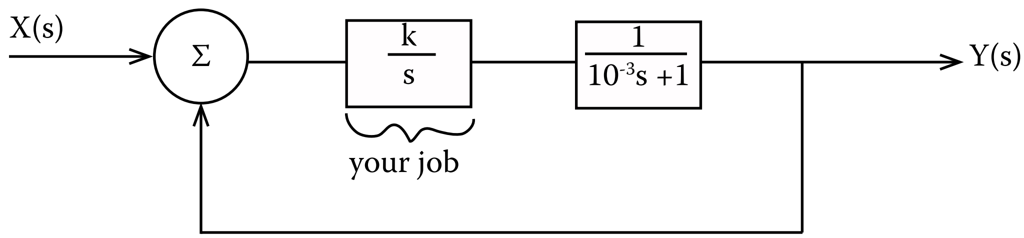
6.302 Feedback Systems

Recitation 15: More Bode Obstacle Course & Compensation
Prof. Joel L. Dawson

Today we need to wrap up some of the Bode Obstacle Course stuff that we didn't finish on Friday. Before we do, let's start with a class exercise that explores a fundamental trade-off between speed of response and stability.

CLASS EXERCISE

Once again, you're asked to control a plant as shown:



This time, you've decided that what this system needs is a pole at the origin. Choose k to meet the following requirements:

1) Such that the system has a phase margin of ≈ 90˚
2) Such that the system has a phase margin of ≈ 45˚

For which k is the system <u>faster</u>?

This problem illustrates a general property of feedback systems. You'll often hear people say things like, "For reasons related to stability, the bandwidth is limited to x."

Now, back to Bode Obstacle Course. Let's return to the example from last time:

We want to design an acceptable L(s) that results in the following closed-loop performance specs:

1) Steady-state error in response to a ramp < 1%
2) Disturbance rejection better than 10:1 for frequencies below 10 rps.
3) Closed-loop bandwidth > 50 rps
4) Magnitude peaking Mp < 1.4
5) Noise rejection better than 40 dB above 10^3 rps

How does this guide our decision?

1) Steady-state error in response to a ramp is bounded, but not zero. This implies one pole @ the origin. Let's write our loop transmission as

$$L(s) = \frac{k}{s} F(s)$$

$$\text{Where } F(s) = \frac{(\tau_{z1}s+1)(\tau_{z2}s+1)\cdots(\tau_{zN}s+1)}{(\tau_{p1}s+1)(\tau_{p2}s+1)\cdots(\tau_{pm}s+1)} \quad \Rightarrow F(0) = 1$$

In response to a ramp, steady-state error is

$$\lim_{s\to 0} s\left(\frac{1}{s^2}\right)\frac{1}{1+\frac{k}{s}F(s)} = \lim_{s\to 0} \frac{1}{s+kF(0)} = \frac{1}{k}$$

