Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.061/6.690 Introduction to Power Systems

Problem Set 9 Solutions April 18, 2011

Problem 1: Chapter 9, Problem 5 The phasor diagram is shown in Figure 1.

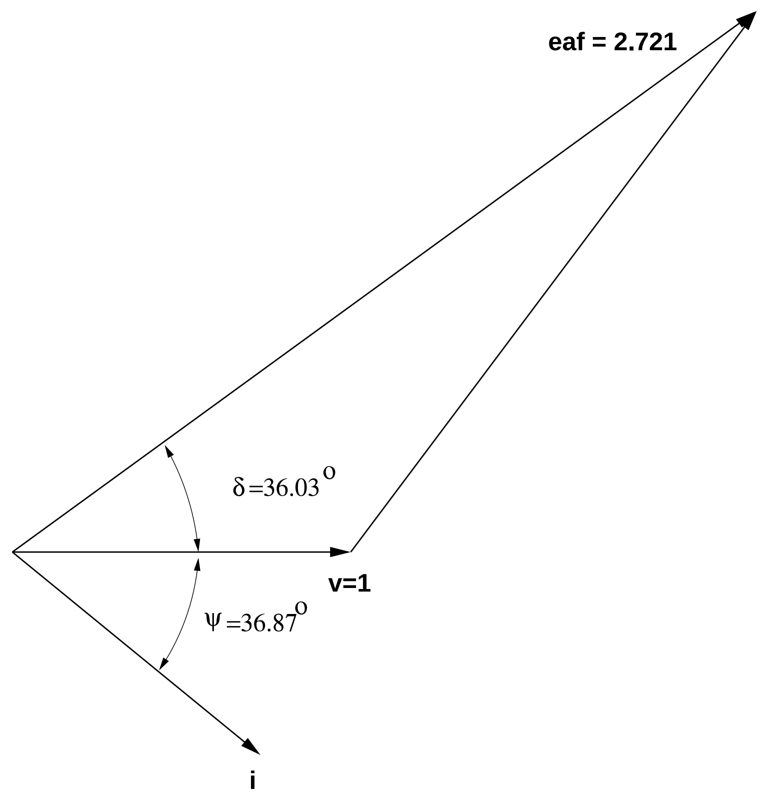


Figure 1: Voltage Vector Diagram of part a

To start, the power factor angle is $\psi = cos^{-1}(0.8) \approx 36.87°$. We can use the law of cosines to find required internal voltage:

$$e_{af}^2 = v^2 + (xi)^2 + 2vxi \sin\psi$$

or, equivalently, we could use the pythagorean theorem:

$$e_{af}^2 = (v + xi\sin\psi)^2 + (xi\cos\psi)^2$$

which are transparently identical.

Torque angle is, pretty directly:

$$\delta = \tan^{-1} \frac{xi \cos\psi}{v + xi \sin\psi}$$

These evaluate to:

$$\begin{aligned} e_{af0} &= \sqrt{1 + 2^2 + 2 \times 2 \times 0.6} \approx 2.721 \\ \delta &= tan^{-1} \frac{.6}{1 + 1.2} \approx 36.03^\circ \end{aligned}$$

If $I_{fnl} = 1000A$, then required field current for that operating point would be 2721 A.

Part b:

Since $p = \frac{ve_{af}}{x_d} \sin\delta$, if we vary field current at constant load:

$$\delta = \sin^{-1} \frac{px_d}{ve_{af}}$$

Note that we are using the symbol p to indicate per-unit real power. The stability limit is reached when $px_d = ve_{af}$, or $e_{af} = 1.6$ per-unit. That makes $I_f = 1,600A$. Angle vs. field current is shown in Figure 2

