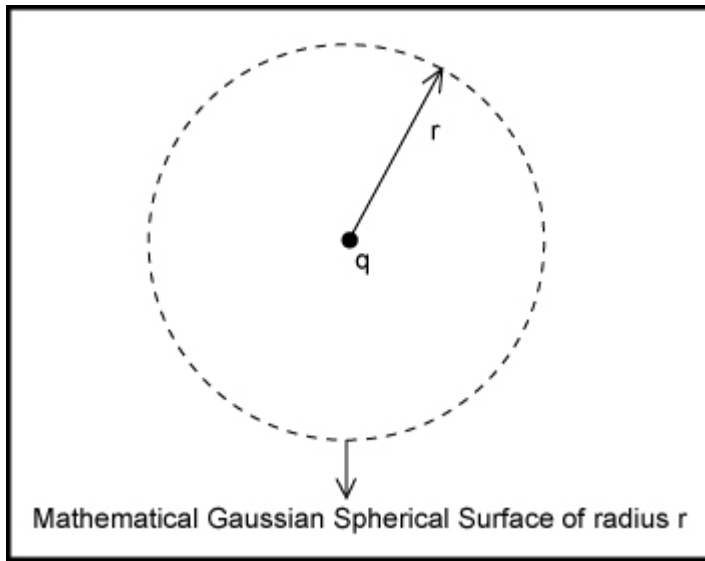
$$\oint_S \varepsilon_0 \bar{E} \cdot \overline{da} = \varepsilon_0 E_r 4\pi r^2 = q$$

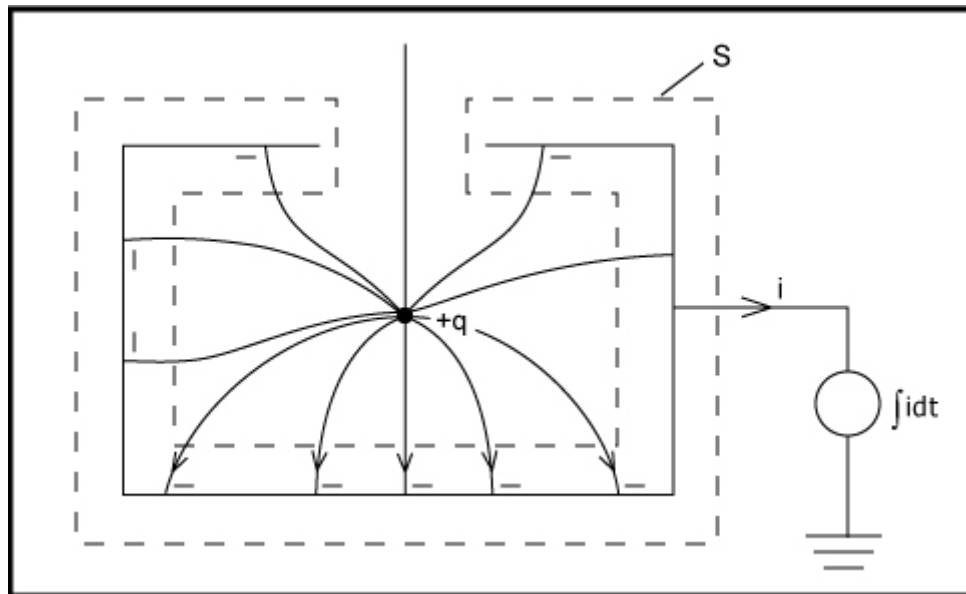$$E_r = \frac{q}{4\pi \varepsilon_0 r^2}$$
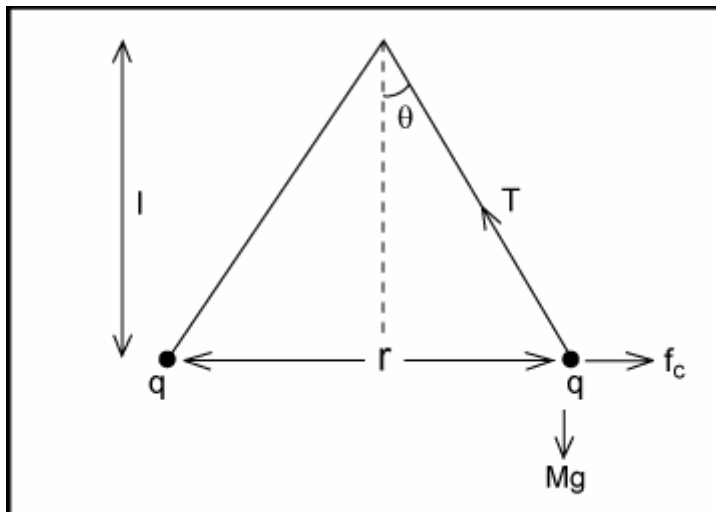
$$T \sin\theta = f_c = \frac{q^2}{4\pi \varepsilon_0 r^2}$$

