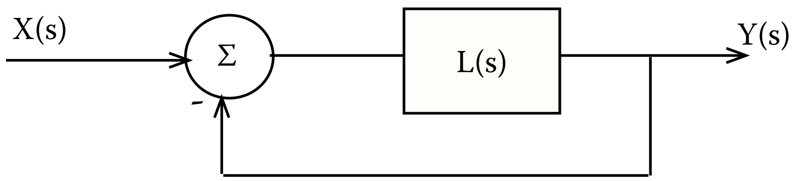
6.302 Feedback Systems

Recitation 12: Phase-locked Loops II
Prof. Joel L. Dawson

Before we jump back into learning about PLLs, we should pause to carefully understand a new term that has crept into our discussions of feedback systems. That term is "loop crossover frequency," or ω_C. What is it, and why is it important?

Consider a unity feedback system:



And we ask, "What happens to the closed-loop response at the crossover frequency?" By definition, the crossover frequency is where $|L(s)|$ is unity, so $L(j\omega_C) = e^{j\phi}$

$$\frac{Y(s)}{X(s)} = \frac{L(s)}{1+L(s)} = \frac{e^{j\phi}}{1+e^{j\phi}}$$

Needs to be massaged:

$$\frac{e^{j\phi}}{1+e^{j\phi}} \cdot \frac{1+e^{-j\phi}}{(1+e^{-j\phi})} = \frac{e^{j\phi}+1}{2+2\cos\phi}$$

Now investigate for different values of ϕ:

$\phi = 0$ => P.M. of 180˚ => $\left|\frac{1+1}{2+2}\right| = ½$

$\phi = -90˚$ => P.M. of 90˚ => $\left|\frac{-j+1}{2}\right| = ½\sqrt{(-j+1)(j+1)} = \frac{\sqrt{2}}{2}$

-3dB point!

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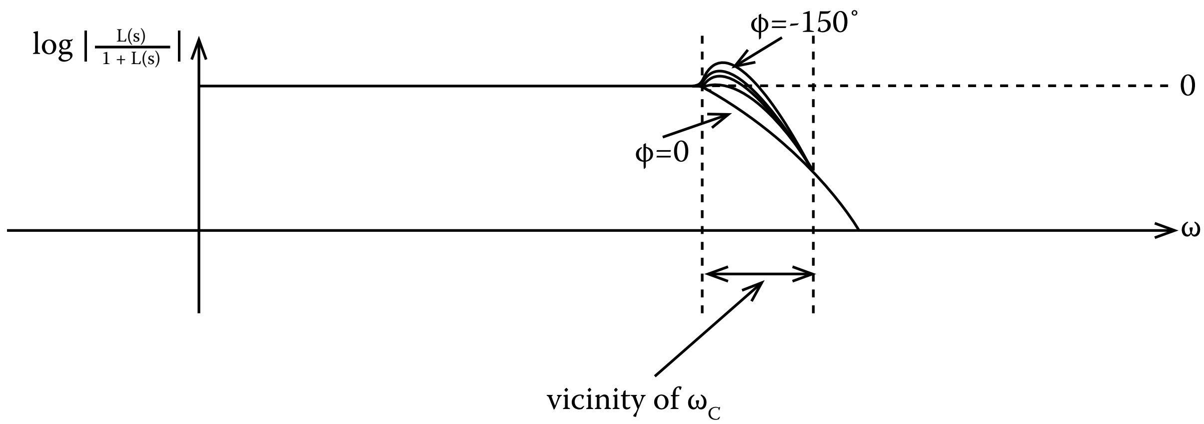
$$\phi = -120^\circ \Rightarrow \text{P.M. of } 60^\circ \Rightarrow \left| \frac{-0.5 - \frac{\sqrt{3}}{2}j + 1}{2 + 2(-.05)} \right| = 1$$

$$\phi = -135^\circ \Rightarrow \text{P.M. of } 45^\circ \Rightarrow \left| \frac{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j + 1}{2 + 2(-\frac{\sqrt{2}}{2})} \right| = 1.3$$

$$\phi = 150^\circ \Rightarrow \text{P.M. of } 30^\circ \Rightarrow \left| \frac{-\frac{\sqrt{3}}{2} - \frac{1}{2}j + 1}{2 + 2(-\frac{\sqrt{3}}{2})} \right| = 1.93$$

