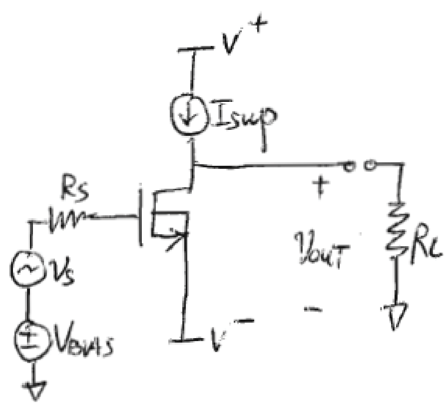
Recitation 22: CS Amplifier Frequency Response

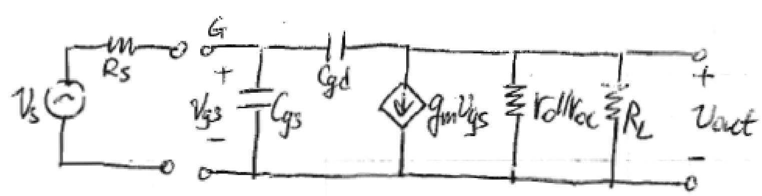
Yesterday, we discussed the frequency response of a CE Amplifier, using the following methods.

- Full analysis (using model Analysis to derive $\frac{V_{\text{out}}}{V_{\text{s}}}$)
- Miller approximation
- Open circuit time constant technique

Today we will look at the frequency response of CS Amplifier using 2-3.



Small signal equivalent circuit model

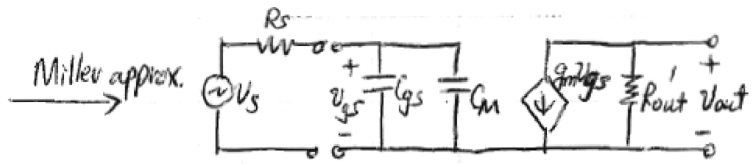


Low frequency voltage gain (ignoring the two capacitors)

$$A_{\text{v,LF}} = \frac{V_{\text{out}}}{V_{\text{s}}} = -g_{\text{m}}R_{\text{out}} \text{ where } R_{\text{out}} = \gamma_{\text{o}}||\gamma_{\text{oc}}||R_{\text{L}}$$

$$(\because V_{\text{gs}} = V_{\text{s}},\ V_{\text{out}} = -g_{\text{m}}R'_{\text{out}}V_{\text{gs}} \implies \frac{V_{\text{out}}}{V_{\text{s}}} = -g_{\text{m}}R'_{\text{out}})$$

Miller Approximation



$$C_{\mathrm{M}} = C_{\mathrm{gd}}(1 - A_{\mathrm{v}_{C_{\mathrm{gd}}}}) = C_{\mathrm{gd}}(1 + g_{\mathrm{m}} R'_{\mathrm{out}})$$

C_{gd} is in the position between input and output

$$
\begin{aligned}
V_{\mathrm{out}} &= -g_{\mathrm{m}} V_{\mathrm{gs}} \cdot R'_{\mathrm{out}} \\
V_{\mathrm{gs}} &= \frac{Z_{\mathrm{c}}}{Z_{\mathrm{c}} + R_{\mathrm{s}}} \cdot V_{\mathrm{s}}, \text{ where } Z_{\mathrm{c}} = \text{ impedance of 2 capacitors} \, (C_{\mathrm{gs}} \,\&\, C_{\mathrm{in}}) \text{ in parallel} \\
Z_{\mathrm{c}} &= \frac{1}{jw(C_{\mathrm{gs}} + C_{\mathrm{M}})} \\
V_{\mathrm{gs}} &= V_{\mathrm{s}} \frac{1/jw(C_{\mathrm{gs}} + C_{\mathrm{M}})}{1/jw(C_{\mathrm{gs}} + C_{\mathrm{M}}) + R_{\mathrm{s}}} = \frac{1}{1 + R_{\mathrm{s}}(jw(C_{\mathrm{gs}} + C_{\mathrm{M}}))} \cdot V_{\mathrm{s}} \\
\therefore \frac{V_{\mathrm{out}}}{V_{\mathrm{s}}} &= -\frac{g_{\mathrm{m}} R'_{\mathrm{out}} \cdot V_{\mathrm{gs}}}{V_{\mathrm{s}}} = -g_{\mathrm{m}} R'_{\mathrm{out}} \frac{1}{1 + jwR_{\mathrm{s}}(C_{\mathrm{gs}} + C_{\mathrm{M}})} \\
w_{3\mathrm{dB}} &= \frac{1}{R_{\mathrm{s}}(C_{\mathrm{gs}} + C_{\mathrm{M}})} = \frac{1}{R_{\mathrm{s}}(C_{\mathrm{gs}} + C_{\mathrm{gd}}(1 + g_{\mathrm{m}} R'_{\mathrm{out}}))}
\end{aligned}
$$

To compare with CE Amplifier,

$$w_{3\mathrm{dB}} = \frac{1}{R'_{\mathrm{in}}(C_\pi + C_\mu(1 + g_{\mathrm{m}} R'_{\mathrm{out}}))} \qquad R'_{\mathrm{in}} = R_{\mathrm{s}} || \gamma_\pi$$

Open Circuit Time Constant Analysis

Assumptions

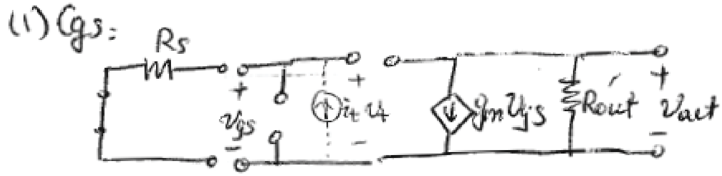
1. No zeros (or zeros can be ignored)
2. One dominant pole ($\frac{1}{\tau_1} \ll \frac{1}{\tau_2}, \frac{1}{\tau_3} \cdots$)