So for our first spec,

$$\frac{1}{k} < 0.01 \rightarrow k > 100$$

2) $\rightarrow |L(j\omega)| > 10$ for $\omega < 10$ rps

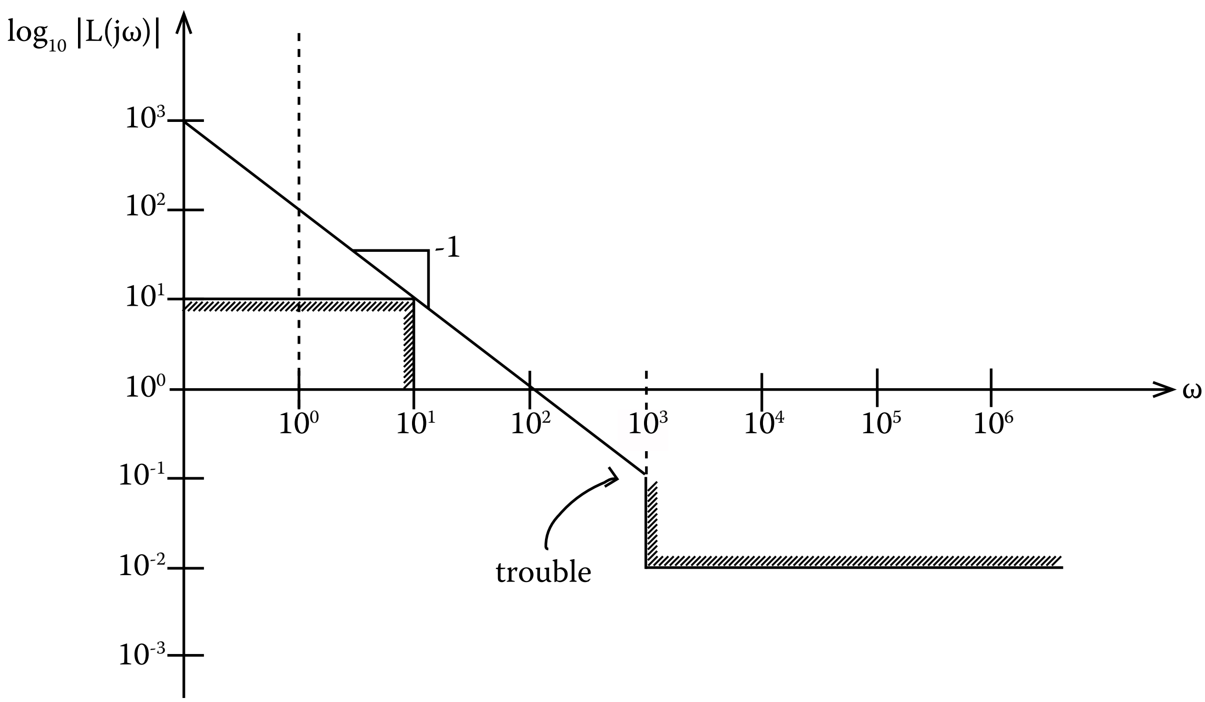
3) $\rightarrow \omega_c > 50$ rps

4) $\rightarrow \phi_m > 45^\circ$

5) $\rightarrow |L(j\omega)| < 0.01$ for $\omega > 10^3$ rps

A a first try, let's follow sound engineering judgement and with the simplest L(s) possible:

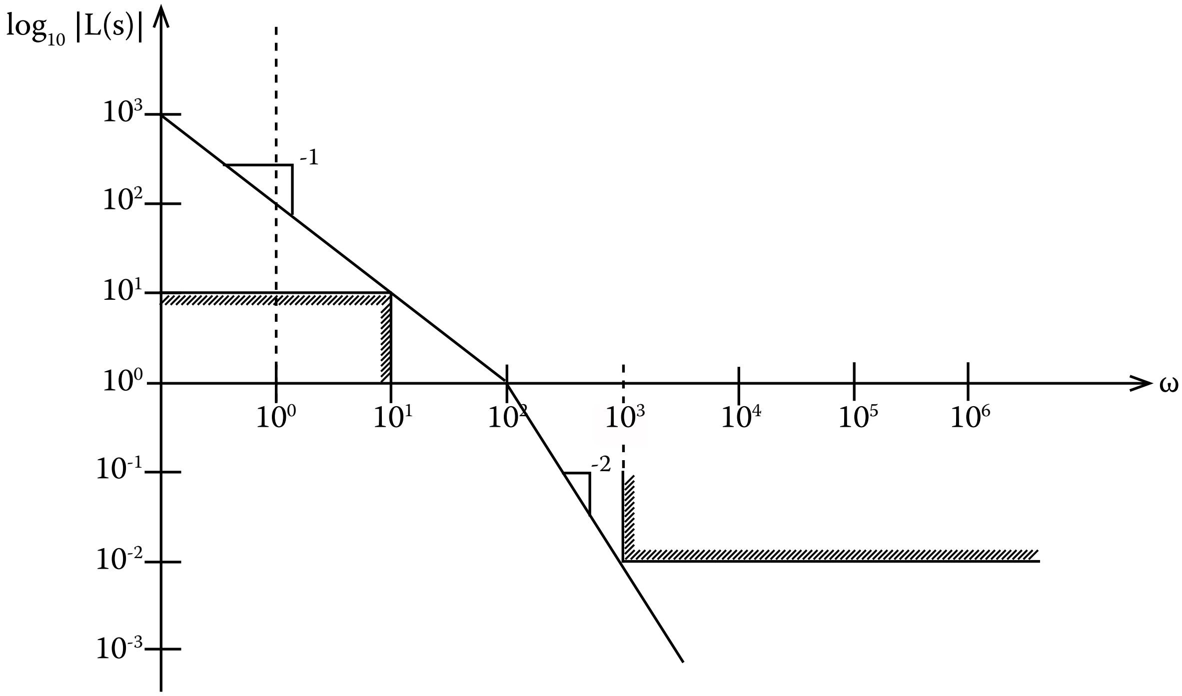
$$L(s) = \frac{100}{s} \quad .$$

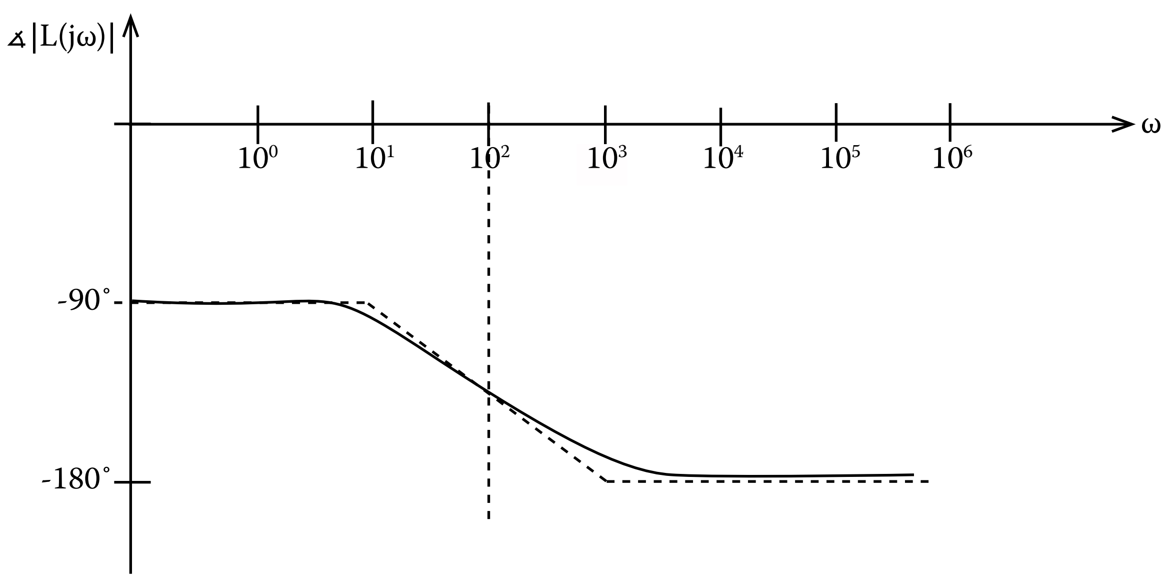


What to do? We need another pole somewhere in order to meet our high-frequency spec. But if we put the pole too low in frequency, we'll lower ω_c and our phase margin.

What about a pole right at 100 rps? Using asymptotes on the bode plot, that would fix ω_c right at 100 rps, and the phase margin would be 45˚....

$$\text{try } L(s) = \frac{100}{s(0.01s + 1)}$$





Actual numbers: $\omega_C \approx 80$ rps, $\phi_m \approx 50°$. Success!

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Remember:

- Use closed-loop specifications to place constraints on L(s)
- Capture as many of those constraints as you can as Bode Obstacles.
- Start simple, and poles/zeros as necessary.

Compensation

The Bode Obstacle course is one tool we have for doing compensation, or "the art of making things better." In our in-class exercise, we added a pole at the origin and made k as large as we could to make things better. And we noticed that there was a tradeoff between crossover frequency and stability.

So in an ideal world, what would we really want? We would want a magic box that allowed us to set its phase response independent of its magnitude response. For example, we could have arbitrary positive phase shift and a magnitude response of unity for all frequencies.

<u>NATURE DOES NOT ALLOW THIS.</u>

But it allows us something of that flavor. Consider a zero:

$$H(s) = \tau s + 1$$

$$\measuredangle H(j\omega) = \tan^{-1}(\tau\omega)$$

$$|H(j\omega)| = \sqrt{1 + (\tau\omega)^2}$$

Over the range of frequencies for which $\tau\omega << 1$:

$$\left.\begin{aligned} \measuredangle H(j\omega) &\approx \tau\omega \\ |H(j\omega)| &\approx 1 + \frac{(\tau\omega)^2}{2} \end{aligned}\right\} \text{ the phase increase is more substantial than the magnitude increase!} \rightarrow \text{ Zeros can help.}$$