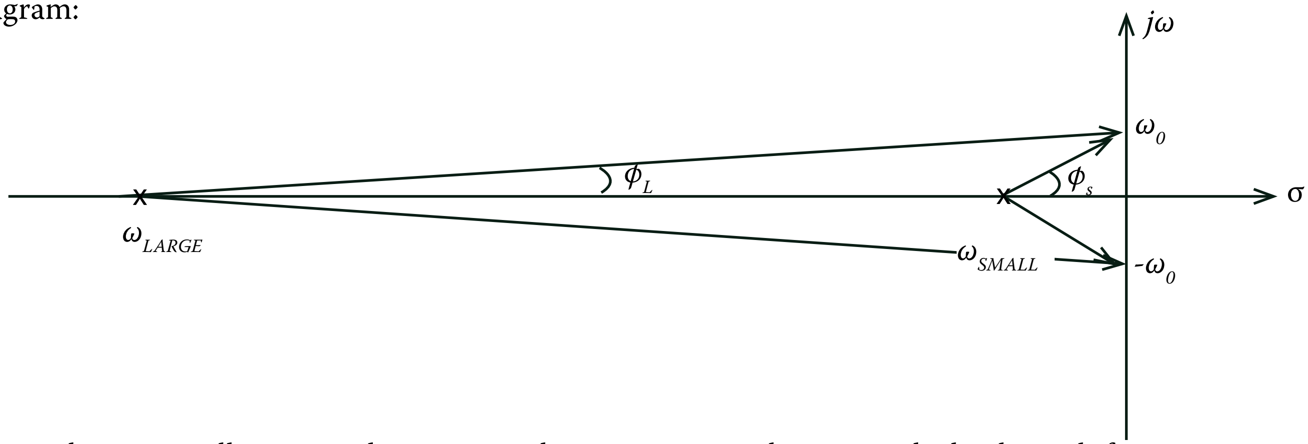
6.302 Feedback Systems

Recitation 5: Op-amp Circuits and Analog Computers

Prof. Joel L. Dawson

In lecture, Prof. Roberge spoke of building elaborate systems using op-amp circuits. When we do this, we habitually make statements like, "...and we'll choose the dynamics such that the poles contributed by the op-amps themselves are negligible." Let's discuss.

Consider a two-pole system whose behavior interests us only in the range from DC to ω_0. A pole-zero diagram:



The angle ϕ_L is small compared to ϕ_S. => pole @ ω_{LARGE} contributes very little phase shift.

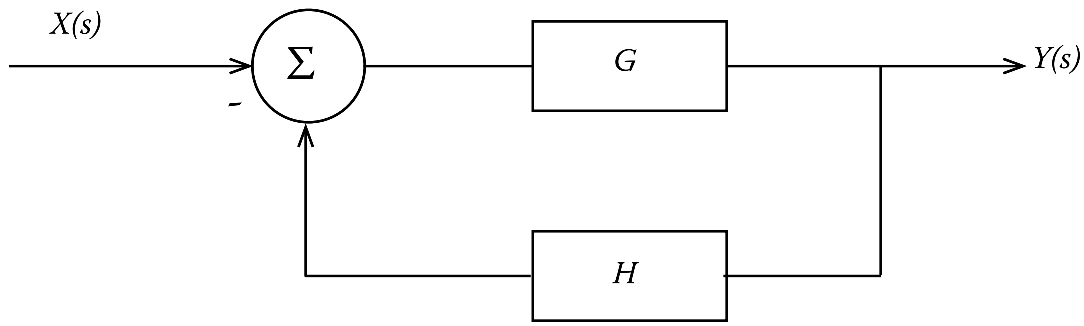
For the frequency response evaluated at $s=j\omega_o$:

$$\left.\frac{1}{\left(\frac{s}{\omega_L}+1\right)\left(\frac{s}{\omega_S}+1\right)}\right|_{s=j\omega_0} = \frac{1}{\left(\frac{j\omega_0}{\omega_L}+1\right)\left(\frac{j\omega_0}{\omega_S}+1\right)}$$

$$\frac{\omega_0}{\omega_L} \ll 1 \qquad , \qquad \frac{\omega_0}{\omega_S} \text{ is on the order of unity (at least)!}$$

So this transfer function is well-approximated by the single-pole transfer function $\frac{1}{\left(\frac{s}{\omega_S}+1\right)}$ for frequencies from DC to ω_o.

