Mathematics for Computer Science

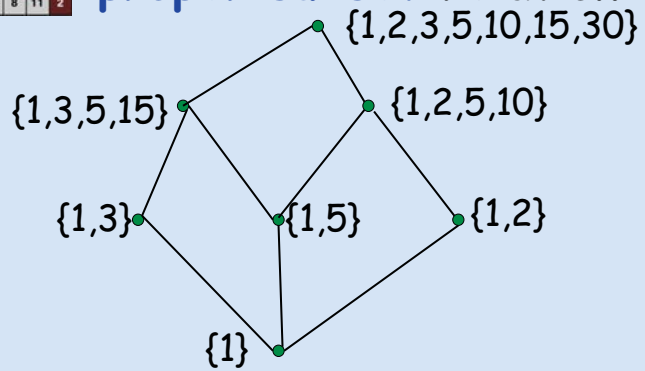
MIT 6.042J/18.062J

Representing Partial Orders

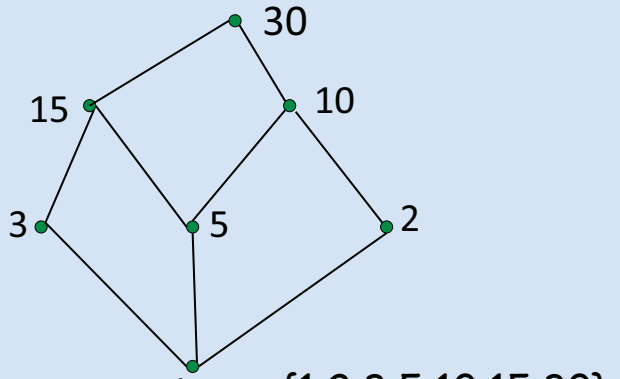
proper subset relation

$A \subset B$ means
B has everything
that A has
and more: $B \not\subset A$

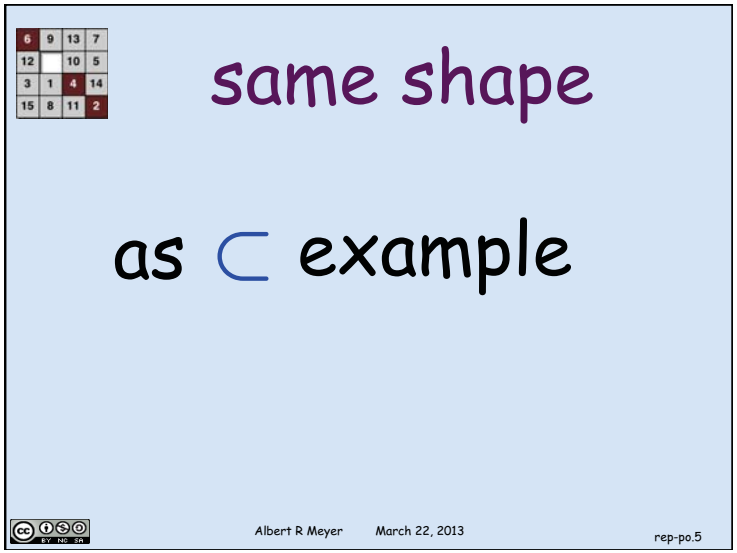
proper subset relation

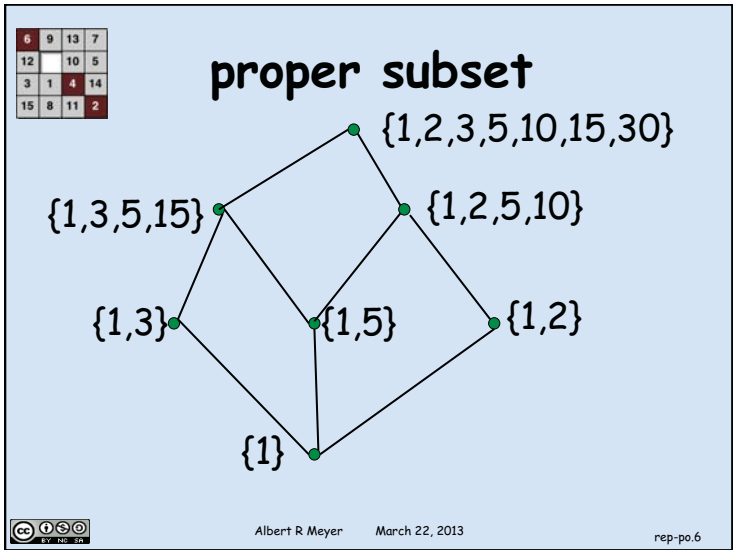


partial order: properly divides



on {1,2,3,5,10,15,30}


same shape
as ⊂ example
Albert R Meyer March 22, 2013
rep-po.5


proper subset
{1,2,3,5,10,15,30}
{1,3,5,15}
{1,2,5,10}
{1,3}
{1,5}
{1,2}
{1}
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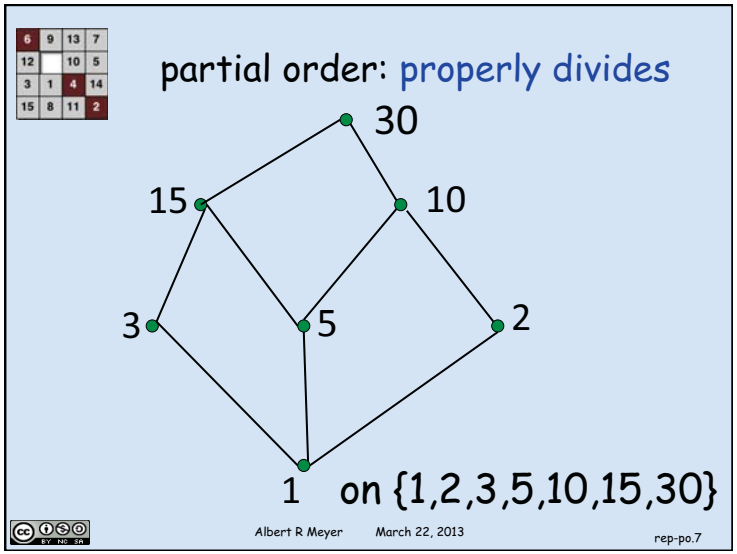
rep-po.6


partial order: properly divides
30
15
10
3
5
2
1
on {1,2,3,5,10,15,30}
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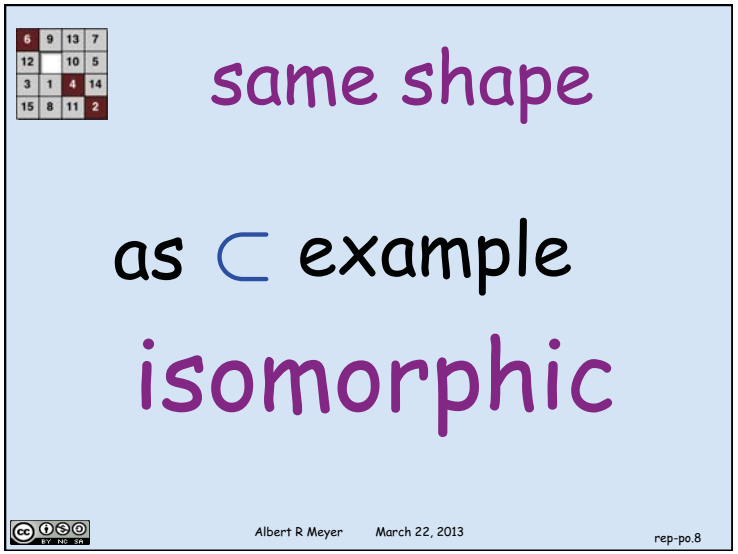
rep-po.7


same shape
as ⊂ example
isomorphic
