Problem Set 5 Solutions

Problem 5.1

(a)

(i) For a quarter wave plate we know that the difference in phase gained from propagating through the plate between the component polarized along the fast axis and the component polarized along the slow axis is $\pi/2$. We know that for a wave with free space wavelength $\lambda_\circ$, the phase gained by propagating through a material of thickness d with $\mu = \mu_\circ$ and $\varepsilon = \varepsilon_r \varepsilon_\circ$ is $\phi = \frac{2\pi}{\lambda_\circ} d\sqrt{\varepsilon_r}$.

$\phi_{fast-slow} = \frac{2\pi}{\lambda_\circ} d\sqrt{\varepsilon_{fast\ r}} - \frac{2\pi}{\lambda_\circ} d\sqrt{\varepsilon_{slow\ r}} = \frac{2\pi}{\lambda_\circ} d\left(\sqrt{2 + 10^{-5}\frac{V}{\varepsilon_\circ}} - \sqrt{2}\right) = \frac{\pi}{2}$

We can solve this for V, the applied voltage, and we get

$$V = \frac{\varepsilon_\circ}{10^{-5}}\left(\left(\frac{\lambda_\circ}{4d} + \sqrt{2}\right)^2 - 2\right) \tag{1}$$

(ii) If we limit ourselves to using the plate as a quarter wave plate, than we can express the output of the system as a function of the input light and the orientation of the QWP. For this we will assume the transmission axis of the first polaroid is along $\hat{x}$ and the transmission axis of the second polaroid is along $\hat{y}$. Let the unit vector in the direction of the fast axis of the waveplate be $\hat{i}_f$ and in the direction of the slow axis be $\hat{i}_s$. Since these vectors are orthogonal, it is possible to decompose the electric field into one component along $\hat{i}_f$ and one along $\hat{i}_s$.

$\hat{i}_f = \hat{x}cos(\Theta) + \hat{y}sin(\Theta)$

$\hat{i}_s = \hat{x}sin(\Theta) - \hat{y}cos(\Theta)$

$\bar{E} = \hat{x}E_\circ cos(\omega t - kz + \phi_\circ) = \hat{i}_f\ (\hat{i}_f \cdot \hat{x}E_\circ cos(\omega t - kz + \phi_\circ)) + \hat{i}_s\ (\hat{i}_s \cdot \hat{x}E_\circ cos(\omega t - kz + \phi_\circ))$

$\bar{E} = \hat{i}_f cos(\Theta)E_\circ cos(\omega t - kz + \phi_\circ) + \hat{i}_s sin(\Theta)E_\circ cos(\omega t - kz + \phi_\circ)$

At the output of the plate, the component of the electric field along the fast axis will have gained phase ϕ_{fast} and the component along the slow axis will have gained phase ϕ_{slow}. Thus at the output we have

$\bar{E} = \hat{i}_f cos(\Theta)E_\circ cos(\omega t - kz + \phi_\circ - \phi_{fast}) + \hat{i}_s sin(\Theta)E_\circ cos(\omega t - kz + \phi_\circ - \phi_{slow})$

Since the ablsolute phase of the components at the input doesn't matter, we can set $\phi_\circ = \phi_{slow}$ to simplify the result (remember that $\phi_{fast-slow} = \pi/2$).

$\bar{E} = \hat{i}_f cos(\Theta)E_\circ cos(\omega t - kz - \pi/2) + \hat{i}_s sin(\Theta)E_\circ cos(\omega t - kz)$

$\bar{E} = \hat{i}_f cos(\Theta)E_\circ sin(\omega t - kz) + \hat{i}_s sin(\Theta)E_\circ cos(\omega t - kz)$

Notice that this is an elliptically polarized wave with semi-major and semi-minor axis $\hat{i}_f$ and $\hat{i}_f$. Which axis is major/minor depends on the input angle Θ.

To get the ouput of the second polaroid we take the dot product of the input and $\hat{y}$ (the direction of the polaroids transmission axis).