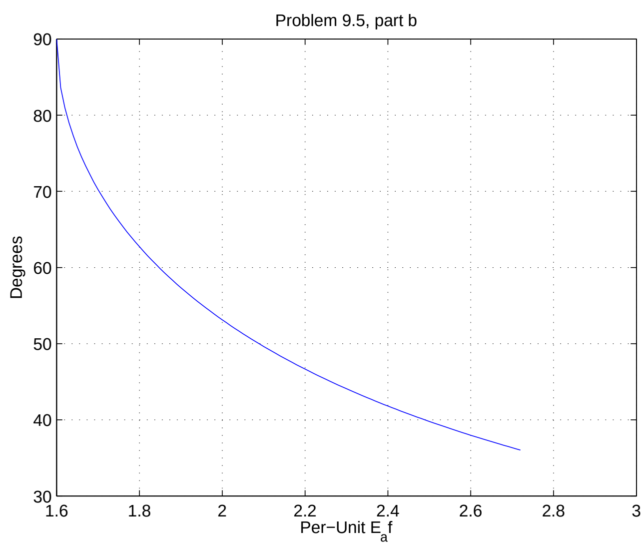


Figure 2: Problem 9.5, Part b

Parts c-e The rest of this problem is carried out in the script that is appended. First, the vee curve from the point we just computed to the overexcited field limit is shown in Figure 3.

The zero-power vee curve has zero real power, so all that is interesting is reactive power: $q = \frac{ve_{af} - v}{x_d}$, and for zero real power the absolute value of this is also per-unit armature current (if $v = 1$).

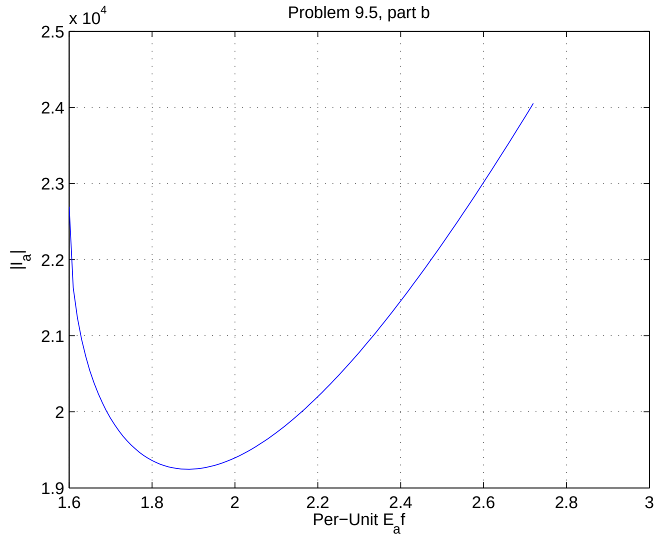


Figure 3: Problem 9.5, part c

Finding the rest of the curves is straightforward: the beginning of the curve is at an angle determined by the real power and by the maximum excitation. The end of the curve is at the stability limit, which for a round rotor machine is at an angle of 90 degrees.

$$\begin{aligned}\delta_{min} &= sin^{-1}\frac{px_d}{ve_{af0}} \\ \delta_{max} &= \frac{\pi}{2}\end{aligned}$$

Then we set up a vector of closely spaced points in angle, find required value of internal voltage to make the right amount of real power and then compute reactive power:

$$\begin{aligned}e_{af} &= \frac{px_d}{v\sin\delta} \\ p &= \frac{ve_{af}}{x_d}\sin\delta \\ q &= \frac{ve_{af}}{x_d\cos\delta} - \frac{v^2}{x_d} \\ i_a &= \sqrt{p^2+q^2}\end{aligned}$$

This calculation can be vectorized in MATLAB, and the set of vee curves that result is shown in figure 4.

Finally, the capability curve can simply be sketched. The vector shown is drawn to the 'rating point', or $p = .8$, $q = .6$. The other limits are just sections of circles. This is shown in Figure 5

Here is the script:

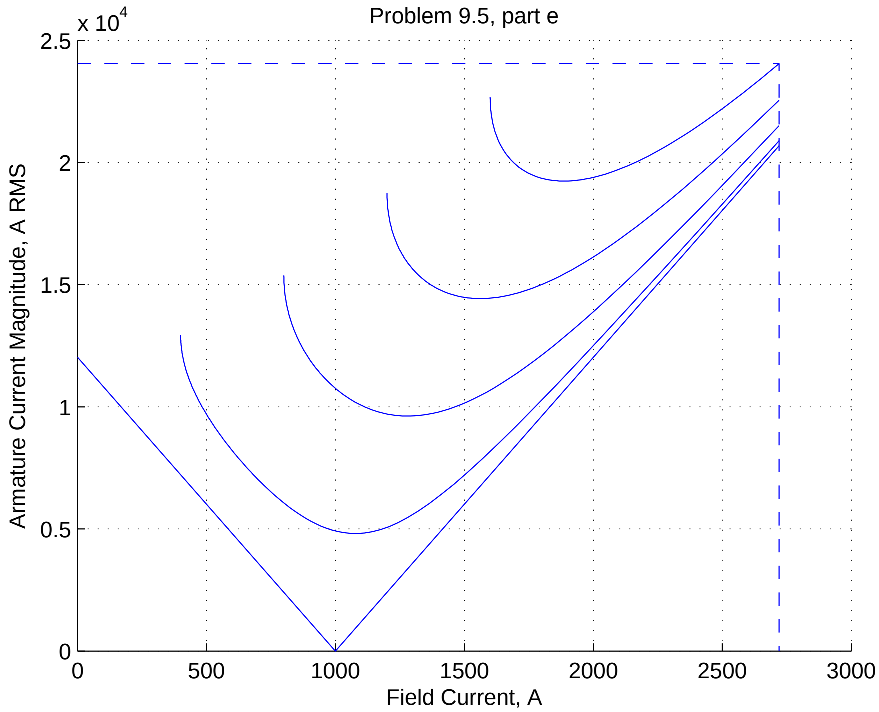


Figure 4: Vee Curves: Part d and e of Problem 9.5

```
% Problem Set 9, Problem 1: Round Rotor Machine

xd = 2;                  % synchronous reactance
Pb = 1e9;                % rated power
Vb = 24000;              % rated terminal voltage, line-line, RMS
Ifnl = 1000;             % field current for unity internal voltage
Ib = Pb/(sqrt(3)*Vb);     % base current
pf = .8;                 % can reach this power factor at rated VA

% Part a:
psi = acos(pf);          % this is the rated power factor angle
eaf = sqrt(1+xd^2+2*xd*sin(psi));    % law of cosines calculation
vr = xd*sin(psi);
vi = xd*cos(psi);

fprintf('eaf = %g  vr = %g  vi = %g\n', eaf, vr, vi)
eang = (180/pi)*atan(vi/(1+vr));
vang = (180/pi)*atan(vi/vr);
fprintf('angles: eaf = %g  vx = %g\n', eang, vang)
```

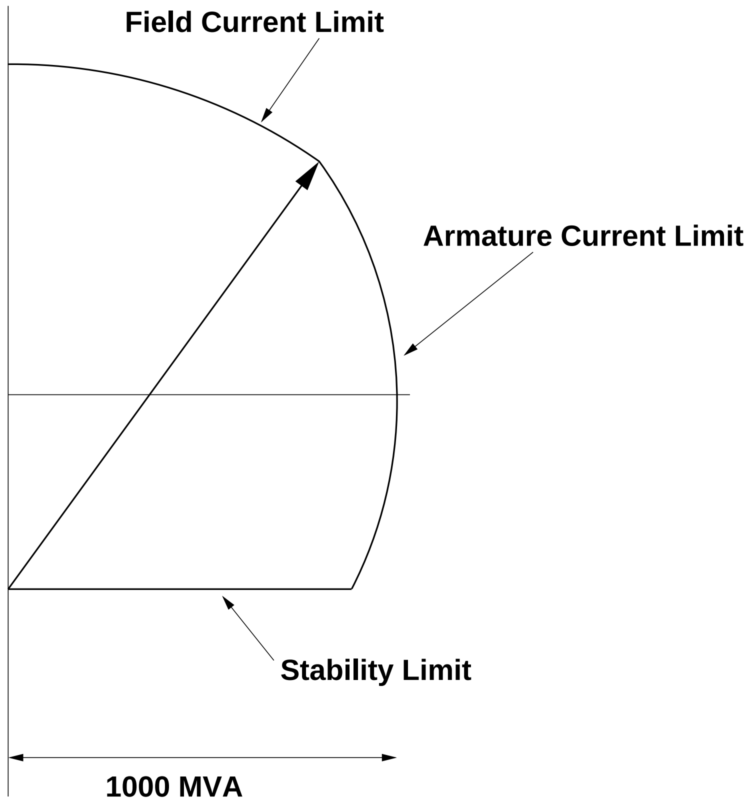
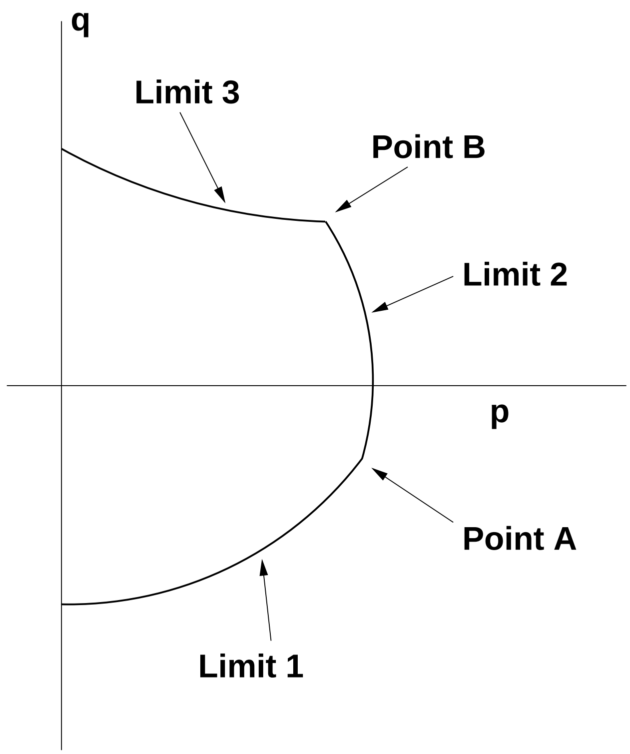


