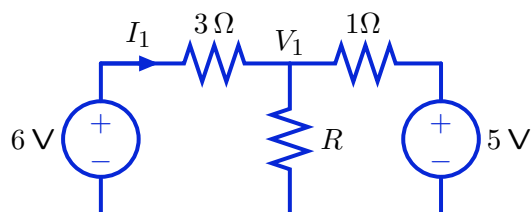


1 Circuits (20 points)

Consider the following circuit where the resistance R is in the range $0 \leq R \leq \infty$.



Part a. Determine I_1 if $R = 0\Omega$.

$$I_1 = \boxed{2 \text{ A}}$$

$$I_1 = \frac{6 \text{ V}}{3 \Omega} = 2 \text{ A}$$

Part b. Determine V_1 if $R = 1\Omega$.

$$V_1 = \boxed{3 \text{ V}}$$

$$\frac{V_1 - 6}{3} + \frac{V_1}{1} + \frac{V_1 - 5}{1} = 0$$

$$\frac{7}{3}V_1 = 7$$

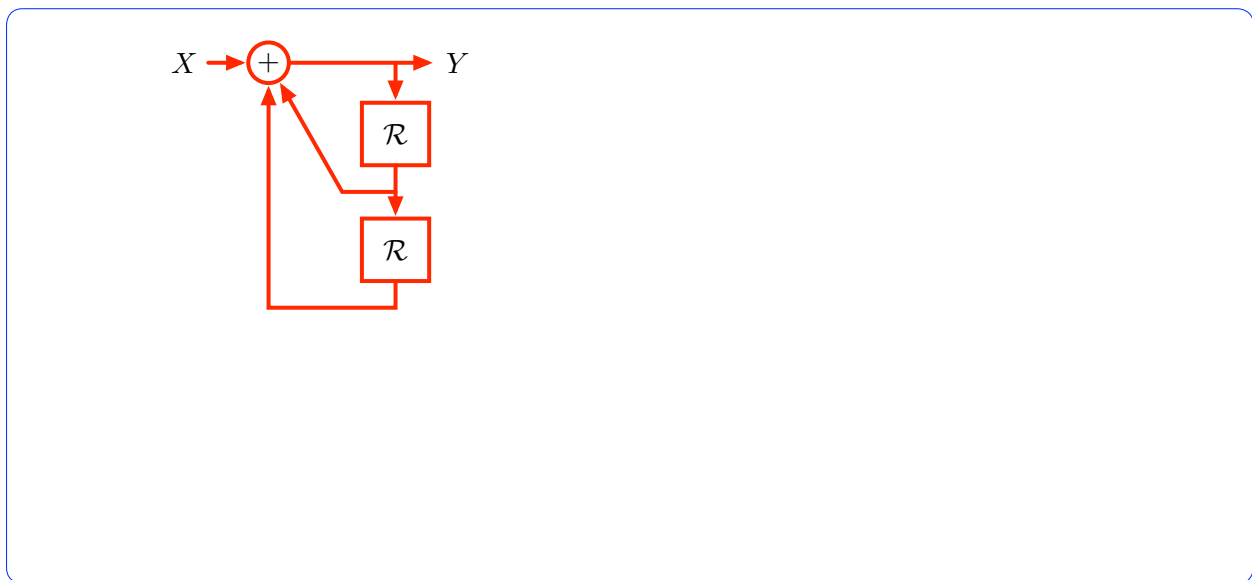
$$V_1 = 3$$

3 State Machine Behaviors (20 points)

Consider the following state machine.

```
class mySystem(sm.SM):
    startState = (0,0)
    def getNextValues(self, state, inp):
        (y1,y2) = state
        y0 = inp + y1 + y2
        return ((y0,y1), y0)
```

Part a. An instance of `mySystem` can be represented by a block diagram that contains only adders, gains, and/or delays. Draw this block diagram in the space below.



Part b. Determine the result of the following Python expression.

```
mySystem().transduce([1,0,0,0,0,0,0,0])
```

Enter the result in the box below.