$E_{out} = \hat{y} \cdot \left[\hat{i}_f cos(\Theta)E_\circ sin(\omega t - kz) + \hat{i}_s sin(\Theta)E_\circ cos(\omega t - kz)\right]$

$E_{out} = sin(\Theta)cos(\Theta)E_\circ sin(\omega t - kz) - cos(\Theta)sin(\Theta)E_\circ cos(\omega t - kz)$

$E_{out} = sin(\Theta)cos(\Theta)E_\circ \left[sin(\omega t - kz) - cos(\omega t - kz)\right]$

$E_{out} = \sqrt{2}cos(\Theta)sin(\Theta)E_\circ cos(\omega t - kz - 3\pi/4)$

Ignoring the time variation, this will be a maximum when $\Theta = 45°$ (the output of the plate is circular polarization) and will have magnitude $E_\circ/\sqrt{2}$. This implies an intensity of 0.5 times the intensity at the input of the wave plate, for a total transmission of 0.25 times the intensity at the input of the system (assuming unpolarized light at the system input).

$F_{max} = 0.25$

(iii) When the waveplate is rotated to give maximum transmission through the system, the fast (or slow) axis of the wave plate is oriented at $\Theta = 45°$ from the transmission axis of either polaroid sheet, as shown above.

(iv) If we apply enough voltage to make the plate behave as a half wave plate, we can rotate a linearly polarized input by 90° (provided the plate is oriented correctly). So, ignoring the reflections at the surfaces of the plate, the total transmission for unpolarized light is 0.5 ($I_{out} = 0.5I_{in}$).

(b) To express the output as a function of the voltage, we need to derive an expression for the electric field at the output of the plate as a function of voltage, and then take the projection of that field onto the transmission axis of the second polarizer. For this solution we'll assume that the fast axis of the plate is along $\hat{x}$, the slow axis is along $\hat{y}$, that the first polarizer is oriented to transmit light polarized along $\frac{\hat{x}+\hat{y}}{\sqrt{2}}$, and the second polarizer is set up to transmit light polarized along $\frac{-\hat{x}+\hat{y}}{\sqrt{2}}$.

If we set the phase at input of the plate to zero, then the E field phasor at the input is $\underline{\bar{E}} = \frac{E_\circ}{\sqrt{2}}(\hat{x}+\hat{y})$. After propagating through the plate this becomes

$\underline{\bar{E}} = \frac{E_\circ}{\sqrt{2}}[\hat{x}\ exp(-jk_f d) + \hat{y}\ exp(-jk_s d))]$,

which gives us an electric field of

$\bar{E} = Re\{exp(j\omega t))\frac{E_\circ}{\sqrt{2}}[\hat{x}\ exp(-jk_f d) + \hat{y}\ exp(-jk_s d))]\} = \frac{E_\circ}{\sqrt{2}}[\hat{x}\ cos(\omega t - k_f d) + \hat{y}\ cos(\omega t - k_s d)]$

The transmission of this E field through an ideal second polarizer would be

$E_{out} = \bar{E}\cdot\frac{-\hat{x}+\hat{y}}{\sqrt{2}} = \frac{E_\circ}{2}[cos(\omega t - k_s d) - cos(\omega t - k_f d)]$

Note that this oscilates from zero to $E_\circ$ as we change k_f (by changing the voltage). The intensity at the output will go as the square of the E field,

$I_{ideal\ out} = \frac{I_\circ}{4}\left[cos(\omega t - k_s d) - cos(\omega t - k_f d)\right]^2 = \frac{I_\circ}{4}\left[cos(\omega t - \frac{2\pi}{\lambda}d\sqrt{2}) - cos\left(\omega t - \frac{2\pi}{\lambda}d\sqrt{2 + V\frac{10^{-5}}{\varepsilon_\circ}}\right)\right]^2$

Since the transmission throught the first polarizer is 0.5 (for unpolarized incident light), we need to scale the intensity to get the desired F.

$$F(V) = \frac{\left[cos(\omega t - \frac{2\pi}{\lambda}d\sqrt{2}) - cos\left(\omega t - \frac{2\pi}{\lambda}d\sqrt{2 + V\frac{10^{-5}}{\varepsilon_\circ}}\right)\right]^2}{8} \tag{2}$$

(c) From equation 9.1.19, we have an expression for the E field reflected at normal incidence. If we observe that the reflected power is simply the square of the ratio of the incident and reflected E field, then we can compute the reflected power.

