6.055J / 2.038J The Art of Approximation in Science and Engineering
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Chapter 1
Preview

An approximate model can be better than an exact model!

This counterintuitive statement suggests a few questions. First, how can approximate models be at all useful? Should we not strive for exactness? Second, what makes some models more useful than others?

On the first question: An approximate answer is all that we can understand because our minds are a small part of the world itself. So when we represent or model the world, we have to throw away aspects of the world in order for our minds to contain the model.

On the second question: Making *useful* models means discarding less important information so that our minds may grasp the important features that remains.

I wrote this book to show you how to discard the less important information and thereby to make the most useful approximate answers. From thinking about and teaching this subject for many years, I find that the most useful techniques fall into three groups:

1. **Divide and conquer** (managing complexity)
   - Hetrogenous hierarchies
   - Homogeneous hierarchies
2. **Symmetry and invariance** (removing spurious complexity)
   - Proportional reasoning
   - Conservation/box models
   - Dimensionless groups
3. **Lying** (discarding complexity)
   - Special cases
   - Spring models
   - Fractional changes
   - Discretization

The two divide-and-conquer techniques help you *manage* complexity. The three symmetry techniques help you *remove* superfluous complexity. These first two groups do lossless compression. The three lying techniques help you *discard* complexity. This third group does lossy compression.

Using these techniques, we will explore the natural and manmade worlds. Applications include:

- turbulent drag: or how falling coffee filters tell us the fuel efficiency of airplanes.
- xylophone acoustics: or why pianos are tuned with the lower notes below the ideal, equal-tempered frequency and with the higher notes above the ideal, equal-tempered frequency.
- the design of compact discs: or how Beethoven's ninth symphony helps you find the spacing between the pits.
- period of a pendulum as a function of amplitude: or how hard it was to navigate 300 years ago.
- the size distribution of eddies in turbulent flow: or how stars twinkle.
- the bending of starlight by the sun: or the size of a black hole.
- biomechanics: how high an animal jumps as a function of its size.

None of these problems has a simple analytic solution. The world – whether manmade or natural – rarely offers problems limited to one field of study, let alone problems whose equations have an analytic solution. To understand aspects of the world even partially, we need to use the preceding techniques, to make models that keep only the important features of a problem.

By making such models, we make understanding and designing more enjoyable. So the hidden although less tangible purpose of this book is to amplify your curiosity about the world.