Recitation 3 Solutions
February 16, 2006

1. Problem 1.35, page 61. See online solutions.

2. Example 1.21, page 37 of text. See solutions in text.

3. (a) $\boxed{\text{True}}$
   If $P(A \mid B) = P(A)$, then A and B are independent. And if B is independent of A, then B is also independent of A^c. This implies, by the definition of independence:
   $$P(B \mid A^c) = P(B)$$

   (b) $\boxed{\text{False}}$
   Since there are only 5 tails out of ten, knowledge of one coin toss provides knowledge about the other coin tosses, which means the two events are not independent. In other words, the knowledge that the first coin toss was a tails influences the probability that the tenth coin toss is a tails.

   (c) $\boxed{\text{True}}$
   Here, all tosses are tails, so knowledge of one coin toss provides no additional knowledge about the tenth coin toss. Therefore the two events are independent.

   (d) $\boxed{\text{False}}$
   On the left hand side of the expression, since A_i's are disjoint,
   $$\begin{aligned} P(B \mid C) &= \frac{P(B \cap C)}{P(C)} \\ &= \sum_{i=1}^{n} \frac{P(A_i)P(B \cap C \mid A_i)}{P(C)} \\ &= \sum_{i=1}^{n} \frac{P(A_i \cap B \cap C)}{P(C)} \end{aligned}$$

   However, the right hand side of the given expression shows,
   $$\begin{aligned} \sum_{i=1}^{n} P(A_i \mid C)P(B \mid A_i) &= \sum_{i=1}^{n} \frac{P(A_i \cap C)}{P(C)} \frac{P(B \cap A_i)}{P(A_i)} \\ &= \sum_{i=1}^{n} \frac{P(A_i \cap B \cap C)}{P(C)P(A_i)} \end{aligned}$$

   where the last line is ONLY TRUE if the events $A_i \cap C$ and $B \cap A_i$ are independent of each other.
   Note also for the expression to be true, $i = 1$ and A_1 has to be the entire sample space, i.e. $P(A_1) = 1$. Therefore, the given expression only holds if $A_i \cap C$ and $B \cap A_i$ are independent and $i = 1$.