But what determines ω_o? Depends on the context, but for feedback systems we often look at the loop transmission.



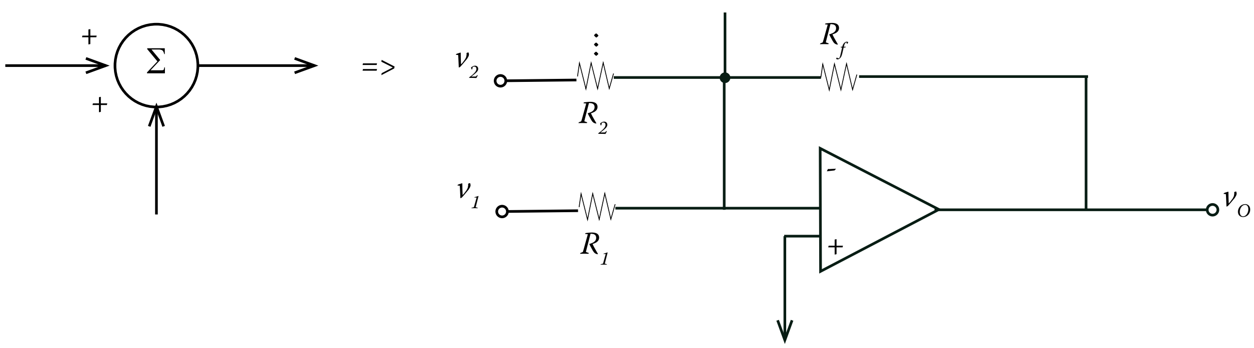
When $|L(s)| = |GH| << 1$, we arguably have an open-loop system:

$$\frac{Y(s)}{X(s)} = \frac{G}{1+GH} \approx G$$

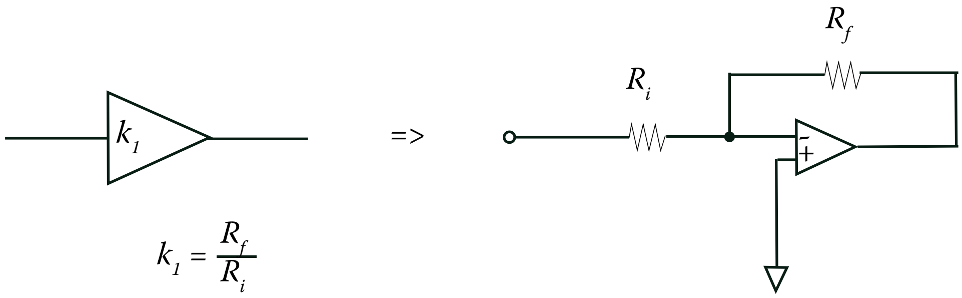
So when we're looking for dynamics to ignore, we will often discard poles that are large compared to the loop crossover frequency, or the frequency at which $|L(S)| = 1$.

Op-amp circuits for modeling our systems

We need an integrator, a summer, an inversion, and a gain.

