6.302 Feedback Systems

The prerequisites for 6.302 are linear systems, circuits, physics, and math. Officially, this requirement is 6.003 or 2.003 or 16.040. Previous laboratory experience with circuits is assumed, such as in 6.002 or 6.071 or 16.040. We also assume that you have not forgotten 8.01 and 8.02, and that you have had some exposure to complex variables (18.04 is helpful, but overkill). This problem set should remind you of what you already know. You should feel comfortable with the following problems if you intend to take 6.302.

**Problem 1:** Sketch (by hand) the Bode plot (magnitude and phase) for each of the following transfer functions.

$$\text{(a) } H_1(s) = \frac{1}{s+1} \qquad \text{(b) } H_2(s) = \frac{s}{s+1} \qquad \text{(c) } H_3(s) = \frac{(s+1)(s+10)}{s(0.01s+1)(0.001s+1)}$$

Problem 2: For a single-pole system of the form

$$A(s) = \frac{10^6}{\tau s + 1}$$

express the following quantities in terms of the time constant (τ): f_b, the 3dB bandwidth in hertz (the frequency at which the magnitude of the frequency response is 0.707 of the DC gain), and t_r, the 10%–90% rise time of the step response.

Problem 3: Find the transfer function $\frac{V_o}{V_i}(s)$ of the following op amp circuit. Assume the op amp is ideal. Use the form ($\tau s + 1$) for the poles and zeros.
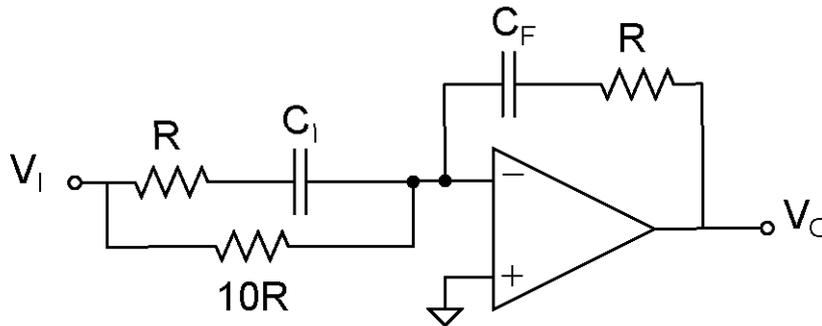
Problem 4: Use the Final Value Theorem to find the steady state response of the following transfer function to an input unit ramp.

$$\frac{E}{R}(s) = \frac{s(s+1)}{s^2 + 2s + 100}$$

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Problem 5: Match each step response to the appropriate pole-zero plot in the figure below.
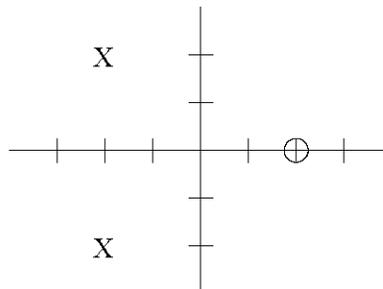
(a)

(1)

(b)

(2)

(c)

(3)

(d)

(4)
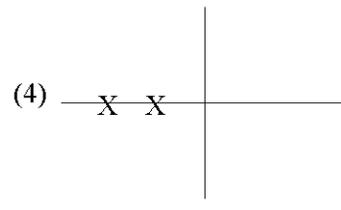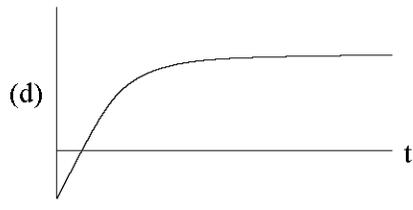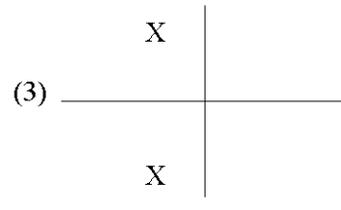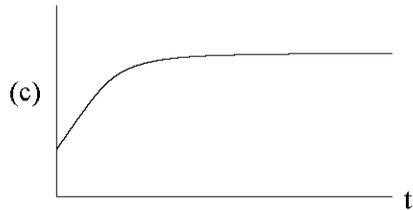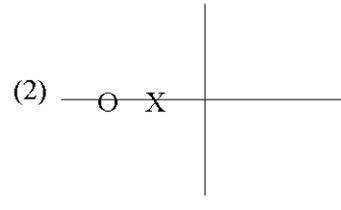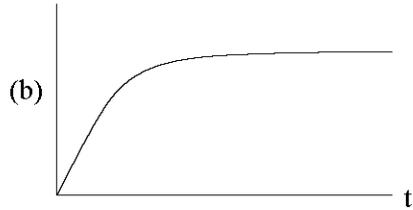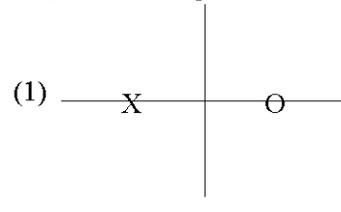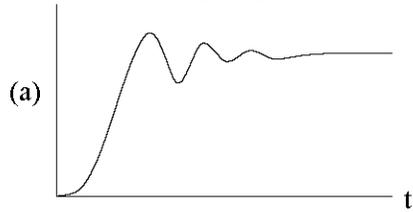
Problem 6: Draw a pole-zero plot for $F(s)$

$$F(s) = \frac{(s-2)(s^2 - 2s + 2)}{s(s+2)(s^2 + 2s + 2)}$$

Using geometry alone, find the angles $\angle F(-3)$, $\angle F(-1)$, $\angle F(1)$, and $\angle F(3)$.

Problem 7: You are given the following pole-zero plot for $G(s)$ and told that $|G(0)| = 2$. Using geometry alone, find $|G(j2)|$.

Problem 8: Find a simple value for

$$p = \frac{2}{j} \ln(j)$$

Problem 9: A solid disc of radius r and mass m, initially at rest, is acted on by a tangential force F. What is the angular velocity $\omega(t)$?

Problem 10: Draw the following plots by hand for the function $L(s)$. Do not use Matlab.

$$L(s) = \frac{100}{s^2 + 2s + 100}$$

(a) The pole-zero plot.

(b) The Bode plot (magnitude versus frequency and angle versus frequency). Be sure to note where the real plot deviates from the straight-line asymptotes.

(c) The polar plot (where the radius $r = |L(j\omega)|$ is the distance from the origin, the angle $\theta = \angle L(j\omega)$ is the angle with respect to the positive real axis, and ω is the parameter).

(d) The gain/phase plot (a plot of $\log_{10} |L(j\omega)|$ versus $\angle L(j\omega)$: this plot can be constructed by examining the above Bode plot and noting how the magnitude and angle change as frequency increases).

(e) Finally, once you have constructed the above plots by hand, generate them with Matlab. You may find the following Matlab commands helpful: `help, plot, polar, abs, angle,` and `bode.`

Comment on the differences between your plots and the Matlab plots (differences are okay, in fact, if there aren't differences, we'll assume that you used Matlab for parts (a)–(d) and deduct points...).