MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.302 Feedback Systems

Fall Term 2004 | Issued : `7:30 pm`
Quiz 1 | Due : `9:30 pm`

Midterm Quiz

October 21, 2003
120 minutes

1. This examination consists of four problems. Work all problems.
2. This examination is closed book. Helpful equations and the root-locus rules appear at the end of this packet.
3. You MUST summarize your solutions in the answer sheets included in this packet. Draw all sketches neatly and clearly where requested. Remember to label ALL important features of any sketches.
4. Make sure that your name is on each answer sheet and on each examination booklet.

We encourage you to do the work for all of the problems in the answer sheets as well. If you find that the answer sheets do not contain enough space for your scratch work, you may do additional work in the accompanying examination booklet. Make sure that you clearly denote which problem is on each page of the examination booklet. Your examination booklet will also be read by the graders, but only if your answers appear on the answer sheets.

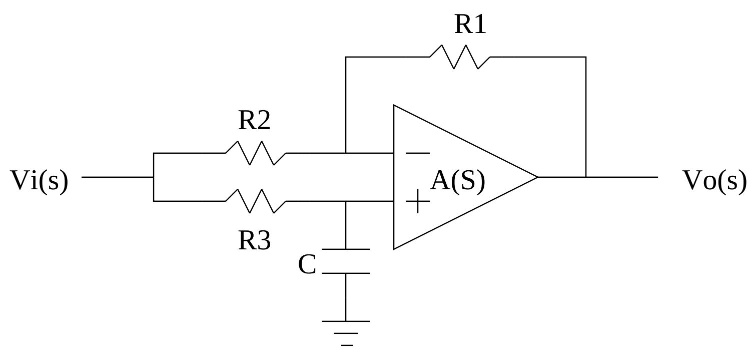
Good luck.

Problem 1 (25%)

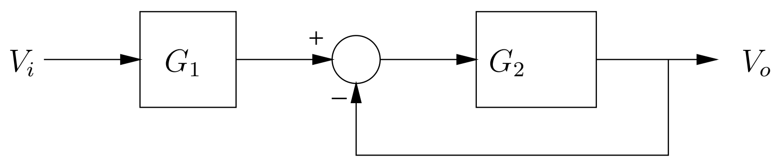
Designing with an OpAmp

For the following questions, assume that the opamp is ideal: has infinite input impedance and very high gain $A(s) = A_o$ across all frequencies.

(a) Find the transfer function, $\frac{V_o(s)}{V_i(s)}$, for the operational amplifier circuit shown below.



(b) Draw the block diagram for the circuit and manipulate it into unity feedback form. What are $G_1(s)$ and $G_2(s)$?

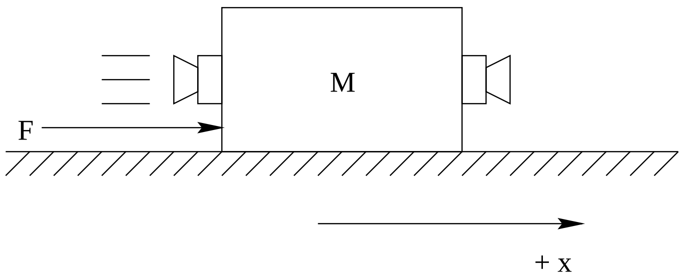


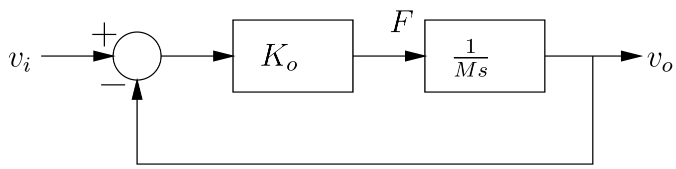
(c) Let $R_1 = 10K\Omega$, $R_2 = 10K\Omega$, $R_3 = 100K\Omega$, $C = 1\mu F$, and sketch the step response of the system. Make sure to label the **Initial** and **Final** values, the **Initial slope** and approximate **risetime**.

Problem 2 (25%)

Rocket Sled

Consider a rocket-propelled mass resting on a frictionless surface. We wish to control the velocity of the block, and adopt a feedback control strategy as shown below.