$r = \frac{(\eta_t/\eta_\circ)-1}{(\eta_t/\eta_\circ)+1} = \frac{\sqrt{\varepsilon_\circ/\varepsilon_t}-1}{\sqrt{\varepsilon_\circ/\varepsilon_t}+1} = \frac{1-\sqrt{\varepsilon_t/\varepsilon_\circ}}{1+\sqrt{\varepsilon_t/\varepsilon_\circ}} = \frac{1-2}{1+2} = -\frac{1}{3}$

$F_r = r^2 = \left(-\frac{1}{3}\right)^2 = \frac{1}{9}$

(d) From equation 9.2.75 we can calculate Brewsters Angle,

$\Theta_B = arctan\left(\sqrt{\frac{\varepsilon_t}{\varepsilon_\circ}}\right) = arctan(2) = 1.107\ [rad]\ = 63.43°$

Problem 5.2

(a) From the course notes we know that the skin depth in a material will be the inverse of the imaginary part of the wavevector $\bar{k}$. For a plasma, we know

$k = \sqrt{\mu\varepsilon}\ (\omega^2 - \omega_p^2)^{\frac{1}{2}}$

In the low frequency limit we know that the frequency ω should be lower than the plasma frequency ω_p, so k will be purely imaginary for real μ and ε.

$\delta = \frac{1}{k_i} = \frac{1}{k} = \frac{1}{\sqrt{\mu\varepsilon}}(\omega_p^2 - \omega^2)^{-\frac{1}{2}}$

Noting that the plasma frequency is given by $\omega_p = \sqrt{\frac{n_e e^2}{m_e \varepsilon}}$, and plugging in the values from the problem statement we get (at $\omega = 0$)

$\delta = \frac{1}{\sqrt{\mu_\circ\varepsilon_\circ}}\sqrt{\frac{m_e\varepsilon_\circ}{n_e e^2}} = \sqrt{\frac{m_e}{\mu_\circ n_e}}\ \frac{1}{e}$

$\delta = \sqrt{\frac{9.1\times10^{-31}}{4\pi\times10^{-7}\ 10^{30}}}\ \frac{1}{1.6\times10^{-19}} = 5.32\times10^{-9}\ [m]$

(b) We have two expressions for skin depth in conductors. They are

$\delta_{(\sigma<<\omega\varepsilon)} = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$

$\delta_{(\sigma>>\omega\varepsilon)} = \sqrt{\frac{2}{\omega\mu\sigma}}$

There's no way to solve the problem using the first expression (since in this limit the skin depth is constant with frequency), so lets try the second expression. We can solve this for ω

$\omega = \frac{2}{\delta^2\mu\sigma} = \frac{2}{(5.32\times10^{-9})^2\ 4\pi\times10^{-7}10^7} = \frac{2}{3.5566\times10^{-16}} = 5.6\times10^{15}$

Now we should check our assumptions. Is $\sigma = 10^7 >> \omega\varepsilon = 5.6\times10^{15}\ 8.85\times10^-12 = 5\times10^4$?

It looks like the assumption wasn't too bad, so we can use this answer for this class. Solving for the exact answer is a little more complicated, and is done below.

Exact Solution for skin depth NOT REQUIRED FOR CREDIT

In conductors we know that the relation between k and ω,μ, and ε is governed by the following set of equations

$k'^2 - k''^2 = \omega^2\mu\varepsilon$, and $2k'k'' = \omega\mu\sigma$

where k' is the real part of k, and k'' is the imaginary part of k. We can use these two equation to get a relation between the imaginary part of k and the frequency of the wave ω.

$k' = \frac{\omega\mu\sigma}{2k''}$

$\left(\frac{\omega\mu\sigma}{2k''}\right)^2 - k''^2 = \omega^2\mu\varepsilon$

$\omega^2\mu^2\sigma^2 - 4k''^4 = 4k''^2\omega^2\mu\varepsilon$

$\omega^2(\mu^2\sigma^2 - 4k''^2\mu\varepsilon) = 4k''^4$

$\omega = 2k''^2(\mu^2\sigma^2 - 4k''^2\mu\varepsilon)^{-\frac{1}{2}}$

$\omega = \frac{2}{\delta^2}(\mu^2\sigma^2 - \frac{4\mu\varepsilon}{\delta^2})^{-\frac{1}{2}}$

For the frequency ω to be real we need the argument of the square root to be positive.

$\mu^2\sigma^2 > \frac{4\mu\varepsilon}{\delta^2}$

$\delta^2 > \frac{4\varepsilon}{\mu\sigma^2}$

$\delta > \sqrt{\frac{\varepsilon}{\mu}}\ \frac{2}{\sigma} = 5.3 \times 10^{-10}$

So there should be a real frequency that will have a skin depth in this conductor that is equal to the skin depth from part (a).