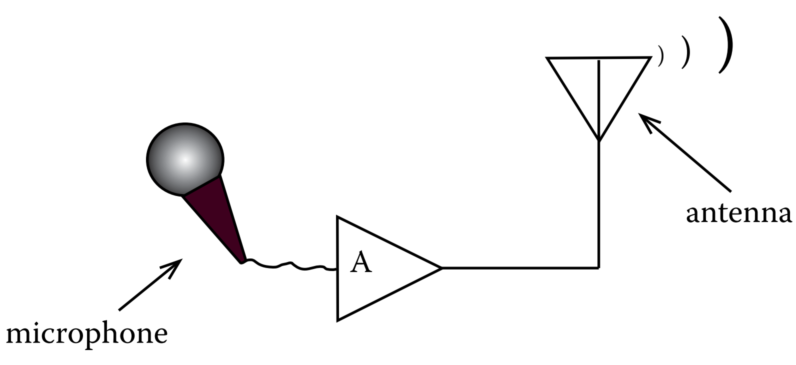


So at loop crossover, we stop getting the desired $\left|\frac{L(s)}{1+L(s)}\right| \approx 1$ behavior.

Now, let's get back to understanding phase-locked loops. How are they actually used?

EXAMPLE 1: FM Radios ("FM" = frequency modulation)

Audio frequencies fall in the region from about 20Hz up to about 20kHz. If you want to transmit this information via electromagnetic waves, your first attempt might look something like this:



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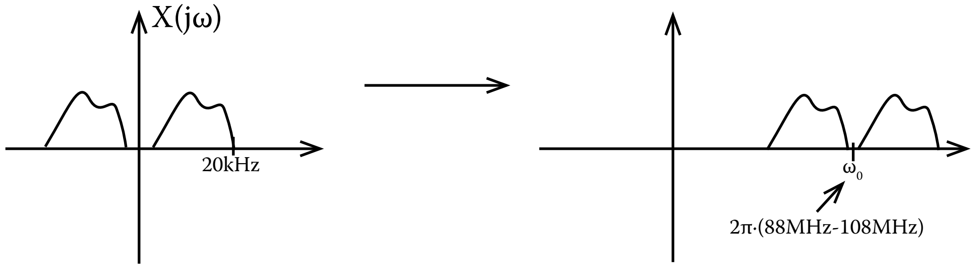
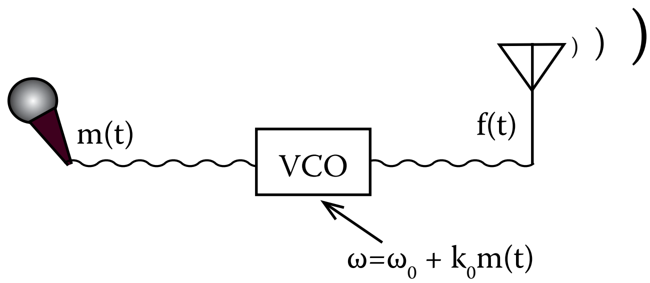
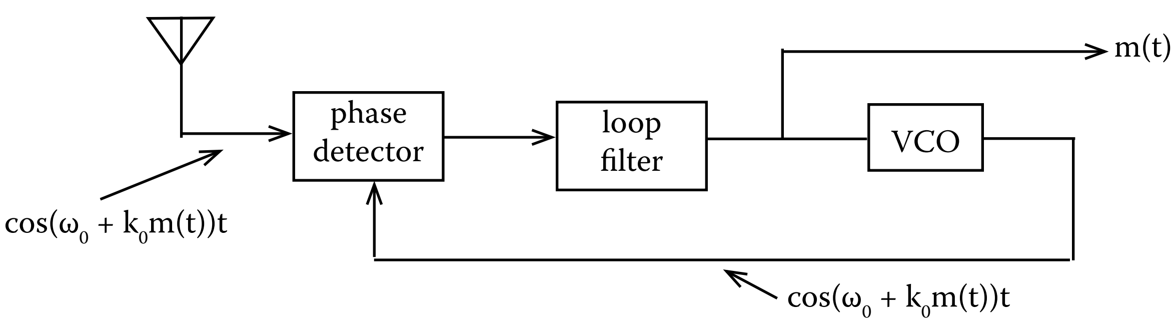
In order for this to work, the antenna would have to be HUGE...the wavelength through free space for a 1kHz E&M wave is 300 km. So we arrange to have the audio spectrum centered about a high-frequency carrier.



