

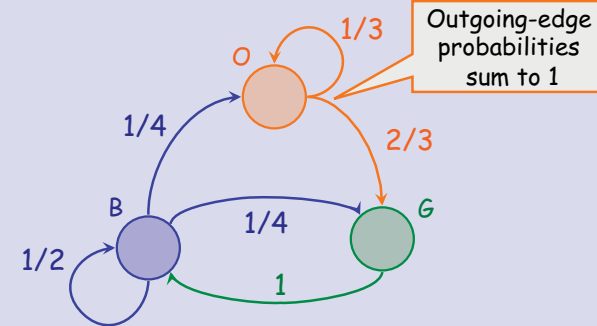
6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Random Walks



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

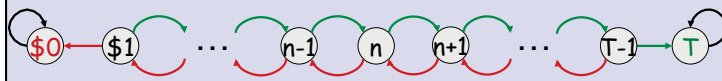
Graph With Probable Transitions



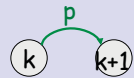
6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: Gambler's Ruin

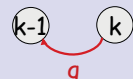
View as random walk on a line.



$p ::= \Pr[\text{win a bet}]$



$q ::= 1-p = \Pr[\text{lose a bet}]$



What is $\Pr[\text{reach } T \text{ before } 0]$?



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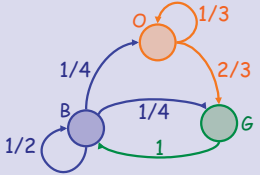
Applications of Random Walk

- Physics — Brownian motion
- Finance — stocks, options
- Algorithms — web search, clustering




6	9	13	7
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Questions

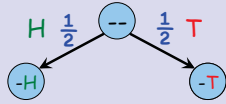


- Pr[reach O in 7 steps | start at B]
- Average # steps from B to O
- Pr[reach G before O | start at B]


 Albert R Meyer, May 13, 2015 random-walk.5

6	9	13	7
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Example: Toss HTH before TTH

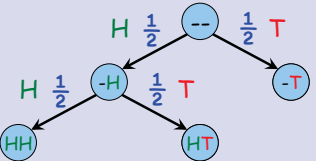



$$\begin{aligned}
 \Pr[\text{win}] &= \Pr[\text{win} | \text{--}] \\
 &= \frac{1}{2} \Pr[\text{win} | \text{-H}] + \frac{1}{2} \Pr[\text{win} | \text{-T}]
 \end{aligned}$$

 Albert R Meyer, May 13, 2015 random-walk.6

6	9	13	7
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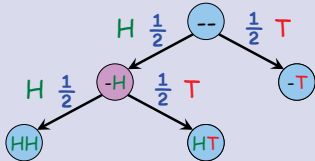
Example: Toss HTH before TTH




 Albert R Meyer, May 13, 2015 random-walk.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: Toss HTH before TTH



$$\begin{aligned}
 \Pr[\text{win} | \text{-H}] &= \frac{1}{2} \Pr[\text{win} | \text{HH}]
 \end{aligned}$$

 Albert R Meyer, May 13, 2015 random-walk.8

Example: Toss **HTH** before **TTH**

$$\Pr[\text{win} | \text{-H}]$$

$$= \frac{1}{2}\Pr[\text{win} | \text{HH}] + \frac{1}{2}\Pr[\text{win} | \text{HT}]$$

random-walk.9

Example: Toss **HTH** before **TTH**

random-walk.10

Example: Toss **HTH** before **TTH**

$$\Pr[\text{win} | \text{-T}]$$

$$= \frac{1}{2}\Pr[\text{win} | \text{TH}] + \frac{1}{2}\Pr[\text{win} | \text{TT}]$$

random-walk.11

Example: Toss **HTH** before **TTH**

random-walk.12

Example: Toss **HTH** before **TTH**

$$\Pr[\text{win} | \text{HH}] = \frac{1}{2}\Pr[\text{win} | \text{HH}] + \frac{1}{2}\Pr[\text{win} | \text{HT}]$$

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Example: Toss **HTH** before **TTH**

$$\Pr[\text{win} | \text{win}] = 1 \quad \Pr[\text{win} | \text{lose}] = 0$$

Now solve system of linear equations for $\Pr[\text{win}]$

Albert R Meyer, May 13, 2015 random-walk.14

Questions

- $\Pr[\text{reach } O \text{ in } 7 \text{ steps} | \text{start at } B]$
- Average # steps from **B** to **O**
- $\Pr[\text{reach } G \text{ before } O | \text{start at } B]$

Just solve systems of linear equations

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