$\omega = \frac{2}{(5.32\times10^{-9})^2}((4\pi \times 10^{-7})^2(10^7)^2 - \frac{4\times4\pi\times10^{-7}\ 8.85\times10^{-12}}{(5.32\times10^{-9})^2})^{-\frac{1}{2}} = 5.65 \times 10^{15}[rad/s] = 8.99 \times 10^{14}[Hz]$

Note that this answer is a little different than the answer we found using the approximation for skin depth.

Problem 5.3

(a) We know that the plasma frequency of the ionosphere has to be greater than the broadcast frequency of the station (otherwise John would have not lost the signal and then picked it back up again). From equation 9.2.30 in the notes we know that the critical angle is given by

$\Theta_c = arcsin\left(\frac{c_i}{c_t}\right)$

where c_i and c_t are the speed of light in the incident medium and transmitted medium.

$sin(\Theta_c) = \frac{c_i}{c_t} = \frac{c_\circ}{c_p}$, $c_p = \frac{c_\circ}{sin(45°)}$

In the plasma we know

$k^2 = \omega^2\mu_\circ\varepsilon_\circ(1 - (\frac{f_p}{f})^2) = \left(\frac{\omega}{c_\circ}\right)^2(1 - (\frac{f_p}{f})^2) = \left(\frac{\omega}{c_p}\right)^2 = \left(\frac{\omega}{c_\circ}\right)^2 sin^2(45°)$

$1 - (\frac{f_p}{f})^2 = sin^2(45°)$

$1 - sin^2(45°) = cos^2(45°) = (\frac{f_p}{f})^2$

$\frac{f_p}{f} = cos(45°)$

$f_p = f * cos(45°) = \frac{1}{\sqrt{2}} \times 10^6 \simeq 707\ [kHz]$

(b) To solve this problem we will assume that we are looking at reflection from a dielectric boundry, at $\Theta = 45°$, and using a TE wave (for simplicity).

We can solve this problem using equation 9.2.35 if we can find values for $\bar{\underline{E}}_t$ and α. Exactly at the critical angle, the transmitted wave has $\Theta_t = 90°$. This gives a real wave vector, $\bar{k}_t$, so we know that $\alpha = 0$. From equation 9.2.24 we know that $\underline{E}_\circ = \underline{E}_r$ (because $cos(\Theta_t) = 0$), so $\underline{E}_t = 2\underline{E}_\circ$. Putting this into equation 9.2.35 and using the notation from Figure 9.2.3 of the notes we get

$\bar{\underline{S}} = \hat{z}\frac{k_z|\bar{\underline{E}}_t|^2}{\omega\mu_t}\ [W/m^2]$

$\bar{\underline{S}} = \hat{z}\frac{\omega\sqrt{\mu_\circ\varepsilon}\ 4\ |\bar{\underline{E}}_\circ|^2}{\omega\mu_\circ}\ [W/m^2]$

From part (a) above, we know that $\varepsilon = \varepsilon_\circ(1 - \frac{f_p}{f}) = \varepsilon_\circ(1 - cos^2(45°)) = 0.5\varepsilon_\circ$. So,

$\bar{\underline{S}} = \hat{z}\ 2\left(\sqrt{\frac{\varepsilon_\circ}{\mu_\circ}}|\bar{\underline{E}}_\circ|^2\right) = \hat{z}\ 4\ [mW/m^2]$

Where the last step was done by recognizing that the term in the brackets was twice the power density of the input wave (which was given by the problem statement as 1 mW/m^2).

Problem 5.4

(a) From the problem statement we know that the electric field phasor will have the form

$\bar{\underline{E}} = \hat{x} E_\circ e^{-jkz}$,

with $k = \omega\sqrt{\mu_\circ \varepsilon_\circ}[1 - j\frac{\sigma}{\omega\varepsilon_\circ}]^{1/2}$.

From Maxwells Equations for the sinusoidal steady state, we can write

$-j\mu_\circ\omega\bar{\underline{H}} = \nabla \times \bar{\underline{E}} = \hat{y}\frac{\partial}{\partial z}E_\circ e^{-jkz} = -\hat{y}jkE_\circ e^{-jkz}$

$H_y = \frac{k}{\mu_\circ\omega}E_\circ e^{-jkz} = \sqrt{\frac{\varepsilon_\circ}{\mu_\circ}}\ [1 - j\frac{\sigma}{\omega\varepsilon_\circ}]^{1/2}\ E_\circ e^{-jkz}$

So the complex electric and magnetic fields are

$\bar{\underline{E}} = \hat{x} E_\circ e^{-jkz}$

$\bar{\underline{H}} = \hat{y}\sqrt{\frac{\varepsilon_\circ}{\mu_\circ}}\ [1 - j\frac{\sigma}{\omega\varepsilon_\circ}]^{1/2}\ E_\circ e^{-jkz}$

For the case when sigma is very small, than the two fields are effectivly in phase. We can break the magnetic field from the above expression into two terms. The real part of the phaser will be in phase with the electric field (in both time and space) while the imaginary part will represent a field 90° out of phase with the electric field. The out of phase component in this example will have a magnitude of 2×10^{-3} times the in phase component, so we'll ignore it and say the magnetic and electric fields are in phase.