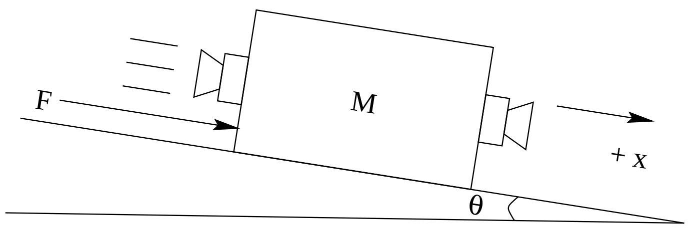
Assume that a velocity sensor is available, and that the mass of the block is constant.

(a) Sketch the response of the system to a step command input, $u(t)$, and compute the steady-state error between the input and output velocities.

(b) After trying out the control strategy, measurements indicate that wind resistance is non-negligible. If the force of the wind resistance is equal to Bv_o, redraw the block diagram to reflect this new effect.

(c) Compute the new steady-state error in response to a step input in velocity.

(d) Now, suppose that the command velocity input is held at zero, and that the system is at rest. Then, the surface is **abruptly** tilted at an angle Θ as shown.

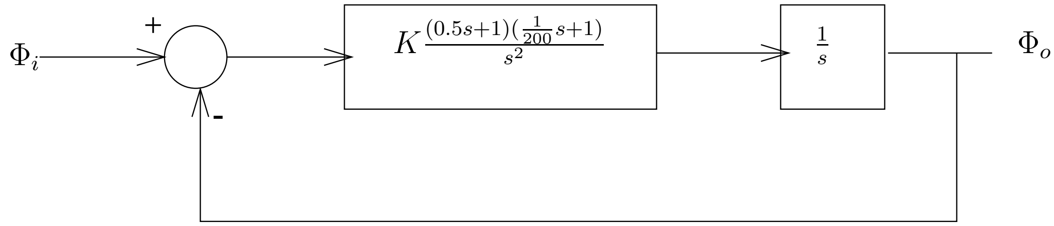


Sketch the resulting velocity v_o. Ignore friction. Use the model of the system from part (a) with any appropriate addition(s). Clearly label the plot with **Initial**, **Final** values and show any other features of interest.

Problem 3 (25%)

Feedback PLL

A phase-lock loop has the block diagram shown.



(a) Draw the root locus diagram for the closed-loop poles of the system for positive K. Label features of the plot such as centroid, asymptotes, angles of departure from multiple poles etc. You DO NOT need to compute point(s) of entry/departure on the real axis (too grungy).

(b) Assume that K is set so that there is a closed-loop pair of poles on the imaginary axis at $s = \pm j\Omega_c$.

What is the value of K?

What is Ω_c?

(c) K is made K_o, and this value results in 3 closed-loop poles in the left-half plane. What steady-state errors result for the following inputs?

1. unit step ($\Phi_i = u(t)$, $t > 0$)
2. unit ramp ($\Phi_i = t$, $t > 0$)
3. unit parabola ($\Phi_i = \frac{t^2}{2}$, $t > 0$)
4. unit cubic ($\Phi_i = \frac{t^3}{6}$, $t > 0$)

Problem 4 (25%)

Assessing system stability via Nyquist and root locus

A feedback system has a forward path $KG(s)$ and unity feedback. All poles and zeros of $G(s)$ are on the real axis with the real part, $\sigma \leq 0$.

- For positive K: the system is stable for $0 < K < K_1$, has two poles in the RHP for $K_1 < K < K_2$, is stable for $K_2 < K < K_3$, and has two poles in the RHP for all $K > K_3$.
- For all negative K: the system has one closed-loop pole in the RHP.

(a) What is the minimum number of poles, P, and zeros, Z, of $G(s)$?

(b) Draw a Nyquist plot for this system for P and Z of part (a). Be sure to indicate the value of $G(s)$ where the plot crosses the real axis.

(c) Sketch a possible root locus diagram for positive K assuming P and Z as found in part (a). (Note: it is a sketch so you do NOT need to compute asymptote angles, points of entry/departure on the real axis etc.)

(d) Sketch a possible root locus diagram for negative K assuming P and Z as found in part (a).