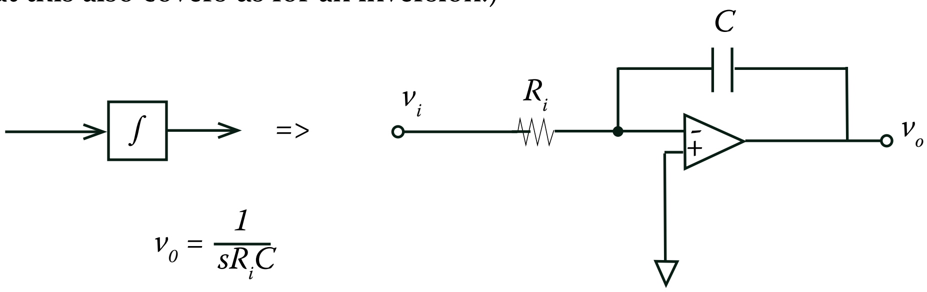


$$V_0 = -\frac{R_f}{R_1} V_1 + \left(-\frac{R_f}{R_2}\right) V_2 + \cdots$$



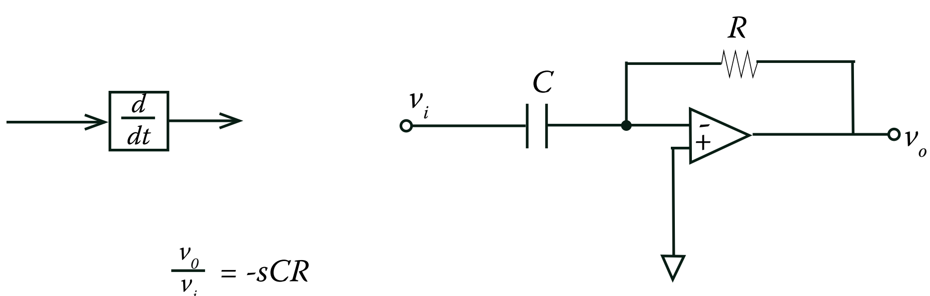
$$k_1 = \frac{R_f}{R_i}$$

(Note that this also covers us for an inversion.)



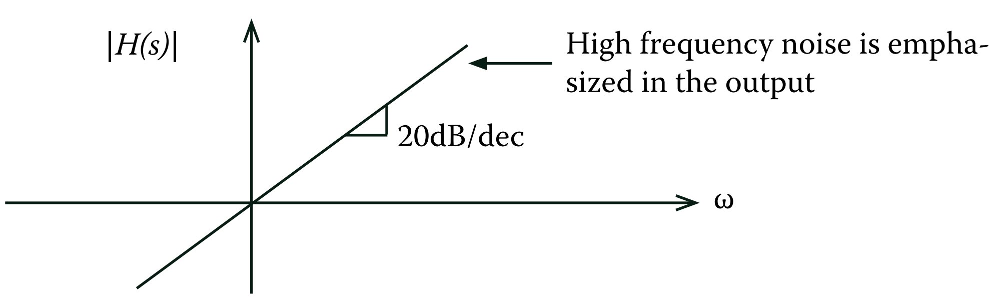
$$v_0 = \frac{1}{sR_iC}$$

And, a block that we do not use:



$$\frac{v_0}{v_i} = -sCR$$

Difficult to manage in a noisy world:



Also against the differentiator: it's hard enough to get high gain at DC. High gain at high frequencies? Forget it.

Now, on to building analog computers. Suppose we have an all-pole system. It begins as a differential equation:

$$a_n \frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + a_{n-2}\frac{d^{n-2}y}{dt^{n-2}} + \ldots\ldots + a_0 y = x$$

Completely general procedure starts with taking the Laplace transform:

$$(a_n s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots.. + a_0)Y(s) = x(s)$$

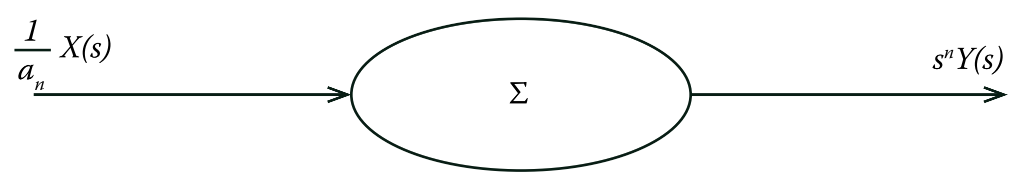
$$\left[\text{The system function, BTW, is: } \frac{Y(s)}{x(s)} = \frac{1}{a_n s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots.. + a_0} \right]$$