[1, 1, 2, 3, 5, 8, 13, 21]

Part c. Determine the magnitude of the dominant pole of the system represented by `mySystem()`, and enter it in the box below.

magnitude of dominant pole:

$$\frac{1 + \sqrt{5}}{2}$$

$$H = \frac{Y}{X} = \frac{1}{1 - \mathcal{R} - \mathcal{R}^2} = \frac{1}{1 - \frac{1}{z} - \frac{1}{z^2}} = \frac{z^2}{z^2 - z - 1}$$

$$\text{poles are at } \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + 1} = \frac{1}{2} \pm \sqrt{\frac{5}{4}} = \frac{1 \pm \sqrt{5}}{2}$$

Part d. In the space below, write a new subclass of `sm.SM` called `newSystem`. Instances of `newSystem` should have a system function H given by

$$H = \frac{Y}{X} = 1 - \mathcal{R}^3$$

```
class newSystem(sm.SM):

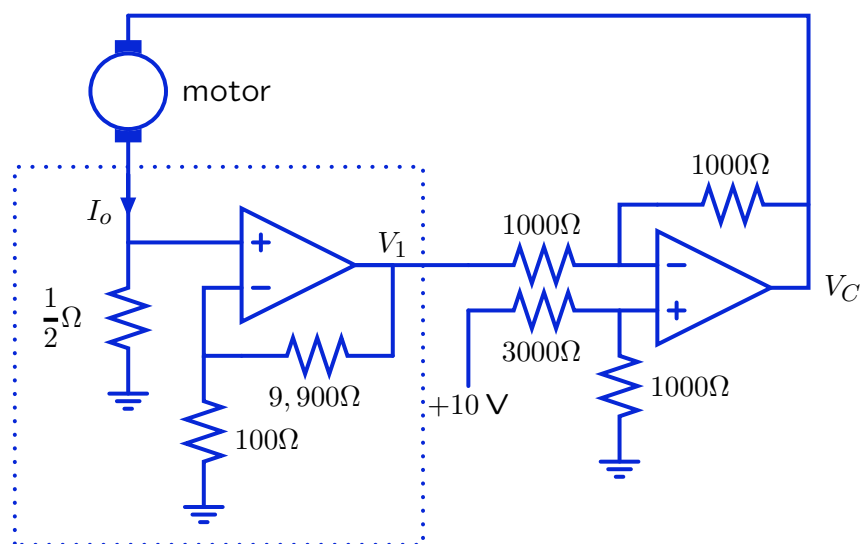
    startState = (0,0,0)
    def getNextValues(self, state, inp):
        (x1,x2,x3) = state
        y0 = inp - x3
        return ((inp,x1,x2), y0)
```

4 Motor Control (20 points)

The following circuit is a proportional controller that regulates the current through a motor by setting the motor voltage V_C to

$$V_C = K(I_d - I_o)$$

where K is the gain (notice that its dimensions are ohms), I_d is the desired motor current, and I_o is the actual current through the motor. Although K and I_d are not explicit in this circuit, their values can be determined from the resistor values below (see part b).



Part a. Consider the circuit inside the dotted rectangle. Determine an expression for V_1 as a function of I_o .

$$V_1 = \frac{1}{2} \Omega \times I_o \times 100$$

Part b. Determine the gain K and desired motor current I_d .

$$K = 50\Omega$$

$$I_d = 0.1 \text{ A}$$

KCL at negative input to right op-amp:

$$\frac{V_C - 2.5 \text{ V}}{1000\Omega} = \frac{2.5 \text{ V} - \frac{1}{2}\Omega \times I_o \times 100}{1000\Omega}$$

Solving:

$$V_C - 2.5 \text{ V} = 2.5 \text{ V} - 50\Omega \times I_o$$

$$V_C = 5 \text{ V} - 50\Omega \times I_o = 50\Omega (0.1 \text{ A} - I_o)$$

5 Members of the Club (20 points)

We would like to make a class to represent a club. A club has a list of members and a scoring function, which takes a member as an input and returns a numerical value. When a member is proposed for addition to the club, it is only added if its score is **greater than the average score of the current members**.