(b) The complex fields for the $\hat{z}$ propagating waves are now

$\bar{\underline{E}}_i = \hat{x}E_\circ e^{-jkz}$ and $\bar{\underline{H}}_i = \hat{y}\sqrt{\frac{\varepsilon_\circ}{\mu_\circ}}E_\circ e^{-jkz}$.

From boundry conditions we can express the reflected waves as

$\bar{\underline{E}}_r = -\hat{x}E_\circ e^{jkz}$ and $\bar{\underline{H}}_r = \hat{y}\sqrt{\frac{\varepsilon_\circ}{\mu_\circ}}E_\circ e^{jkz}$.

The total field is the sum of the incident and reflected fields, which are

$\bar{\underline{E}} = \bar{\underline{E}}_i + \bar{\underline{E}}_r = \hat{x}\ E_\circ(e^{-jkz} - e^{jkz}) = -\hat{x}j2E_\circ sin(kz)$

$\bar{\underline{H}} = \bar{\underline{H}}_i + \bar{\underline{H}}_r = \hat{y}\sqrt{\frac{\varepsilon_\circ}{\mu_\circ}}E_\circ(e^{-jkz} + e^{jkz}) = \hat{y}2\sqrt{\frac{\varepsilon_\circ}{\mu_\circ}}E_\circ cos(kz)$

In this case the waves are not in phase in either space or time. From the expression for the phasors we can see that the electric field will have a spacial dependance of $sin(kz)$, while the magnetic field will have a spacial dependance of $cos(kz)$. The electric field is also 90° out of phase in time with the magnetic field due to the presence of a $+j$ term in the electric field phasor which is not in the magnetic field phasor.

(c) In this case, the polarization of the electric field has changed (from $\hat{x}$ to $\frac{\hat{x}+\hat{z}}{\sqrt{2}}$) and the direction of propagation has changed from $\hat{z}$ to $\frac{\hat{z}-\hat{x}}{\sqrt{2}}$. So the incident complex electric field is now

$\bar{\underline{E}}_i = \frac{\hat{x}+\hat{z}}{\sqrt{2}}E_\circ e^{-jk_z z}e^{-jk_x x} = \frac{\hat{x}+\hat{z}}{\sqrt{2}}E_\circ e^{-jkz/\sqrt{2}}e^{jkx/\sqrt{2}}$,

with $k = \omega\sqrt{\mu_\circ\varepsilon_\circ}$. So the incident complex magnetic field is now

$\bar{\underline{H}}_i = \hat{y}\sqrt{\frac{\varepsilon_\circ}{\mu_\circ}}E_\circ e^{-jkz/\sqrt{2}}e^{jkx/\sqrt{2}}$

To get the reflected fields, we need to look up the boundry conditions for a perfect conductor (equations

2.6.14 - 2.6.17 in the notes). The incident magnetic field is parallel to the boundry, and there can be a current on the surface of a perfect conductor, so equations 2.1.14 and 2.6.17 don't help us. Equation 2.6.16 tells us that the component of the electric field parallel to the boundry must be equal to zero, so $E_{rx} = -E_{ix}$. We also know that the polarization of the reflected wave must be perpendicular to the direction of the reflected wave ($-\frac{\hat{x}+\hat{z}}{\sqrt{2}}$). So the reflected complex electric field will be

$$\underline{\bar{E}}_r = \frac{\hat{z}-\hat{x}}{\sqrt{2}} E_\circ e^{-jk_z z} e^{-jk_x x} = \frac{\hat{z}-\hat{x}}{\sqrt{2}} E_\circ e^{jkz/\sqrt{2}} e^{jkx/\sqrt{2}},$$

and the reflected complex magnetic field will be

$$\underline{\bar{H}}_i = \hat{y}\sqrt{\frac{\varepsilon_\circ}{\mu_\circ}} E_\circ e^{jkz/\sqrt{2}} e^{jkx/\sqrt{2}}$$

