

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mathematics for Computer Science
 MIT 6.042J/18.062J

Spanning Trees

Albert R Meyer, April 8, 2013

spanning.1

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Spanning Subgraphs

A **spanning subgraph** of graph G is a subgraph that has all the vertices of G .

Albert R Meyer, April 8, 2013

spanning.2

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Spanning Subgraphs

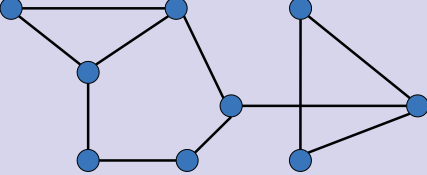
A **spanning subgraph** of graph G is a subgraph that has all the vertices of G . A **spanning tree** is a spanning subgraph that is a tree.

Albert R Meyer, April 8, 2013

spanning.3


6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

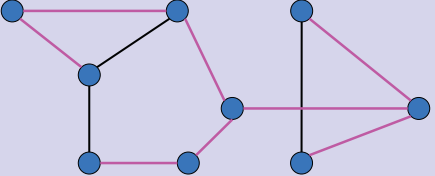
Spanning Trees




Albert R Meyer, April 8, 2013


spanning.4

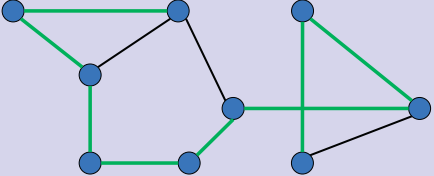

Spanning Trees




a spanning tree



 Albert R Meyer, April 8, 2013 spanning.5


Spanning Trees



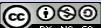
another spanning tree
(can have many)



 Albert R Meyer, April 8, 2013 spanning.6


Spanning Trees

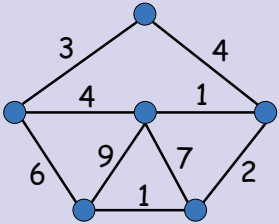
Lemma: G connected implies G has a spanning tree

Pf: Namely, any minimum edge connected spanning graph.



 Albert R Meyer, April 8, 2013 spanning.7


Minimum Weight Spanning Trees

Suppose edges have weights:





Find min weight spanning tree?


 Albert R Meyer, April 8, 2013 min-tree.9

6	9	13	7
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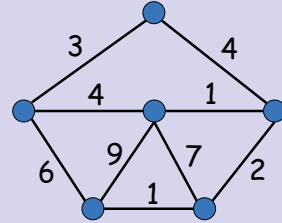
Build MST using gray edges


- Start with vertices
- Color components black & white
- gray edge ::= 
- add min weight gray edge


Albert R Meyer, April 8, 2013
min-tree.10

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

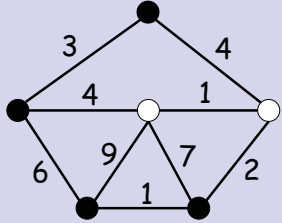
Minimum Spanning Trees color components





Albert R Meyer, April 8, 2013
min-tree.11

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

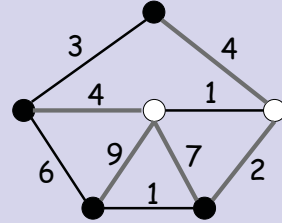
Minimum Spanning Trees color components






Albert R Meyer, April 8, 2013
min-tree.12

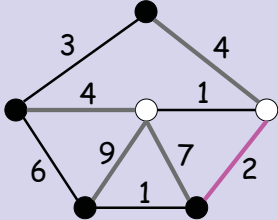
6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2


Minimum Spanning Trees gray edges




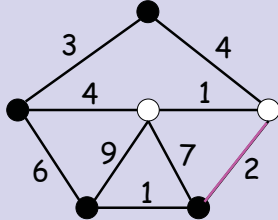

Albert R Meyer, April 8, 2013
min-tree.13



Minimum Spanning Trees
 gray edges: min weight




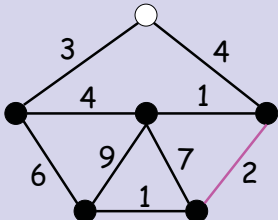

 Albert R Meyer, April 8, 2013 min-tree.14

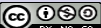

Minimum Spanning Trees
 re-color components




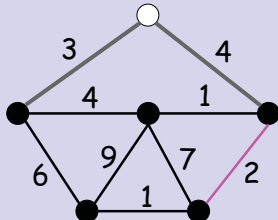

 Albert R Meyer, April 8, 2013 min-tree.15



Minimum Spanning Trees
 re-color components




 Albert R Meyer, April 8, 2013 min-tree.16


Minimum Spanning Trees
 gray edges




 Albert R Meyer, April 8, 2013 min-tree.17

Minimum Spanning Trees
gray edges: min weight

Albert R Meyer, April 8, 2013 min-tree.18

Minimum Spanning Trees
re-color components

Albert R Meyer, April 8, 2013 min-tree.19

Minimum Spanning Trees
re-color components

Albert R Meyer, April 8, 2013 min-tree.20

Minimum Spanning Trees
etc

Albert R Meyer, April 8, 2013 min-tree.21

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Ways to grow an MST

- start at any vertex, keep building one tree. (Prim)
- keep choosing min weight edge between diff components (Kruskal)
- grow trees in parallel (Meyer)



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6.042J / 18.062J Mathematics for Computer Science
Spring 2015

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