Solve for the highest order derivatve:

$$a_n s^n Y(s) = X(s) - (a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots.. + a_0)Y(s)$$

$$s^n Y(s) = \frac{1}{a_n}X(s) - \left[\frac{a_{n-1}}{a_n}s^{n-1} + \frac{a_{n-2}}{a_n}s^{n-2} + \ldots\ldots + \frac{a_0}{a_n} \right] Y(s)$$

Put down a big summing junction:

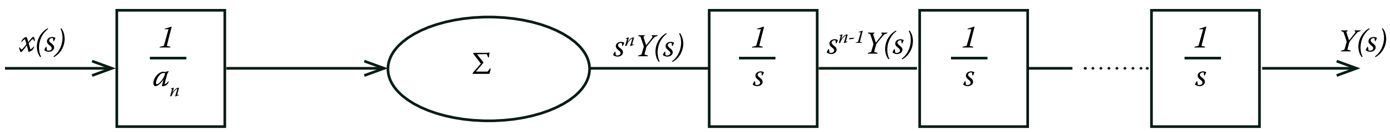


6.302 Feedback Systems

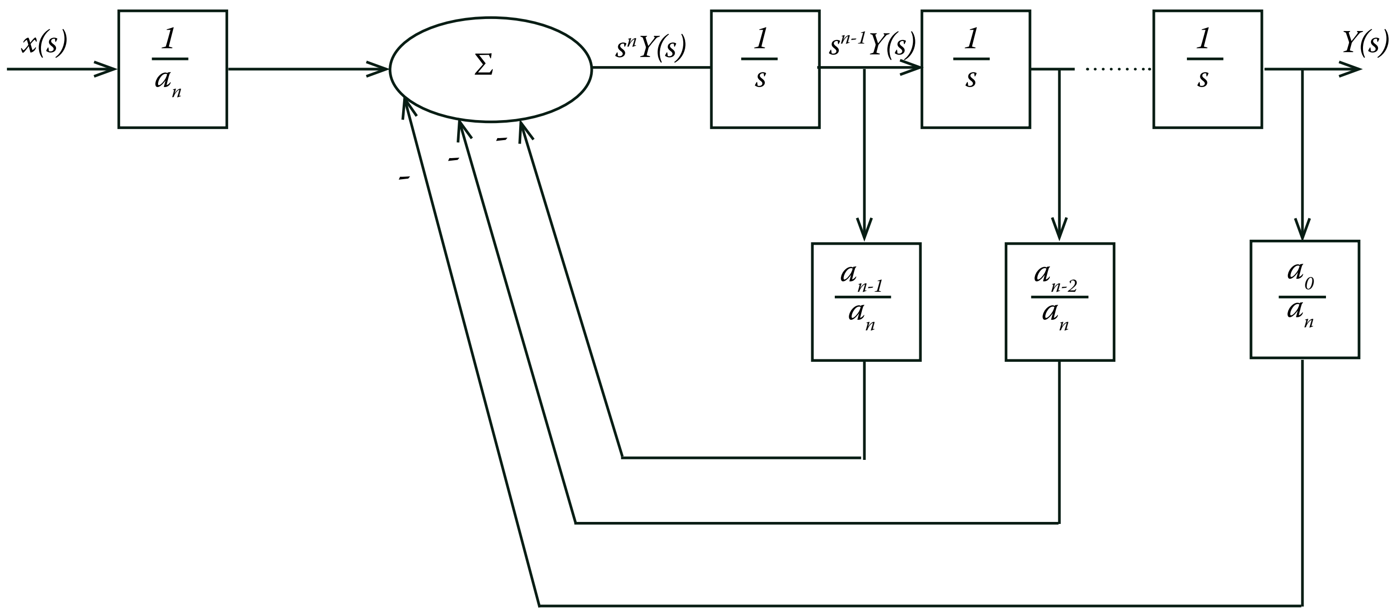
Recitation 5: Op-amp Circuits and Analog Computers