5.1 Join the club

We would like to make a club of basketball players, who are scored on their height. Here is a simple `BallPlayer` class.

```
class BallPlayer:
    def __init__(self, name, height):
        self.name = name
        self.height = height

    def getHeight(self):
        return self.height

    def __str__(self):
        return 'BallPlayer('+self.name+', ' + str(self.height) +')
```

The `Club` class has an `__init__` method that takes two arguments: an initial member, and a scoring function (that takes a single member as input and returns a numerical value).

Write an expression below that will create a new club. The first member is person named 'Wilt' whose height is 84 inches. The scoring function for club members should return their height.

```
c = Club(BallPlayer('Wilt', 84), BallPlayer.getHeight)
```

Now, imagine that we try, successively, to add the following new players, whose names and heights are listed below, to club `c`:

- 'Shorty', 60
- 'Walt', 86

- 'Stilt', 90
- 'Larry', 85

List the resulting membership of club c.

```
'Wilt', 'Walt', 'Stilt'
```

5.2 Implementation

Fill in the definition of the Club class. Use a list comprehension in the averageScore method.

```
class Club:
    def __init__(self, firstMember, scoreFunction):
        self.members = [firstMember]
        self.scoreFunction = scoreFunction

    # Returns average score of current members.
    def averageScore(self):

        n = float(len(self.members))
        return sum([self.scoreFunction(m) for m in self.members])/n

    # Adds member if it meets the criterion. Returns True if the
    # member was added and False, if not.
    def proposeMember(self, member):

        if self.scoreFunction(member) > self.averageScore():
            self.members.append(member)
            return True
        else:
            return False
```

5.3 Histogram

Write a procedure to compute a histogram of the member scores, that is, a count of how many members' scores fall within some specified ranges. We specify ranges by a list of N upper bounds. For example, the following bound list ($N = 3$):

```
[3, 9, 12]
```

specifies the following $N + 1 = 4$ ranges:

```
x < 3, 3 <= x < 9, 9 <= x < 12, 12 <= x
```

Given a list of scores that is already sorted from smallest to largest, such as: [1, 1, 4, 5, 7, 15] the resulting histogram would be: [2, 3, 0, 1] That is, 2 scores less than 3, 3 scores between 3 and 9, 0 scores between 9 and 12 and 1 score above 12.

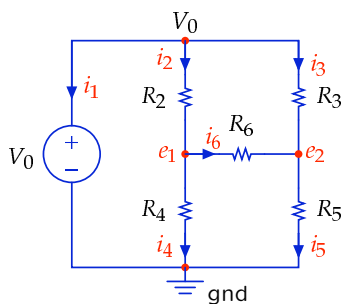
```
>>> histogram([1, 1, 4, 5, 7, 15], [3, 9, 12])  
[2, 3, 0, 1]
```

The output should always have $N + 1$ values; values should be zero if there are no scores in the appropriate range.

```
def histogram(scores, bounds):  
  
    counts = []           # answer list  
    index = 0            # index into scores  
    n = len(scores)      # total no of scores  
    for x in bounds:     # for each bound  
        count = 0        # initialize count  
        while index < n and scores[index] < x:  
            count += 1    # increment count  
            index += 1    # move to next score  
        counts.append(count) # store count in answer  
    counts.append(n-index) # count any remaning scores  
    return counts
```


1 Short-Answer Questions (10 points)

Part a. Consider the following circuit.



Determine if the following equations and/or statements are

– **Always True** – i.e., true for all possible values of the resistors $R_2 - R_6$ and voltage V_0

or

– **NOT Always True** – i.e., false for some or all resistor and voltage values.