This frequency shift is called modulation. For an FM modulation:



How do we demodulate this signal? With a PLL of course!



For a demanding audio application like high fidelity FM, we probably want a PLL that tracks well (has zero steady-state error) and isn't wider bandwidth than necessary. Let's review our PLL performance from last recitation and see if we can improve it.

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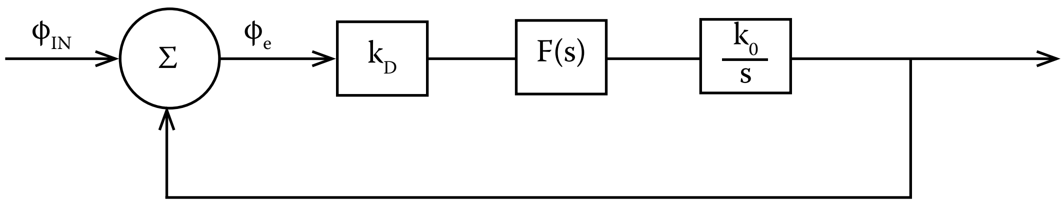
First-order Loop



For the first-order loop, we chose F(s) = 1. What did we find? We checked the steady-state error in response to a phase ramp $\phi_{IN} = \omega_0 t$ ($\longrightarrow \cos \omega_0 t$):

$$\lim_{t\to\infty} \phi_e = \lim_{s\to 0} s \cdot \frac{\omega_0}{s^2}\left(\frac{1}{1+L(s)}\right) = \lim_{s\to 0} \frac{\omega_0}{s}\left[\frac{1}{1+\frac{k_D k_0}{s}}\right]$$

$$\boxed{= \frac{\omega_0}{k_D k_0}}$$

Look at loop crossover for this PLL: $|L(s)| = 1 \rightarrow \omega_0 = k_D k_0$ so our steady-state error and bandwidth are tightly coupled. Small error implies large bandwidth → high susceptibility to noise.

How to improve? Use a ...

Second-order loop

$$F(s) = \frac{\tau s + 1}{s} \qquad\qquad L(s) = \frac{k_D k_0(\tau s+1)}{s^2}$$

What's the steady-state error?

$$\lim_{t\to\alpha} \phi_e = \lim_{s\to 0} s \cdot \frac{\omega_0}{s^2}\left[\frac{1}{1+\frac{k_D k_0(\tau s+1)}{s^2}}\right]$$

$$= \lim_{s\to 0} s \cdot \frac{\omega_0}{\cancel{s}}\left(\frac{s^{\cancel{2}}}{s^2 + k_D k_0(\tau s+1)}\right) = \underline{\underline{0}}$$

