6.055J / 2.038J The Art of Approximation in Science and Engineering
Spring 2008

6.055J/2.038J (Spring 2008)

Homework 4

Do the following warmups and problems. Due in class on ***Friday, 04 Apr 2008****.*

Open universe: *Collaboration, notes, and other sources of information are* ***encouraged.*** *However, avoid looking up answers until you solve the problem (or have tried hard). That policy helps you learn the most from the problems.*

Bring a photocopy to class on the due date*, trade it for a solution set, and figure out or ask me about any confusing points. Your work will be graded lightly: P (made a reasonable effort), D (did not make a reasonable effort), or F (did not turn in).*

Warmups

1. Minimum power

In lecture we estimated the flight speed that minimizes energy consumption. Call that speed v_{E}. We could also have estimated v_{P}, the speed that minimizes power consumption. What is the ratio $v_{\mathrm{P}}/v_{\mathrm{E}}$?

2. Solitaire

You start with the numbers 3, 4, and 5. At each move, you choose any two of the three numbers – call the choices a and b – and replace them with $0.8a - 0.6b$ and $0.6a + 0.8b$. The goal is to reach 4, 4, 4. Can you do it? If yes, give a move sequence; if no, show that you cannot.

Problems

3. Bird flight

a. For geometrically similar animals (same shape and composition but different size), how does the minimum-energy speed v depend on mass M and air density ρ? In other words, what are the exponents α and β in $v \propto \rho^{\alpha} M^{\beta}$?

b. Use that result to write the ratio $v_{747}/v_{\mathrm{godwit}}$ as a product of dimensionless factors, where v_{747} is the minimum-energy speed of a 747, and v_{godwit} is the minimum-energy speed of a bar-tailed godwit. Then estimate the dimensionless factors and their product. Useful information: $m_{\mathrm{godwit}} \sim 0.4\,\mathrm{kg}$.

c. Use v_{747}, from experience or from looking it up, to find v_{godwit}. Compare with the speed of the record-setting bar-tailed godwit, which made its 11,570 km journey in 8 days, 12 hours.

4. Hovering and hummingbirds

A simple model of hovering is that the animal or helicopter (mass M and wingspan L) forces air downward to stay aloft.

a. Estimate the downward air speed v_{down} needed to hover.

b. Show that the power required to hover is

$$P \sim \frac{(Mg)^{3/2}}{\rho^{1/2}L}.$$

c. Estimate P and v_{down} for a person hovering by flapping or waving his or her arms.

d. How does P depend on M for geometrically similar animals (same composition and shape but varying size)? In other words, give the exponent β in

$$P \propto M^{\beta}.$$

e. What fraction of its body weight does a hummingbird ($M \sim 3\,\text{g}$) eat every day in order to hover for a working day (8 hours)? Compare to the fraction for a person in a typical day. [Hummingbirds eat nectar, which is roughly equal parts sugar and water.]

Optional

5. Inertia tensor

[For those who know about inertia tensors.] Here is the inertia tensor (the generalization of moment of inertia) of a particular object, calculated in a lousy coordinate system:

$$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 4 & 5 \end{pmatrix}$$

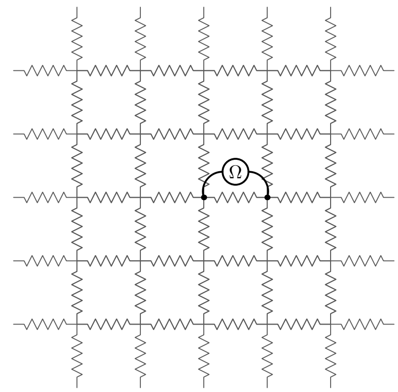
a. Change coordinate systems to a set of principal axes. In other words, write the inertia tensor as

$$\begin{pmatrix} I_{\text{xx}} & 0 & 0 \\ 0 & I_{\text{yy}} & 0 \\ 0 & 0 & I_{\text{zz}} \end{pmatrix}$$

and give I_{xx}, I_{yy}, and I_{zz}. *Hint:* What properties of a matrix are invariant when changing coordinate systems?

b. Give an example of an object with a similar inertia tensor. On Friday in class we'll have a demonstration.

6. Resistive grid



In an infinite grid of 1-ohm resistors, what is the resistance measured across one resistor?

To measure resistance, an ohmmeter injects a current I at one terminal (for simplicity, say $I = 1\,\text{A}$), removes the same current from the other terminal, and measures the resulting voltage difference V between the terminals. The resistance is $R = V/I$.

Hint: Use symmetry. But it's still a hard problem!