Check the appropriate box for each of the following:

NOT
Always
True

Always
True

If $\frac{R_2}{R_4} = \frac{R_3}{R_5}$ then $i_6 = 0$

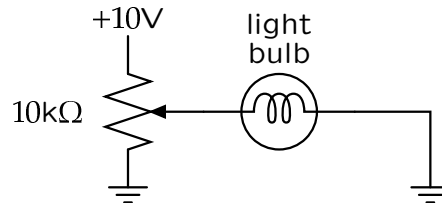
$i_2 + i_3 = i_4 + i_5$

$i_2 + i_6 = i_3$

$e_1 = \frac{R_4}{R_2 + R_4} V_0$

If $i_6 = 0$ then $\frac{R_2}{R_2 + R_4} = \frac{R_3}{R_3 + R_5}$

Part b. Our goal is to design a dimmer for a light bulb that can be modelled as a constant resistor of 10Ω . We have a single 10V power supply. Our first design uses a $10k\Omega$ potentiometer, with the goal of controlling the current through the lamp so that the current is **proportional** to the potentiometer setting, with a maximum current of 1A.



Briefly (using fewer than 50 words) describe problems with this circuit.

If the potentiometer is turned nearly all the way to the +10V side, then the current through the light bulb will be near 1A. Otherwise, the resistance of the potentiometer will limit the current through the light bulb to $< 1\text{mA}$. This the potentiometer will work more like an on/off switch than like a dimmer.

Suggest a better circuit. Draw it in the following box.

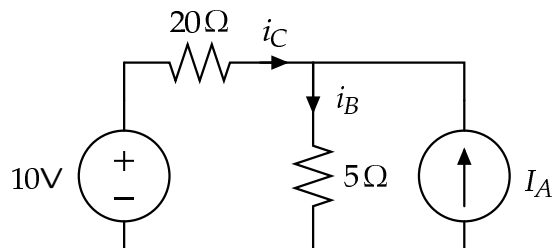
potentiometer -> unity buffer -> bulb -> ground

Explain briefly why your circuit is better.

The current into the op-amp buffer will be nearly zero. Therefore, the potentiometer will act as a voltage divider, and its output voltage will be proportional to the angle of its shaft. The op-amp buffer will drive its output to match the potentiometer voltage. Since we are modelling the light bulb as a resistor, its current will be proportional to the potentiometer shaft angle.

6 Circuits (20 points)

Consider the following circuit.



Part a. Find i_C if $I_A = 0$.

$$i_C = \boxed{0.4\text{A}}$$

$$i_C = \frac{10\text{V}}{20\Omega + 5\Omega} = \frac{2}{5}\text{A} = 0.4\text{A}$$

Part b. Find i_B if $I_A = 5\text{A}$.

$$i_B = \boxed{4.4\text{A}}$$

$$i_B = \frac{10\text{V}}{20\Omega + 5\Omega} + \frac{20\Omega}{5\Omega + 20\Omega} \times 5\text{A} = 4.4\text{A}$$

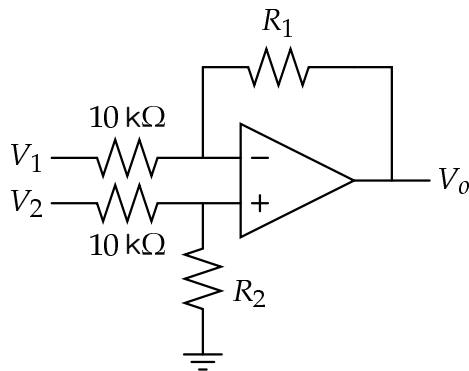
Part c. Find I_A so that $i_C = 0$.

$$I_A = \boxed{2\text{A}}$$

$$i_C = 0 = \frac{10\text{V}}{20\Omega + 5\Omega} - \frac{5\Omega}{5\Omega + 20\Omega} \times I_A = \frac{2}{5}\text{A} - \frac{1}{5}I_A$$

$$I_A = 2\text{A}$$

Part e. Consider the following op-amp circuit.



Fill in the values of R_1 and R_2 required to satisfy the equations in the left column of the following table. The values must be non-negative (i.e., in the range $[0, \infty]$). If the equation is impossible to implement with non-negative resistors, then write “impossible” for both resistor values.