Figure 5: Capability Curve for Problem 9.5

```
% part b:
eafmin = pf*xd;         % this will be the stability limit
Eaf = eafmin:.01:eaf;  % to generate a plot
Delta = asin(xd*pf./ Eaf);
Deltad = (180/pi) .* Delta;

figure(1)
plot(Eaf, Deltad)
title('Problem 9.5, part b')
ylabel('Degrees')
xlabel('Per-Unit E_af')
grid on

% part c:
ppu = Eaf .* sin(Delta) ./ xd;
qpu = Eaf .* cos(Delta) ./ xd -1/xd;
I_a = Ib .* sqrt(ppu .^2 + qpu .^2);

figure(2)
plot(Eaf, I_a)
```

```
title('Problem 9.5, part b')
ylabel('|I_a|')
xlabel('Per-Unit E_{af}')
grid on

% pard d and e: generate vee curve
% first, get zero power curve: need only corners
iamax = (eaf-1)/xd;
iamin = 1/xd;
I_az = Ib .* [iamin 0 iamax];
E_az = Ifnl .* [0 1 eaf];

figure(3)
clf
hold on

plot(E_az, I_az)

P_pu = [.2 .4 .6 .8];

for k = 1:length(P_pu)
    p = P_pu(k);
    deltm = asin(xd*p/eaf);  % delta for the top of each curve
    Delt = deltm:.01:pi/2;   % from there to stability limit
    E = p*xd ./sin(Delt);    % required per-unit excitation
    ppu = E .* sin(Delt) ./ xd;
    qpu = E .* cos(Delt) ./ xd - 1/xd;
    I_a = Ib .* sqrt(ppu .^2 + qpu .^2);
    I_f = Ifnl .* E;
    plot(I_f, I_a)
end
% now, just to be fancy, we generate the limits
ylim = Ib .* [1 1 0];
xlim = Ifnl*eaf .* [0 1 1];
plot(xlim, ylim, '--')

hold off
grid on
title('Problem 9.5, part e')
ylabel('Armature Current Magnitude, A RMS')
xlabel('Field Current, A')
```

Problem 2: Chapter 9, Problem 6 Revisited Here we find the capability curve for this salient pole machine. As with the round rotor machine, we have three basic limits to capability:



1. Field Current Limit (heating)
2. Armature Current Limit (heating)
3. Underexcited Stability Limit

These are shown in Figure . The three limits are called out with their numbers as above. There are two interesting points that are the places where two limts meet: Point A is the point that field and armature current limits have in common. It is also the point that establishes machine rating. Point B is where armature current limit is also at the stability limit. This problem, too, is carried out using the script appended.

To start, use the vector diagram for salient pole motor operation shown in Figure 6, which is also Figure 9.10 of the text.