Albert R Meyer March 22, 2013
rep-po.8

Isomorphism

All that matters are the connections: graphs with the same connections are isomorphic

Isomorphism

two graphs are isomorphic when there is an edge-preserving bijection of their vertices.

Formal Def of Graph Isomorphism

G_1 isomorphic to G_2 iff

$\exists$ bijection $f: V_1 \to V_2$ with

$u \to v$ in E_1 IFF $f(u) \to f(v)$ in E_2

p.o. represented by $\subset$

Theorem: Every strict partial order is isomorphic to a collection of subsets partially ordered by $\subset$.

p.o. isomorphic to $\subset$

proof: map element, a, to the set of elements below it.

a maps to $\{b \in A \mid bRa \text{ OR } b = a\}$

p.o. isomorphic to $\subset$

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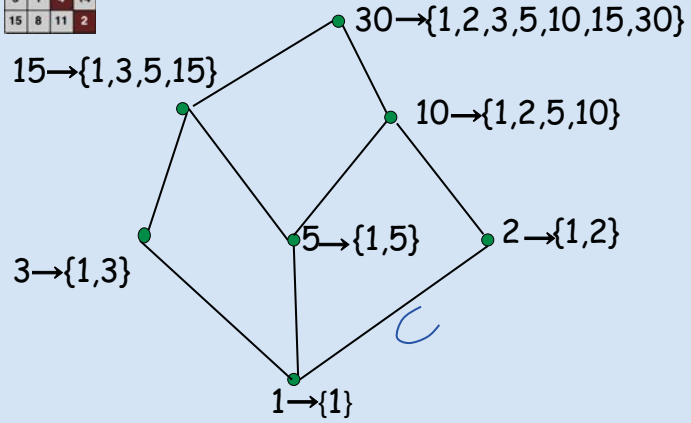
$$f(a) ::= R^{-1}(a) \cup \{a\}$$

subsets from divides



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