	R_1	R_2
$V_o = 2V_2 - 2V_1$	$20\text{k}\Omega$	$20\text{k}\Omega$
$V_o = 2V_2 - V_1$	$10\text{k}\Omega$	∞
$V_o = V_2 - 2V_1$	$20\text{k}\Omega$	$5\text{k}\Omega$
$V_o = 4V_2 - 2V_1$	$20\text{k}\Omega$ or impossible	impossible

$$V_+ = \frac{R_2}{10\text{k}\Omega + R_2} V_2 = V_- = \frac{R_1}{10\text{k}\Omega + R_1} V_1 + \frac{10\text{k}\Omega}{10\text{k}\Omega + R_1} V_o$$

$$V_o = \frac{10\text{k}\Omega + R_1}{10\text{k}\Omega + R_2} \times \frac{R_2}{10\text{k}\Omega} \times V_2 - \frac{R_1}{10\text{k}\Omega} \times V_1$$

Solving the first and third rows requires just simple algebra.

Solving the algebra for the fourth row suggests that R_2 is negative. Therefore, this condition cannot be realized with non-negative resistors.

Solving the second row is a bit more tricky, since the algebraic solution suggests

$$2R_2 = 2R_2 + 20\text{k}\Omega$$

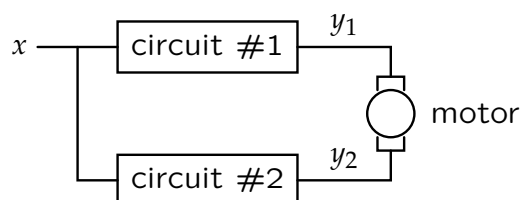
which can only be satisfied in the limit as $R_1 \rightarrow \infty$.

3. Circuits (30 / 100 points)

Motor driver

When we built the robot head, we made the motor move in both directions by connecting one side of the head motor to an op amp circuit and the other side to a buffered voltage divider that produced +5V. This method limited the peak speeds of the motor because the full +10V that is available from the power supply never appeared across the motor.

Our goal is to build two circuits, one to drive each of the two motor wires. Let x represent the input voltage and let y_1 and y_2 represent the voltages applied to the two motor wires. Assume that you have a single 10-volt power supply, so that only +10V and 0V are available.



Question 13: Recall the supply voltage constraints: **ALL** voltages (including x , y_1 and y_2) must be between 0V and +10V. Determine expressions for y_1 and y_2 so that the voltage across the motor is given by

$$y_1 - y_2 = \begin{cases} 10 & \text{if } x = 10 \\ -10 & \text{if } x = 0 \end{cases}$$

and both y_1 and y_2 have the form $y_i = m_i x + b_i$, where m_i and b_i are constants.

Solution: Since the voltages are limited to the range $[0,10]$, there is only one way to make $y_1 - y_2 = 10$: $y_1 = 10$ and $y_2 = 0$. Similarly, to make $y_1 - y_2 = -10$: $y_1 = 0$ and $y_2 = 10$. Thus y_1 ramps from 0 to 10 as x ramps from 0 to 10, so that

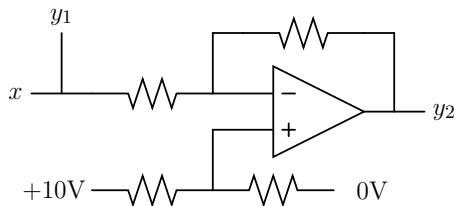
$$y_1 = x$$

and y_2 ramps down from 10 to 0 as x ramps from 0 to 10, so that

$$y_2 = 10 - x.$$

Question 14: Design circuits to implement your solutions to question 14 using resistors, op amps, and a single +10V power supply. You can assume that x is buffered (i.e., it is the output of an op amp). Draw the circuits and label them clearly.

