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6.055J / 2.038J The Art of Approximation in Science and Engineering
Spring 2008

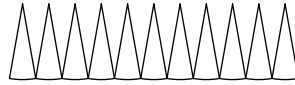
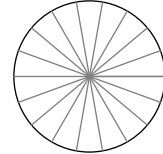
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The second follow-up question is: Does the 3 occur in other problems and for the same reason? A related place is the volume of a sphere

$$V = \frac{4}{3}\pi r^3.$$

The ancient Greeks showed that the 3 in the $4/3$ is the same 3 as in the pyramid volume. To explain their picture, I'll use method to find the area of a circle then use it to find the volume of a sphere.

Divide a circle into many pie wedges. To find its area, cut somewhere on the circumference and unroll it into this shape:



Each pie wedge is almost a triangle, so its area is $bh/2$, where the height h is approximately r . The sum of all the bases is the circumference $2\pi r$, so $A = 2\pi r \times r/2 = \pi r^2$.

Now do the same procedure with a sphere: Divide it into small pieces that are almost pyramids, then unfold it. The unfolded sphere has a base area of $4\pi r^2$, which is the surface area of the sphere. So the volume of all the mini pyramids is

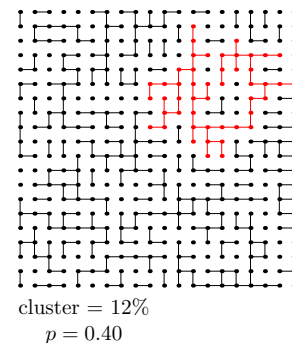
$$V = \frac{1}{3} \times \underbrace{\text{height}}_r \times \underbrace{\text{basearea}}_{4\pi r^2} = \frac{4}{3}\pi r^3.$$

Voilà! So, if you remember the volume of a sphere – and most of us have had it etched into our minds during our schooling – then you know that the volume of a pyramid contains a factor of 3 in the denominator.

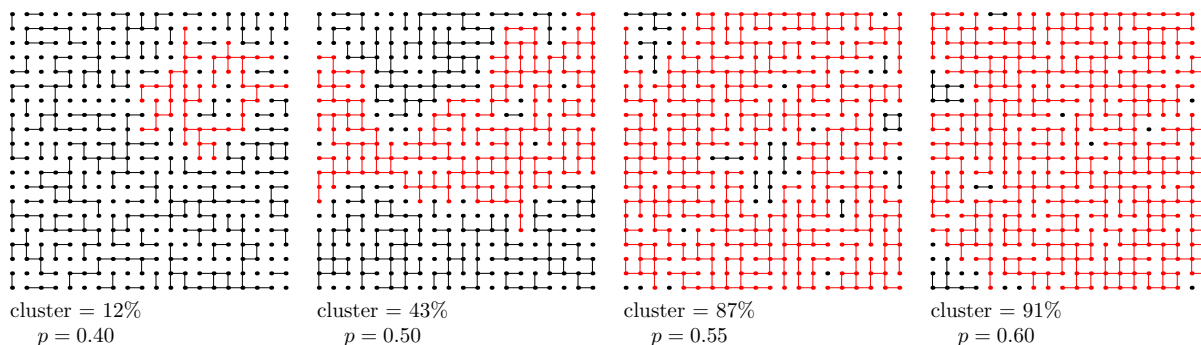
11.3 Percolation model

The moral of the preceding examples is to build connections. A physical illustration of this process is *percolation*. Imagine how oil diffuses through rock. The rock has pores through which oil moves from zone to zone. However, many pores are blocked by mineral deposits. How does the oil percolate through that kind of rock?

That question has led to an extensive mathematics research on the following idealized model. Imagine an infinite two-dimensional lattice. Now add bonds between neighbors (horizontal or vertical, not diagonal) with probability p_{bond} . The figure shows an example of a finite subsection of a percolation lattice where $p_{\text{bond}} = 0.4$. Its largest cluster – the largest set of points connected to each other – is marked in red, and contains 13% of the points.

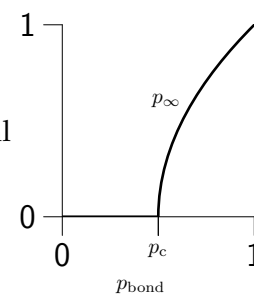


Here is what happens as p_{bond} increases from 0.40 to 0.50 to 0.55 to 0.60:



The largest cluster occupies more and more of the lattice.

For an infinite lattice, a similar question is: What is the probability p_{∞} of finding an infinite connected sublattice? That probability is zero until p_{bond} reaches a critical probability p_c . The critical probability depends on the topology (what kind of lattice and how many dimensions) – for the two-dimensional square lattice, $p_c = 1/2$ – but its existence is independent of topology. When $p_{\text{bond}} > p_c$, the probability of a finding an infinite lattice becomes nonzero and eventually reaches 1.0.



An analogy to learning is that each lattice point (each dot) is a fact or formula, and each bond links two facts. For long-lasting learning, you want the facts to support each other via their connections. Let's say that you want the facts to become part of an infinite and therefore self-supporting lattice. However, if your textbooks or way of learning means that you just add more dots – learn just more facts – then you decrease p_{bond} , so you decrease the chance of an infinite clusters. If the analogy is more exact than I think it is, you might even eliminate infinite clusters altogether.

The opposite approach is to ensure that, with each fact, you create links to facts that you already know. In the percolation model, you add bonds between the dots in order to increase p_{bond} . A famous English writer gave the same advice about life that I am giving about learning:

Only connect! That was the whole of her sermon... Live in fragments no longer!
[E. M. Forster, *Howard's End*]

The ways of reasoning presented in this book offer some ways to build those connections. Bon voyage as you learn and discover new ideas and the links between them!