Prof. Joel L. Dawson

Generate the derivatives that you need:



Complete the mapping:



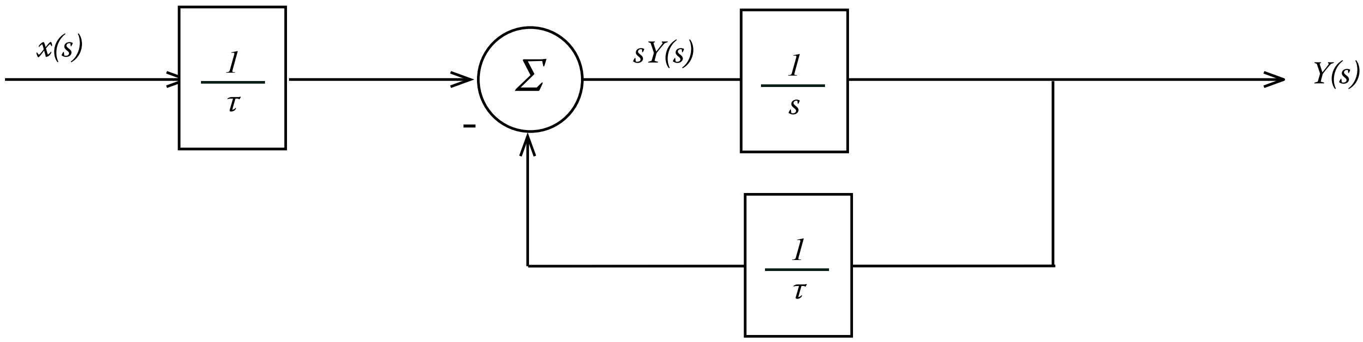
EXAMPLE: First order system

$$\frac{Y(s)}{x(s)} = \frac{1}{\tau s+1}$$

$$(\tau s+1)Y(s) = x(s)$$

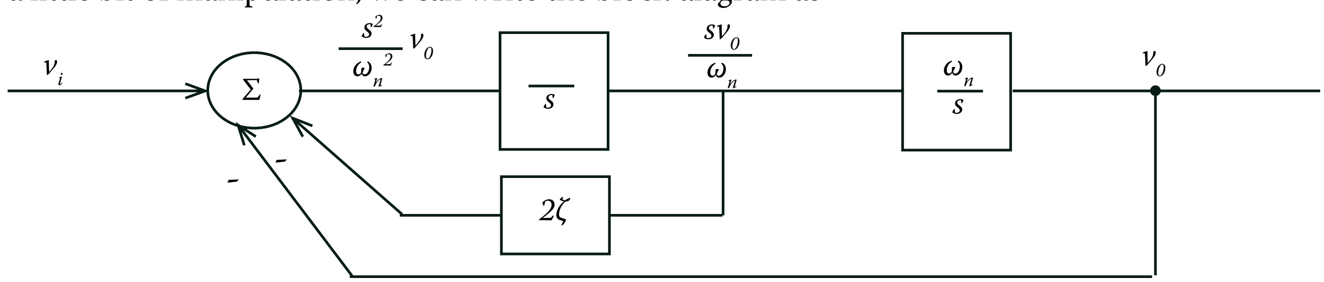
$$sY(s) = \frac{1}{\tau}x(s) - \frac{1}{\tau}Y(s)$$

=>

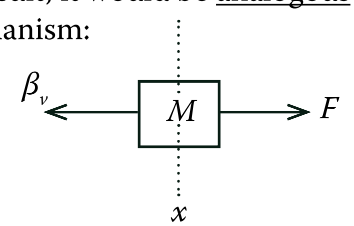


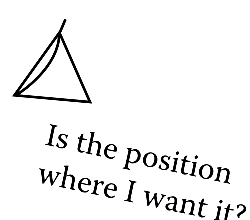
2nd order system from class: $$\frac{v_0}{v_i} = \frac{1}{\frac{s^2}{\omega_n^{\,2}} + \frac{2\zeta s}{\omega_n} + 1}$$