$$T \cos\theta = Mg$$

$$\tan\theta = \frac{q^2}{4\pi \varepsilon_0 r^2 Mg} = \frac{r}{2l}$$

$$q = \left[\frac{2\pi \varepsilon_0 r^3 Mg}{l} \right]^{\frac{1}{2}}$$

III. Faraday Cage

$$\oint_S \bar{J} \cdot \overline{da} = i = -\frac{d}{dt} \int \rho \, dV = -\frac{d}{dt}(-q) = \frac{dq}{dt}$$

$$\int i \, dt = q$$

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IV. Boundary Conditions
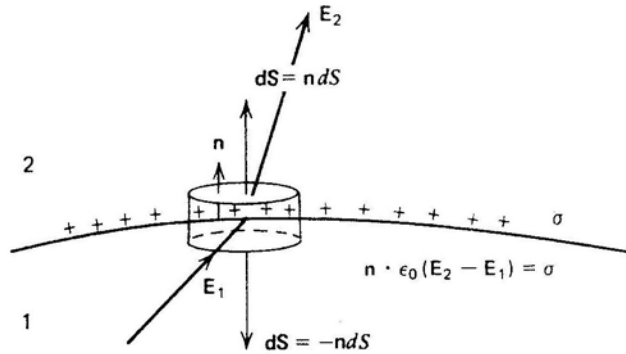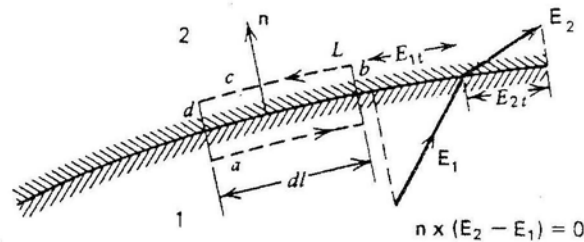
    1. Gauss' Continuity Condition

Figure 2-19 Gauss's law applied to a differential sized pill-box surface enclosing some surface charge shows that the normal component of $\varepsilon_0 \mathbf{E}$ is discontinuous in the surface charge density.

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$$\oint_S \varepsilon_0 \bar{E} \cdot \overline{da} = \int_S \sigma_s dS \Rightarrow \varepsilon_0 \left(E_{2n} - E_{1n} \right) dS = \sigma_s dS$$

$$\varepsilon_0 \left(E_{2n} - E_{1n} \right) = \sigma_s \Rightarrow \bar{n} \cdot \left[\varepsilon_0 \left(\bar{E}_2 - \bar{E}_1 \right) \right] = \sigma_s$$

    2. Continuity of Tangential $\bar{E}$

(a)

Figure 3-12 *(a)* Stokes' law applied to a line integral about an interface of discontinuity shows that the tangential component of electric field is continuous across the boundary.
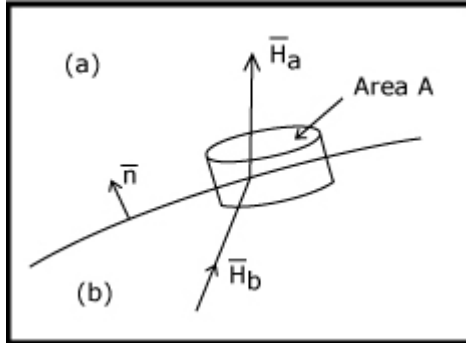
Courtesy of Krieger Publishing. Used with permission.