Point A

Assuming this is operating at rated armature current (Per-unit $i = 1$), rated internal voltage and torque angle are:

$$\begin{aligned}
\psi &= cos^{-1} PF \\
\underline{i} &= i \cos \psi + j i \sin \psi \\
\underline{e}_1 &= v - j x_q \underline{i} \\
\delta &= \angle \underline{e}_1
\end{aligned}$$

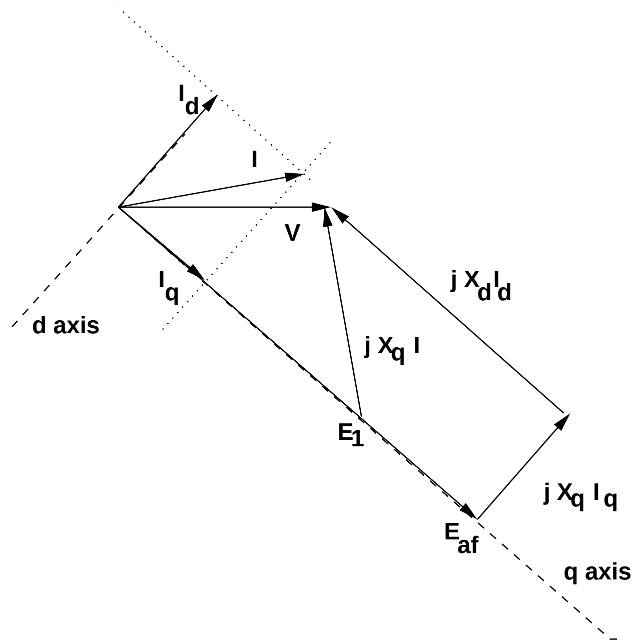


Figure 6: Vector Diagrem for Salient Pole Machine

$$
\begin{aligned}
i_d &= i \sin(\psi - \delta) \\
e_{af} &= |\underline{e}_1 + (x_d - x_q)\, i_d
\end{aligned}
$$

Actually, what is calculated is actually the negative of i_d, but that is just a little bit more convenient.

Point B

This is the intersection of the stability limit, where

$$e_{af} = v \left(1 - \frac{x_d}{x_q}\right) \frac{\cos 2\delta}{\cos \delta}$$

and the armature current limit, where —p+jq— = 1;

As it turns out, this can be found by searching over the angle δ and, calculating e_{af} for each value of δ, finding where armature current is rated.

These two points give us the appropriate limits. Then for segment 1, we set up a vector of closely spaced points in angle from $\delta_A < \delta < 0$. (Note that δ_A is negative.) Internal voltage is constant, and real and reactive power are calculated.

For segment 2, between $\delta_B < \delta < \delta_A$, it is straightforward to set up a vector of closely spaced points in power factor angle ψ, and $p = \cos\psi$ and q $= \sin\psi$.

The last segment is actually most interesting.As it turns out, the angle does go back to zero and internal field voltage crosses zero and becomes negative. The stability limit is as we calculated it above, and that can be calculated for each of a closely spaced vector of points: $\delta_B < \delta < 0$. Real and reactive power are calculated at each point.

To generate the plot, the three segments are plotted separately, and the result is shown in Figure 7.

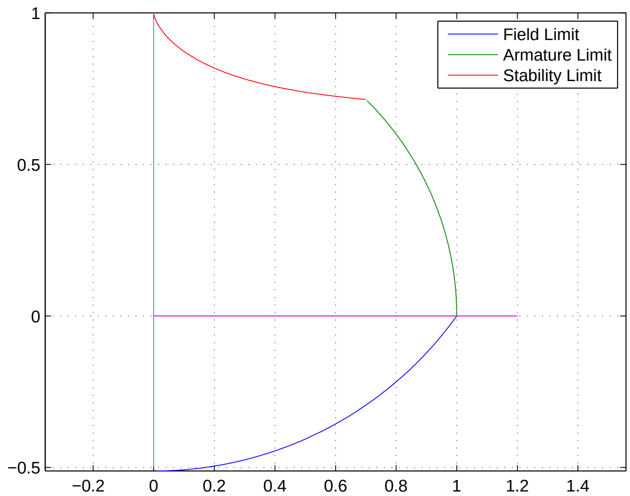


Figure 7: Capability Curve for Salient Pole Machine (per-unit)