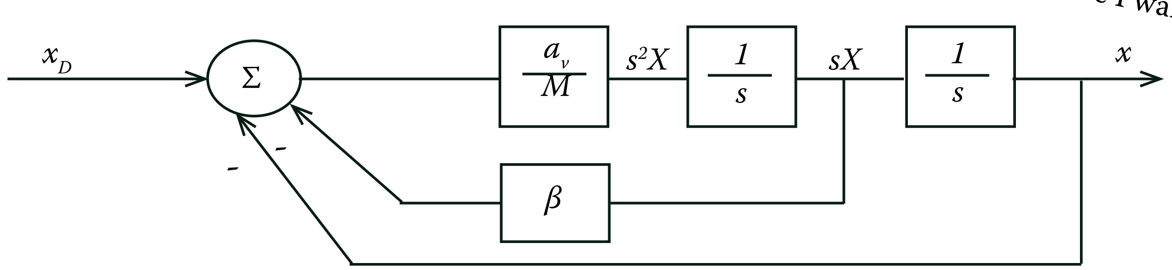
With a little bit of manipulation, we can write the block diagram as



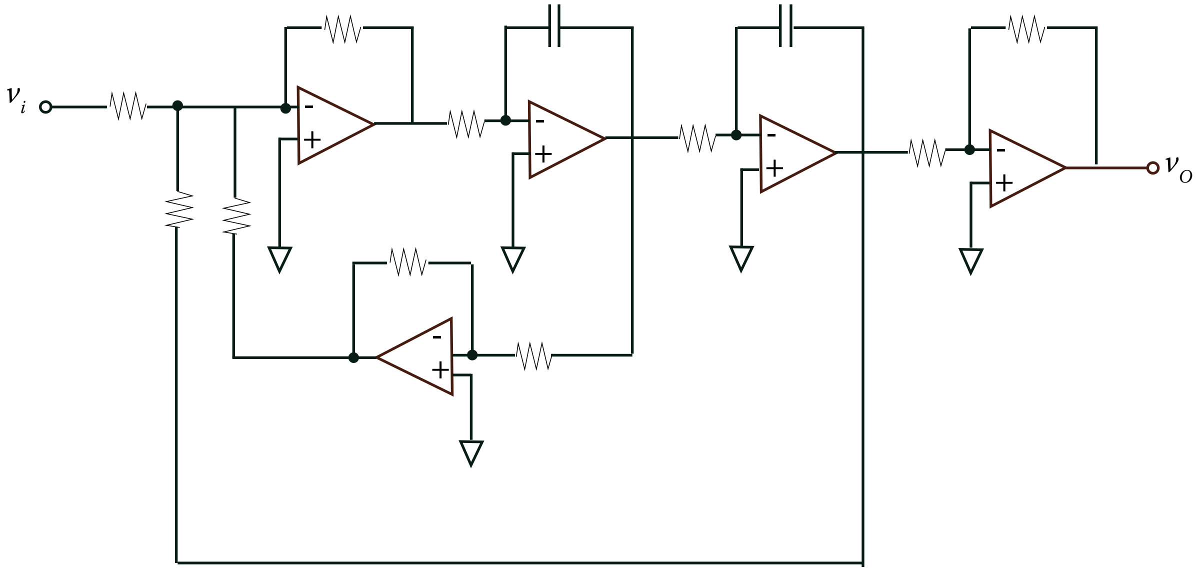
If we built this as an electronic circuit, it would be <u>analogous</u> to our mechanical system consisting of a mass, a viscous fluid, and a forcing mechanism:







An analog computer might look something like this:



Make sure that ω_n is small compared to the parasitic poles of the op-amps. Then, we get a very good analog.