Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.061/6.690 Introduction to Power Systems

Problem Set 7 Solutions April 2, 2011

Problem 1: Load Flow There is no single 'right' answer to this one, but you should have inserted the right bus incidence matrix, which is this:

```
% This is the node-incidence Matrix
% l  1  2  3  4  5  6  7  8
NI=[ 1  0  1  0  0  0  0  0;  % Bus 1
    -1 -1  0  0  0  0  0 -1;  % Bus 2
     0  0  0  0  1  1  0  0;  % Bus 3
     0  0  0  0  0  0 -1  0;  % Bus 4
     0  1 -1  1 -1  0  0  0;  % Bus 5
     0  0  0 -1  0 -1  1  1]; % Bus 6
```

With the real and reactive power loads in the template script, you should have gotten voltage magnitudes like this:

```
Magnitudes

ans =

    0.8950
    0.8903
    1.0000
    1.0000
    0.9126
    0.9188
```

I modified the script to permit selective injection of reactive power thus:

```
% Added Injection of reactive power to try to smooth
% out voltage
Q_i = [0.5 0.25 0 0 0.25 0.1];
Q = [0 -.5 0 0 -.5 0]' + Q_i';
```

The complete script is appended. Anyway, with this set of injections, voltages at the various buses are:

```
Magnitudes

ans =

    1.0102
    0.9807
    1.0000
    1.0000
    0.9849
    0.9882
```

You should do a little playing with the script to see what happens.

Problem 2 Inductance is:

$$L = \mu_0 N^2 W \left(\frac{x}{g-h} + \frac{D-x}{g} \right)$$

Using co-energy force is:

$$f^e = \frac{2}{2} \frac{\partial L}{\partial x} = \frac{\mu_0 N^2 I^2 W}{2} \left(\frac{1}{h-g} - \frac{1}{g} \right)$$

Problem 3 Inductance is:

$$L = \frac{\mu_0 N^2 W D}{2g} = \frac{4\pi \times 10^{-7} \times 10^4 \times .02 \times .04}{.001} \approx 10\text{mH}$$

The current limit will be:

$$I_{\text{sat}} = \frac{2g B_{\text{sat}}}{\mu_0 N} = \frac{1.8 \times .001}{100 \times 4\pi 10^{-7}} \approx 14.3A$$

For the 6.690 part of the problem, we design this to make the saturation current limit be the same as the heating current limit. The ampere-turn limit would then be:

$$NI = 2HW_w J = \frac{2B_{\text{sat}} g}{\mu_0}$$

Where $J = 2 \times 10^6$ is our current limit. To make the gap equal to:

$$g = \frac{\mu_0 NI}{2B_{\text{sat}}} = \frac{\mu_0 HW_w J}{B_{\text{sat}}} = \frac{4\pi \times 10^{-7} \times .02 \times .02 \times 2 \times 10^6}{1.8} \approx .0005585\text{m}$$

Then we pick the number of turns:

$$N^2 = \frac{2Lg}{\mu_0 WD} = \frac{2L}{\mu_0 WD} \frac{\mu_0 HW_w J}{B_{\text{sat}}} = \frac{2LJ}{B_{\text{sat}}} \frac{HW_w}{WD} \approx \frac{2 \times .01 \times 2 \times 10^6}{1.8} \frac{.02 \times .02}{.02 \times .04} \approx 1.111 \times 10^4$$

or $N = 105$.

A calculation of inductance yields $L \approx 9.9176mH$, which is pretty close to our objective. The current limit is:

$$I_{\text{lim}} = \frac{2 \times .02 \times .02 \times 2 \times 10^6}{105} \approx 15.2\text{A}$$

Problem 4 Rail area is $2 \times 12 \times .1 = 2.4m^2$, and lift force required is $9.812 \times 40,000 \approx 392,480N$, so force density is 163,533 Pa. This is provided by the attractive force: $F = \frac{1}{2}\mu_0 H_g^2$, or

$$H_g = \sqrt{\frac{2F}{\mu_0}} = \sqrt{\frac{163,533}{2\pi \times 10^{-7}}} \approx 5.102 \times 10^5 \mathrm{A/m}$$

This means that $NI = gH_g = 5,102$A-T/pole.

With constant current, we know H_g will be inversely proportional to gap: $H_g = \frac{NI}{g}$, so lift force density will be inversely proportional to the square of gap:

$$F = F_0 \left(\frac{g_0}{g}\right)^2$$

This implies instability because the slope of the lift force is negative: if gap decreases, force increases, causing a further decrease in gap.

If current is constant, the magnetic coenergy is just magnetic coenergy density times gap volume:

$$W_m' = \frac{1}{2}\mu_0 H_g^2 \times 2Lg\,(W - x)$$

where x is displacement, assumed positive (to the right). Force would then be

$$f_x = \frac{\partial W_m'}{\partial x} = mu_0 H_g^2 Lg = -2 \times 163,533 \times 12 \times .01 \approx 39,248N$$

This force is restoring, as shown in Figure 1.

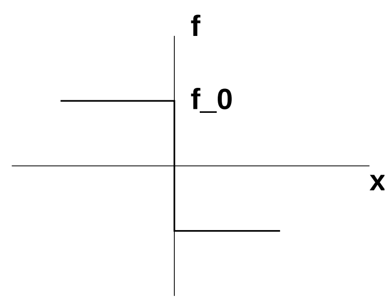

Figure 1: Lateral Force vs. Lateral Displacement

Note, however, that to maintain constant lift, the control system will be forced to increase current with lateral displacement, so restoring force would rise with displacement.

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