$$\oint_C \bar{E} \cdot \overline{ds} = \left(E_{1t} - E_{2t} \right) dl = 0 \Rightarrow E_{1t} - E_{2t} = 0$$

$$\bar{n} \times \left(\bar{E}_1 - \bar{E}_2 \right) = 0$$

Equivalent to $\Phi_1 = \Phi_2$ along boundary

3. Normal H

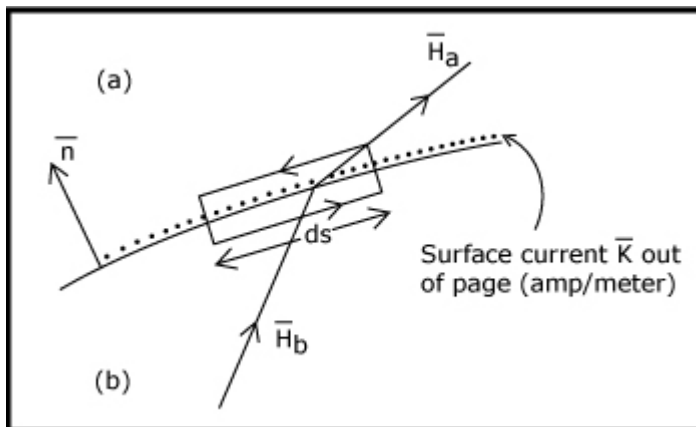$$\nabla \cdot \mu_0 \overline{H} = 0 \Rightarrow \oint_S \mu_0 \ \overline{H} \cdot \overline{da} = 0$$

(a)

$\overline{H}_a$

Area A

$\overline{n}$

$\overline{H}_b$

(b)

$\mu_0 \left(H_{an} - H_{bn} \right) A = 0$

$H_{an} = H_{bn}$

$\overline{n} \cdot \left[\overline{H}_a - \overline{H}_b \right] = 0$

4. Tangential $\overline{H}$

$$\nabla \times \overline{H} = \overline{J} \Rightarrow \oint_C \overline{H} \cdot \overline{ds} = \int_S \overline{J} \cdot \overline{da}$$

$\overline{H}_a$

(a)

$\overline{n}$

$\overline{ds}$

Surface current $\overline{K}$ out
of page (amp/meter)

$\overline{H}_b$

(b)

$H_{bt} ds - H_{at} ds = K ds$

$H_{bt} - H_{at} = K$

$\overline{n} \times \left[\overline{H}_a - \overline{H}_b \right] = \overline{K}$

5. Conservation of Charge Boundary Condition

$\overline{J}_a$

(a)

$\overline{n}$

+++++++++

(b)

$\overline{J}_b$
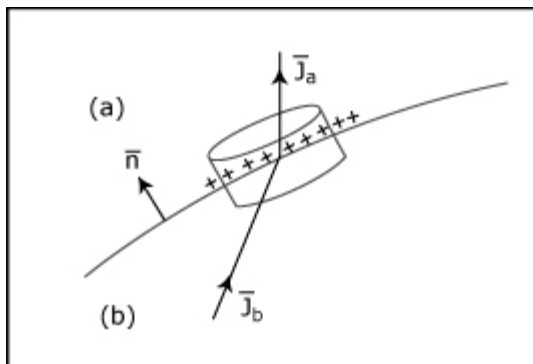
$\nabla \cdot \overline{J} + \dfrac{\partial \rho}{\partial t} = 0$

$\oint_S \overline{J} \cdot \overline{da} + \dfrac{d}{dt} \int_V \rho dV = 0$

$\overline{n} \cdot \left[\overline{J}_a - \overline{J}_b \right] + \dfrac{\partial}{\partial t} \sigma_s = 0$

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