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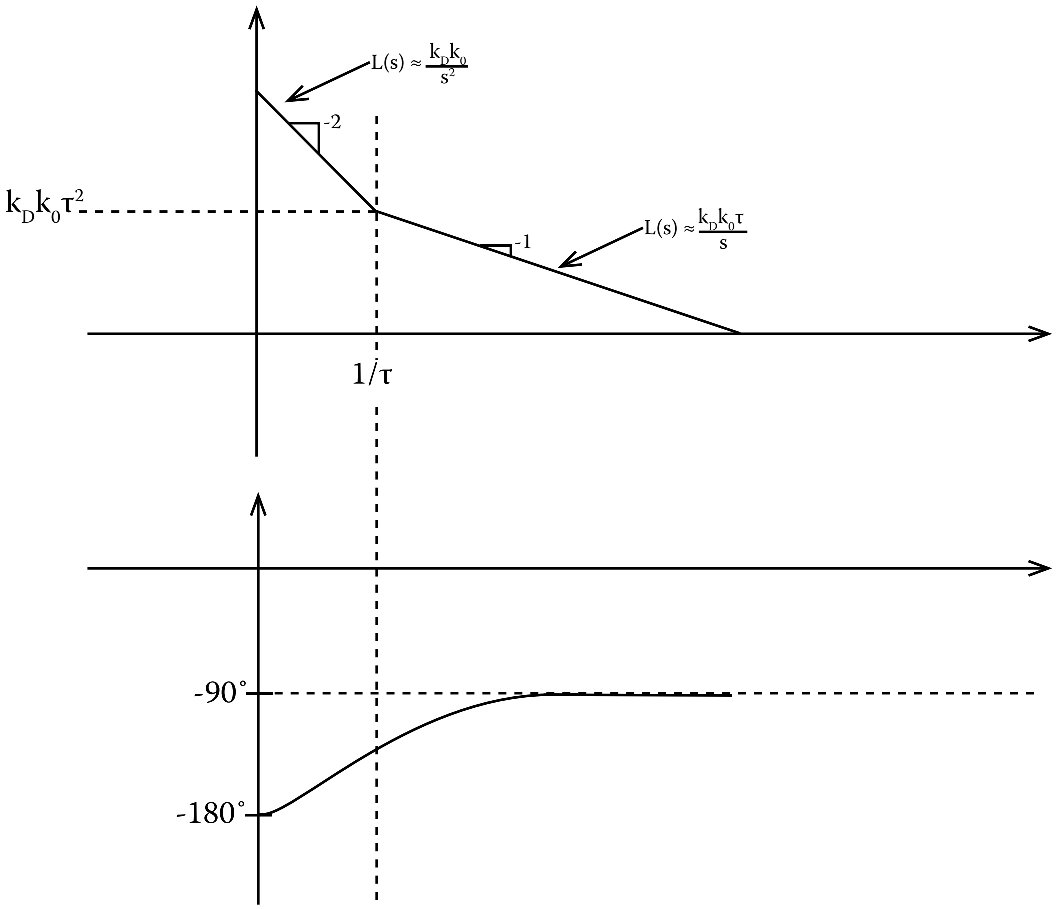
Steady-state error is perfect! What about the loop crossover frequency ω_C?



In order to have good phase margin, we will choose crossover to occur in the region where L(s) is approximated by $L(s) \approx \frac{k_D k_0 \tau}{s}$. Thus:

$$\omega_C = k_D k_0 \tau$$

Comparison:

	steady-state error	loop crossover
first-order loop	$\frac{\omega_0}{k_0 k_D}$	$k_0 k_D$
second-order loop	0	$k_0 k_D \tau$

For second order loop, zero error plus bandwidth of our choosing.