for a total field (for $z < 0$) of

$$\underline{\bar{E}} = \underline{\bar{E}}_i + \underline{\bar{E}}_r = \frac{E_\circ}{\sqrt{2}} e^{jkx/\sqrt{2}} [\hat{x}(e^{-jkz/\sqrt{2}} - e^{jkz/\sqrt{2}}) + \hat{z}(e^{-jkz/\sqrt{2}} + e^{jkz/\sqrt{2}})]$$

$$\underline{\bar{E}} = \frac{E_\circ}{\sqrt{2}} e^{jkx/\sqrt{2}} (-\hat{x}2jsin(kz/\sqrt{2}) + \hat{z}2cos(kz/\sqrt{2}))$$

$$\underline{\bar{H}} = \underline{\bar{H}}_i + \underline{\bar{H}}_r = \hat{y}\sqrt{\frac{\varepsilon_\circ}{\mu_\circ}} E_\circ e^{jkx/\sqrt{2}} (e^{jkz/\sqrt{2}} + e^{-jkz/\sqrt{2}}) = \hat{y}2\sqrt{\frac{\varepsilon_\circ}{\mu_\circ}} E_\circ e^{jkx/\sqrt{2}} cos(kz/\sqrt{2})$$

(d) First find the real electric field.

$$E(x,y,z,t) = Re\{\underline{\bar{E}} e^{j\omega t}\} = Re\{\frac{E_\circ}{\sqrt{2}} e^{jkx/\sqrt{2}} e^{j\omega t} (-\hat{x}2jsin(kz/\sqrt{2}) + \hat{z}2cos(kz/\sqrt{2}))\}$$

$$E(x,y,z,t) = \hat{x}\sqrt{2}E_\circ sin(\omega t + \frac{kx}{\sqrt{2}}) sin(\frac{kz}{\sqrt{2}}) + \hat{z}\sqrt{2}E_\circ cos(\omega t + \frac{kx}{\sqrt{2}}) cos(\frac{kz}{\sqrt{2}})$$

At $t = 0$ this becomes

$$E(x,y,z,t) = \hat{x}\sqrt{2}E_\circ sin(\frac{kx}{\sqrt{2}}) sin(\frac{kz}{\sqrt{2}}) + \hat{z}\sqrt{2}E_\circ cos(\frac{kx}{\sqrt{2}}) cos(\frac{kz}{\sqrt{2}})$$

For a wave with wavelength λ, we can use $k = \frac{2\pi}{\lambda}$ to get

$$E(x,y,z,t) = \hat{x}\sqrt{2}E_\circ sin(\sqrt{2}\pi\frac{x}{\lambda}) sin(\sqrt{2}\pi\frac{z}{\lambda}) + \hat{z}\sqrt{2}E_\circ cos(\sqrt{2}\pi\frac{x}{\lambda}) cos(\sqrt{2}\pi\frac{z}{\lambda})$$

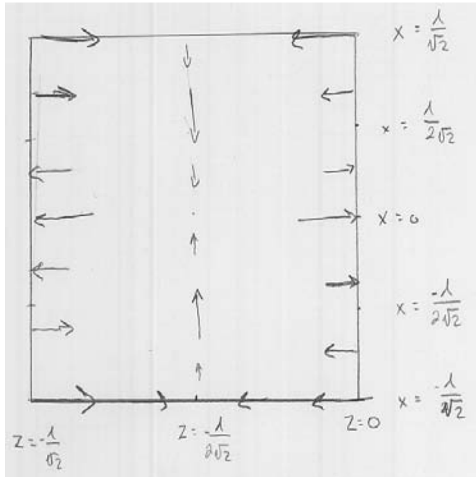
We want to sketch the electric field lines. We do this in a couple of steps, first identifing lines where the field is completly along $\hat{x}$ or $\hat{z}$, then sketching the components along the line, and finally connecting the sketched lines.

Step 1

From the above expression for the real electric field we know that the field will be only along $\hat{z}$ when $z = n\frac{\lambda}{\sqrt{2}}$, and the field will be completly along $\hat{x}$ when $z = (2n-1)\frac{\lambda}{2\sqrt{2}}$.

Step 2

Along these lines we sketch the electric field vectors (just pick a few interesting points). See sketch.



Step 3

Now we connect the arrows drawn in step 2 to see the electric field lines. See sketch below.