MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2010)

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Examples 8.2, 8.7, 8.12, and 8.15 in the textbook

Romeo and Juliet start dating, but Juliet will be late on any date by a random amount X, uniformly distributed over the interval $[0, \theta]$. The parameter θ is unknown and is modeled as the value of a random variable Θ, uniformly distributed between zero and one hour.

(a) Assuming that Juliet was late by an amount x on their first date, how should Romeo use this information to update the distribution of Θ?

(b) How should Romeo update the distribution of Θ if he observes that Juliet is late by $x_1, \ldots, x_n$ on the first n dates? Assume that Juliet is late by a random amount $X_1, \ldots, X_n$ on the first n dates where, given θ, $X_1, \ldots, X_n$ are uniformly distributed between zero and θ and are conditionally independent.

(c) Find the MAP estimate of Θ based on the observation $X = x$.

(d) Find the LMS estimate of Θ based on the observation $X = x$.

(e) Calculate the conditional mean squared error for the MAP and the LMS estimates. Compare your results.

(f) Derive the linear LMS estimator of Θ based on X.

(g) Calculate the conditional mean squared error for the linear LMS estimate. Compare your answer to the results of part (e).