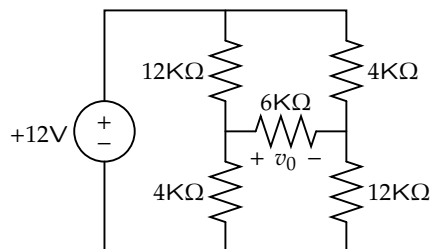
Solution: The y_1 output is equal to the x input (which is buffered), so y_1 can be connected to x with a wire. The y_2 output decreases as x increases, which can be implemented with an inverting amplifier. To get the endpoints right (e.g., $x = 0$ maps to $y_2 = 10$ and $x = 10$ maps to $y_2 = 0$) we must make the positive input to the op amp be 5V, which can be obtained with a voltage divider. The following figure shows the result:



where all of the resistors have equal values (e.g., $10\text{K}\Omega$).

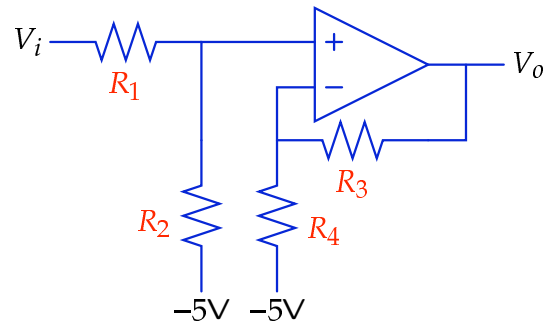
Analysis

A “bridge” circuit consisting of five resistors is connected to a 12 volt source as shown below.



Bridge circuit

Part 2: Op Amps. Consider the following circuit:



where all of the resistors have the same value $R = 1 \text{ k}\Omega$.

If $V_i = 3 \text{ V}$, then $V_o = 3 \text{ V}$

If $V_i = 7 \text{ V}$, then $V_o = 7 \text{ V}$

If $V_i = 9 \text{ V}$, then $V_o = 9 \text{ V}$

Because no current flows into the positive input to an op-amp, all of the current through R_1 flows through R_2 . Therefore, the voltage drops across these resistors are equal. It follows that the voltage V_+ is half way between V_i and -5V :

$$V_+ = \frac{V_i - 5}{2}$$

Similarly, the voltage V_- is half way between V_o and -5V :

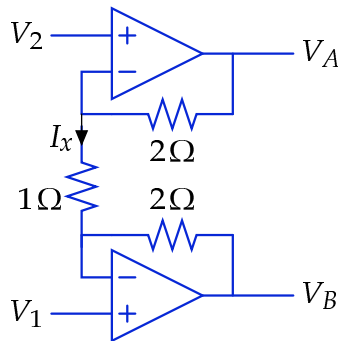
$$V_- = \frac{V_o - 5}{2}$$

Since $V_+ = V_-$, it follows that $V_o = V_i$.

NanoQuiz 10 Makeup

Name:

Part 1: Op Amps. Assume the op-amps in the following circuit are "ideal."



Determine the current I_x when $V_1 = 1\text{ V}$ and $V_2 = 2\text{ V}$.

$$I_x = 1\text{ A}$$

Determine the voltage V_A when $V_1 = 1\text{ V}$ and $V_2 = 2\text{ V}$.

$$V_A = 4\text{ V}$$

Determine a general expression for V_A in terms of V_1 and V_2 .

$$V_A = -2 * V_1 + 3 * V_2$$

The plus and minus input voltages must be equal for each op-amp. Therefore, the voltage across the 1Ω resistor is $V_2 - V_1$, and the current through the 1Ω resistor is

$$I_x = \frac{V_2 - V_1}{1\Omega} = \frac{2 - 1}{1} = 1\text{ A}.$$

The voltage V_A equals the voltage at the minus input of the top op-amp plus the voltage drop across the top 2Ω resistor. Since the current into the minus input to the op-amp is zero, then the same current (I_x) flows through the 2Ω resistor as well. Therefore

$$V_A = V_2 + I_x \times 2\Omega = 2 + 1 \times 2 = 4\text{ V}$$

More generally,

$$V_A = V_2 + I_x \times 2\Omega = V_2 + \frac{V_2 - V_1}{1\Omega} \times 2\Omega = V_2 + 2(V_2 - V_1) = -2V_1 + 3V_2$$

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