Script for Problem 2

```
% Capability Curve for a Salient Generator

global xd xq v eq d
%per-unit
xd = 1.5;
xq = 1.0;
v=1;
Pb = 1e6;
Vb = 4200/sqrt(3);
Ib = Pb/(3*Vb);
Ifnl = 100;
pf0 = 1.0;
% step 1: find rated eaf and angle at rating point (Point A)
psi0 = acos(pf0);

Im = 1;
I = Im * (cos(psi0) + j*sin(psi0));

e1 = v - j*xq*I;
```

```
delt0 = angle(e1);
I_d = Im * sin(psi0-delt0);       % actually -I_d
eaf0 = abs(e1) + I_d * (xd - xq);
fprintf('Point A: eaf = %g  delta = %g\n', eaf0, delt0)

% step 2 find point B Intersection of rated current and stability

delt1 = fzero('ff', [-pi/2 0]);
eaf1 = v*(1-xd/xq)*cos(2*delt1)/cos(delt1);
fprintf('Point B: eaf = %g  delta = %g\n', eaf1, delt1)
p1 = -.5*v^2*(1/xq-1/xd)*sin(2*delt1)-v*eaf1*sin(delt1)/xd;
q1 = .5*v^2*(1/xq+1/xd) - .5*v^2*(1/xq-1/xd)*cos(2*delt1) - v*eaf1*cos(delt1)/xd;
psi1 = atan(q1/p1);

fprintf('Point B: P = %g  Q = %g  psi = %g\n', p1, q1, psi1)

% now run it out
% region 1 is constant eaf: field limit

Delt1 = delt0:.01:0;
eaf = eaf0;

P1 = -.5*v^2*(1/xq-1/xd) .* sin(2 .* Delt1)...
    -(v*eaf/xd) .* sin(Delt1);
Q1 = .5*v^2*(1/xq+1/xd) -.5*v^2*(1/xq-1/xd) .* cos(2 .* Delt1) ...
    -(v*eaf/xd) .* cos(Delt1);

% region 2 is constant armature current magnitude

psi2 = -psi0:.01:psi1;

P2 = cos(psi2);
Q2 = sqrt(1-P2 .^2);

% region 3 is the stability limit

Delt3 = delt1:.01:0;
eaf3 = v*(1-xd/xq)*cos(2 .* Delt3) ./ cos(Delt3);

P3 = -.5*v^2*(1/xq-1/xd) .* sin(2 .* Delt3)...
    -(v .*eaf3 ./xd) .* sin(Delt3);
Q3 = .5*v^2*(1/xq+1/xd) -.5*v^2*(1/xq-1/xd) .* cos(2 .* Delt3) ...
    -(v .*eaf3 ./xd) .* cos(Delt3);
```

```
L1x = [0 0];
L1y = [-.5 1];
L2x = [0 1.2];
L2y = [0 0];
figure(1)
plot(P1, Q1, P2, Q2, P3, Q3, L1x, L1y, L2x, L2y)
axis square
axis equal
grid on
legend('Field Limit', 'Armature Limit', 'Stability Limit')
---------------
function deltb = ff(delt)

global xd xq v eq d

eaf = v*(1-xd/xq)*cos(2*delt)/cos(delt);

p = -.5*v^2*(1/xq-1/xd)*sin(2*delt)-v*eaf*sin(delt)/xd;
q = .5*v^2*(1/xq+1/xd) - .5*v^2*(1/xq-1/xd)*cos(2*delt) - v*eaf*cos(delt)/xd;

deltb = 1-abs(p+j*q);
```

Problem 3 With 48 slots and a four pole machine, the number of slots per pole per phase is:

$$m = \frac{48}{6 \times 2} = 4$$

And the *electrical* angle between slots is $\gamma = 2 \times \frac{360}{48} = 15°$. The breadth factor is:

$$k_b = \frac{\sin \frac{m\gamma}{2}}{m \sin \frac{\gamma}{2}} = \frac{\sin 30°}{4 \times \sin 7.5°} \approx .957$$

For the full pitch winding, of course, $k_p = 1$. Short pitching the winding reduces the pitch angle by the number of slots times γ, so that, for one slot short the angle is $\alpha = 165°$ and for two short it is $\alpha = 150°$. So:

$$\begin{array}{rcl} \text{full} \quad k_w &=& .957 \times 1.0 \approx .957 \\ \text{1 short} \quad k_w &=& .957 \times .991 \approx .948 \\ \text{2 short} \quad k_w &=& .957 \times .966 \approx .924 \end{array}$$

In this context we refer to the one slot short pitched winding as '11/12' and the two slot short pitch as '5/6'.