6.641 Electromagnetic Fields, Forces, and Motion
Spring 2005

6.641 — **Electromagnetic Fields, Forces, and Motion** Spring 2005

Quiz 1 - Solutions 2004

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Problem 1

A

Question: What is the general form of the solution for $\chi(x,y)$ in the regions $x<0$ and $0<x<d$?

Solution:

$$\chi(x,y) = \begin{cases} A\cos(ay)e^{ax} & x<0 \\ B\cos(ay)\cosh\left(a\left(x-d\right)\right) & 0<x<d \end{cases}$$

B

Question: What boundary conditions must be satisfied?

Solution:

$$H_y(x=0_+) - H_y(x=0_-) = K_z$$
$$\mu H_x(x=0_-) = \mu_0 H_x(x=0_+)$$
$$H_x(x=d) = 0$$

C

Question: Solve $\chi(x,y)$ for $x<0$ and $0<x<d$.
Hint: To minimize algebraic complexity, think about the best way to write the general form of the solution for $\chi(x,y)$ to automatically satisfy one of the boundary conditions for part (b).

Solution:

$$H_y = -\frac{\partial\chi}{\partial y} = \begin{cases} aA\sin(ay)e^{ax} & x<0 \\ aB\sin(ay)\cosh(a(x-d)) & 0<x<d \end{cases}$$

$$H_x = -\frac{\partial\chi}{\partial x} = \begin{cases} -aA\cos(ay)e^{ax} & x<0 \\ -aB\cos(ay)\sinh(a(x-d)) & 0<x<d \end{cases}$$

$$\cancel{a}B\cancel{\sin(ay)}\cosh(ad) - \cancel{a}A\cancel{\sin(ay)} = \frac{K_0}{a}\cancel{\sin(ay)}$$
$$-\mu\cancel{a}A\cancel{\cos(ay)} = +\mu_0\cancel{a}B\cancel{\cos(ay)}\sinh(ad)$$
$$A = -\frac{\mu_0}{\mu}B\sinh(ad)$$
$$B\left[cosh(ad) + \frac{\mu_0}{\mu}\sinh(ad)\right] = \frac{K_0}{a}$$
$$B = \frac{K_0}{a\left[\cosh(ad) + \frac{\mu_0}{\mu}\sinh(ad)\right]}$$
$$A = -\frac{\mu_0\sinh(ad)K_0}{\mu a\left[\cosh(ad) + \frac{\mu_0}{\mu}\sinh(ad)\right]}$$

$$\chi(x,y) = \begin{cases} -\frac{\mu_0 K_0\sinh(ad)\cos(ay)e^{ax}}{\mu a\left[\cosh(ad)+\frac{\mu_0}{\mu}\sinh(ad)\right]} & x < 0 \\ \frac{K_0\cos(ay)\cosh(a(x-d))}{a\left[\cosh(ad)+\frac{\mu_0}{\mu}\sinh(ad)\right]} & 0 < x < d \end{cases}$$

D

<u>Question:</u> What is the surface current that flows on the $x = d$ interface?

<u>Solution:</u>

$$K_z(x=d) = -H_y(x=d) = -Ba\sin(ay)$$
$$= \frac{-K_0\sin(ay)}{\left[\cosh(ad) + \frac{\mu_0}{\mu}\sinh(ad)\right]}$$

E

<u>Question:</u> What is the force per unit $y - z$ area on the $x = d$ interface?

<u>Solution:</u>

$$\frac{\bar{f}}{\text{area}} = \frac{1}{2}\left[\bar{K}\times\mu_0\bar{H}\right]\Big|_{x=d} = \frac{1}{2}\mu_0 K_z(x=d)H_y(x=d)\bar{i}_z\times\bar{i}_y$$
$$= -\bar{i}_x\frac{\mu_0}{2}K_z(x=d)(-K_z(x=d))$$
$$= +\frac{\mu_0}{2}K_z^2(x=d)\bar{i}_x$$
$$= \frac{\mu_0}{2}\frac{K_0^2\sin^2(ay)}{\left[\cosh(ad) + \frac{\mu_0}{\mu}\sinh(ad)\right]^2}$$

Problem 2

A

Question: What is the general form of solution for the electric scalar potential $\Phi(r,\phi)$ for $r < R_1$ and $R_1 < r < R_2$?

Solution:

$$\Phi(r,\phi) = \begin{cases} Ar^2\sin(2\phi) & r < R_1 \\ \left(Br^2 + \frac{C}{r^2}\right)\sin(2\phi) & R_1 < r < R_2 \end{cases}$$

B

Question: What boundary conditions must be satisfied?

Solution:

$$\Phi(r=0,\phi) \text{ is finite}$$
$$\Phi(r=R_{1-},\phi) = \Phi(r=R_{1+},\phi) = V_0\sin(2\phi)$$
$$\Phi(r=R_2,\phi) = 0$$

C

Question: What is the potential distribution for $r < R_1$ and $R_1 < r < R_2$?

Solution:

$$AR_1^2\cancel{\sin(2\phi)} = V_0\cancel{\sin(2\phi)} \Rightarrow A = \frac{V_0}{R_1^2}$$
$$\left(BR_2^2 + \frac{C}{R_2^2}\right)\sin(2\phi) = 0 \Rightarrow B = -\frac{C}{R_2^4}$$
$$\left(BR_1^2 + \frac{C}{R_1^2}\right)\cancel{\sin(2\phi)} = V_0\cancel{\sin(2\phi)} \Rightarrow C\left(\frac{1}{R_1^2} - \frac{R_1^2}{R_2^4}\right) = V_0$$
$$\Phi(r,\phi) = \begin{cases} \frac{V_0 r^2}{R_1^2}\sin(2\phi) & r < R_1 \\ C\left(-\frac{r^2}{R_2^4} + \frac{1}{r^2}\right)\sin(2\phi) = \frac{V_0R_1^2R_2^4}{R_2^4 - R_1^4}\sin(2\phi)\left(\frac{1}{r^2} - \frac{r^2}{R_2^4}\right) & R_1 < r < R_2 \end{cases}$$

D

Question: What are the surface charge distributions at $r = R_1$ and $r = R_2$?

Solution:

$$\sigma_s(r=R_1) = -\left[\epsilon_2 \left.\frac{\partial\Phi}{\partial r}\right|_{r=R_{1+}} - \epsilon_1 \left.\frac{\partial\Phi}{\partial r}\right|_{r=R_{1-}}\right] = \left[-\frac{\epsilon_2V_0R_1^2R_2^4}{R_2^4 - R_1^4}\left(-\frac{2}{R_1^3} - \frac{2R_1}{R_2^4}\right) + \frac{\epsilon_1V_02R_1}{R_1^2}\right]\sin(2\phi)$$
$$= \frac{2V_0\sin(2\phi)}{R_1}\left[\epsilon_1 + \epsilon_2\frac{(R_1^4 + R_2^4)}{-R_1^4 + R_2^4)}\right]$$

$$\sigma_2(r = R_2) = +\epsilon_2 \left.\frac{\partial \Phi}{\partial r}\right|_{r=R_2} = \frac{\epsilon_2 V_0 R_1^2 R_2^4}{R_2^4 - R_1^4}\left(-\frac{2}{R_2^3} - \frac{2R_2}{R_2^4}\right) = \frac{-4\epsilon_2 V_0 R_1^2 R_2}{R_2^4 - R_1^4}$$