Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.111 - Introductory Digital Systems Laboratory

Problem Set 1 Solutions

Issued: Lecture 4 Day

Problem 1:

Not Graded.

1) $a + 0 = a$
2) $\overline{a}.0 = 0$
3) $a + \overline{a} = 1$
4) $a + a = a$
5) $a + ab = a(1 + b) = a$
6) $a + \overline{a}b = (a + \overline{a})(a + b) = a + b$
7) $a(\overline{a} + b) = a\overline{a} + ab = ab$
8) $ab + \overline{a}b = b(a + \overline{a}) = b$
9) $(\overline{a} + \overline{b})(\overline{a} + b) = \overline{a}\overline{a} + \overline{a}b + \overline{b}\overline{a} + \overline{b}b = \overline{a} + \overline{a}b + \overline{a}\overline{b} = \overline{a}(1 + b + \overline{b}) = \overline{a}$
10) $a(a + b + c...) = aa + ab + ac + ... = a + ab + ac + ... = a$
11) $f(a, b, ab) = a + b + ab = a + b$
12) $f(a, b, \overline{a}\overline{b}) = a + b + \overline{a}\overline{b} = a + b + \overline{a} = 1$
13) $f(a, b, \overline{(ab)}) = a + b + \overline{(ab)} = a + b + \overline{a} + \overline{b} = 1$
14) $y + y\overline{y} = y$
15) $xy + x\overline{y} = x(y + \overline{y}) = x$
16) $\overline{x} + y\overline{x} = \overline{x}(1 + y) = \overline{x}$
17) $(w + \overline{x} + y + \overline{z})y = y$
18) $(x + \overline{y})(x + y) = x$
19) $w + (w + (wx)) = w$
20) $x(x + (xy)) = x$
21) $\overline{(\overline{x} + \overline{x})} = x$
22) $\overline{(x + \overline{x})} = 0$
23) $w + (w\overline{x}yz) = w(1 + \overline{x}yz) = w$
24) $\overline{w}\overline{(wxyz)} = \overline{w}(\overline{w} + \overline{x} + \overline{y} + \overline{z}) = \overline{w}$
25) $xz + \overline{x}y + zy = xz + \overline{x}y$
26) $(x + z)(\overline{x} + y)(z + y) = (x + z)(\overline{x} + y)$
27) $\overline{x} + \overline{y} + xy\overline{z} = \overline{x} + \overline{y} + \overline{z}$

Problem 2:

1. $$f = (a + (\bar{b} + \bar{c})) \cdot (\bar{c} + (a + b + d) \cdot (\bar{a} + \bar{b} + \bar{d}))$$

i) truth table

a	b	c	d	f
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

ii) Karnaugh map

cd \ ab	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	0	0	1
10	0	0	1	1

iii) MPS

cd \ ab	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	0	0	1
10	0	0	1	1

$$f = \bar{c} + \bar{b} \cdot d + a \cdot \bar{d}$$

iv) MSP

cd \ ab	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	0	0	1
10	0	0	1	1

$$f = (\bar{b} + \bar{c} + \bar{d}) \cdot (a + \bar{c} + d)$$

2.

$$f = (\bar{c} + a \cdot b) \cdot (\bar{c} + (a + \bar{d}) \cdot (b + \bar{d})) \cdot (c + (a + \bar{b}) \cdot (b + \bar{d}))$$

i) truth table

a	b	c	d	f
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

ii) Karnaugh map

cd \ ab	00	01	11	10
00	1	0	1	1
01	0	0	1	0
11	0	0	1	0
10	0	0	1	0

iii) MSP

cd \ ab	00	01	11	10
00	1	0	1	1
01	0	0	1	0
11	0	0	1	0
10	0	0	1	0

$$f = a \cdot b + \bar{b} \cdot \bar{c} \cdot \bar{d}$$

iv) MPS

cd \ ab	00	01	11	10
00	1	0	1	1
01	0	0	1	0
11	0	0	1	0
10	0	0	1	0

$$f = (a + \bar{b}) \cdot (b + \bar{d}) \cdot (b + \bar{c})$$

3.

$$f = \overline{w} \cdot y + w \cdot \overline{x} \cdot y + \overline{w} \cdot x \cdot \overline{z}$$

i) truth table

w	x	y	z	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

ii) Karnaugh map

yz \ wx	00	01	11	10
00	0	1	0	0
01	0	0	0	0
11	1	1	0	1
10	1	1	0	1

iii) MSP

yz \ wx	00	01	11	10
00	0	1	0	0
01	0	0	0	0
11	1	1	0	1
10	1	1	0	1

$$f = \overline{w} \cdot x \cdot \overline{z} + \overline{x} \cdot y + \overline{w} \cdot y$$

iv) MPS

yz \ wx	00	01	11	10
00	0	1	0	0
01	0	0	0	0
11	1	1	0	1
10	1	1	0	1

$$f = (\overline{w} + \overline{x}) \cdot (y + \overline{z}) \cdot (x + y)$$

Problem 3:

1. MSP

a)

cd \ ab	00	01	11	10
00	0	0	0	1
01	0	0	0	X
11	0	0	0	1
10	0	0	0	1

$$f = a \cdot \bar{b}$$

b)

cd \ ab	00	01	11	10
00	0	0	X	1
01	0	0	0	0
11	0	0	0	1
10	0	0	X	1

$$f = a \cdot \bar{d} + a \cdot \bar{b} \cdot c$$

2. MPS

a)

cd \ ab	00	01	11	10
00	0	0	0	1
01	0	0	0	X
11	0	0	0	1
10	0	0	0	1

$$f = a \cdot \bar{b}$$

b)

cd \ ab	00	01	11	10
00	0	0	X	1
01	0	0	0	0
11	0	0	0	1
10	0	0	X	1

$$f = a \cdot \bar{b} \cdot (c + \bar{d})$$

3. The solutions are unique given that we want the minimal equations.

4. The MSP and MPS in part a are equal. The MSP and MPS in part b are not equal as the don't cares in part 1 are assumed to be 1 and the don't cares in part 2 are assumed to be 0.

Problem 4:

1. $$\overline{\overline{(\bar{a} + c)} \cdot \overline{(b + c)}} = (\bar{a} + c) + (b + c) = \bar{a} + b + c$$

2. $$\overline{a \cdot b \cdot \bar{c}} = \bar{a} + \bar{b} + c$$

3. $$\overline{(b + \bar{c})} \cdot \overline{(\bar{a} + c)} \cdot \overline{(\bar{a} + \bar{b})} = (\bar{b} \cdot c) \cdot (a \cdot \bar{c}) \cdot (a \cdot b) = 0$$

Problem 5:

```
library ieee;
use ieee.std_logic_1164.all;

entity pset_1_problem_5 is
  port (
    a, b, c, d : in  std_logic;
    p1, p2     : out std_logic);
end pset_1_problem_5;

architecture structure of pset_1_problem_5 is
begin

  p1 <= (a and c) or (not a and (b or not c));

  p2 <= (not b and not c and d)
          or (not a and b and d)
          or (a and not c and d)
          or (not a and not c and not d);

end structure;
```