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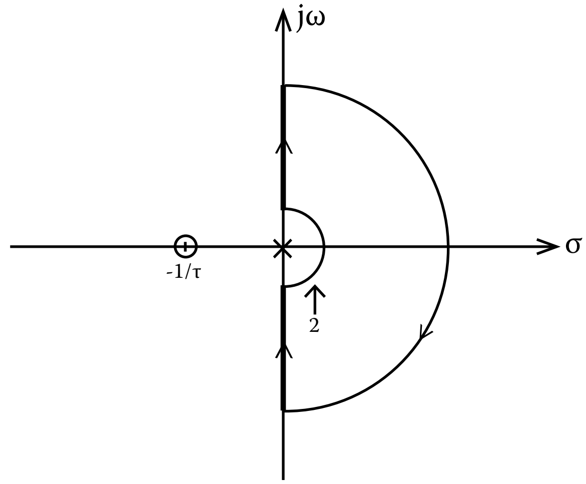
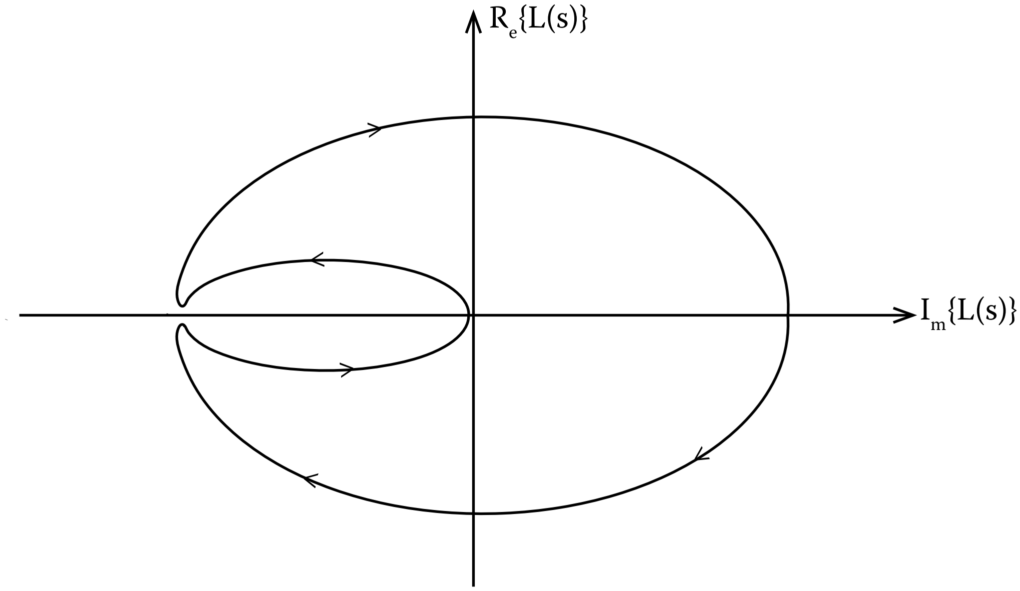
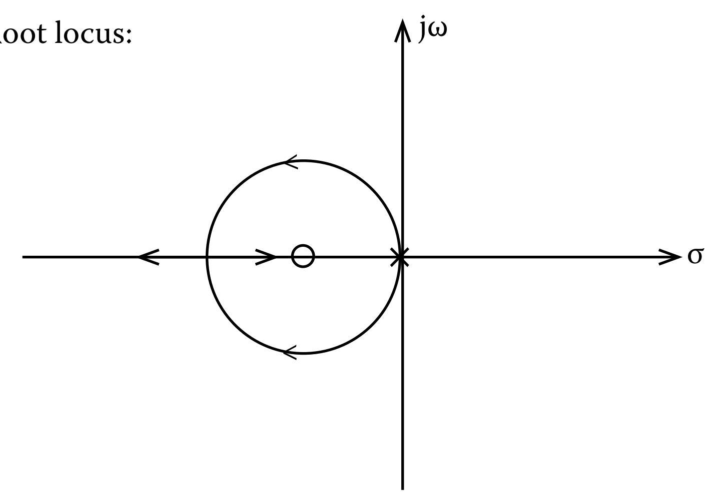
Look at Nyquist Plot for second order loop:





Root locus:



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