Procedures

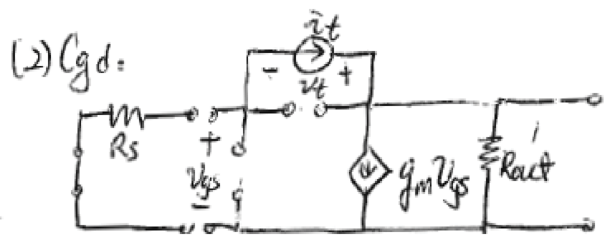
1. Open circuit all capacitors

2. Turn off all independent sources, find Thevenin resistance for each capacitor

3. Sum up the $R_{\mathrm{TH}} \cdot C_{\mathrm{i}} \implies b_1 = \sum_i C_i R_{\mathrm{TH_i}},\ w_{3\mathrm{dB}} \simeq \dfrac{1}{b_1}$



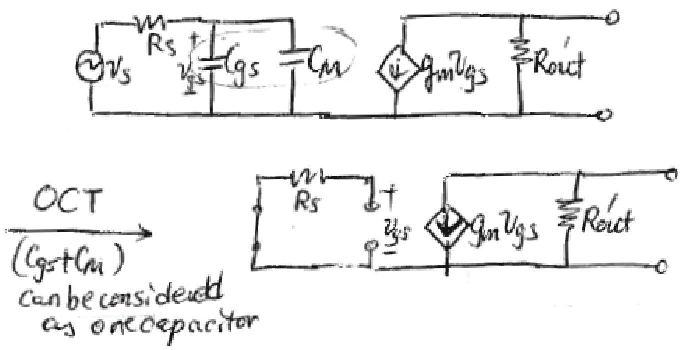
$$R_{\mathrm{TH_1}} = R_{\mathrm{s}}$$



$$\begin{aligned}
i_{\mathrm{t}} &= -\frac{V_{\mathrm{gs}}}{R_{\mathrm{s}}},\ \ i_{\mathrm{t}} = g_{\mathrm{m}} V_{\mathrm{gs}} + \frac{V_{\mathrm{t}} + V_{\mathrm{gs}}}{R'_{\mathrm{out}}} \\
\implies i_{\mathrm{t}} &= g_{\mathrm{m}}(-R_{\mathrm{s}} \cdot i_{\mathrm{t}}) + \frac{V_{\mathrm{t}} + (-R_{\mathrm{s}} \cdot i_{\mathrm{t}})}{R'_{\mathrm{out}}} \\
i_{\mathrm{t}} \cdot R'_{\mathrm{out}} &= (-g_{\mathrm{m}} R'_{\mathrm{out}} \cdot R_{\mathrm{s}} - R_{\mathrm{s}}) \cdot i_{\mathrm{t}} + V_{\mathrm{t}} \\
V_{\mathrm{t}} &= i_{\mathrm{t}}(R'_{\mathrm{out}} + R_{\mathrm{s}}(1 + g_{\mathrm{m}} R'_{\mathrm{out}})) \\
R_{\mathrm{TH_2}} &= \frac{V_{\mathrm{t}}}{i_{\mathrm{t}}} = R'_{\mathrm{out}} + R_{\mathrm{s}}(1 + g_{\mathrm{m}} R'_{\mathrm{out}}) \\
\implies b_1 &= C_{\mathrm{gs}} \cdot R_{\mathrm{TH_1}} + C_{\mathrm{gd}} \cdot R_{\mathrm{TH_2}} \\
&= C_{\mathrm{gs}} \cdot R_{\mathrm{s}} + C_{\mathrm{gd}} \cdot (R'_{\mathrm{out}} + R_{\mathrm{s}}(+g_{\mathrm{m}} R'_{\mathrm{out}})) \\
w_{3\mathrm{dB}} &\simeq \frac{1}{b_1} = \frac{1}{R_{\mathrm{s}}(C_{\mathrm{gs}} + C_{\mathrm{gd}}(1 + g_{\mathrm{m}} R'_{\mathrm{out}})) + R'_{\mathrm{out}} \cdot C_{\mathrm{gd}}}
\end{aligned}$$

This is actually also the result if we do full analysis

Miller + OCT



$$
\begin{aligned}
R_{\mathrm{TH}} &= R_{\mathrm{s}} \\
\Longrightarrow b_1 &= (C_{\mathrm{gs}} + C_{\mathrm{M}}) \cdot R_{\mathrm{TH}} = R_{\mathrm{s}}(C_{\mathrm{gs}} + C_{\mathrm{M}}) = R_{\mathrm{s}}(C_{\mathrm{gs}} + C_{\mathrm{gd}}(1 + g_{\mathrm{m}} R'_{\mathrm{out}})) \\
w_{3\mathrm{dB}} &= \frac{1}{R_{\mathrm{s}}(C_{\mathrm{gs}} + C_{\mathrm{gd}}(1 + g_{\mathrm{m}} R'_{\mathrm{out}}))} \quad \text{same as the Miller approximation analysis, but a lot easier}
\end{aligned}
$$

The comparison of w_{T} (or f_{T}) & $w_{3\mathrm{dB}}$

$$
\begin{aligned}
f_{\mathrm{T}} &= \frac{1}{2\pi} \frac{g_{\mathrm{m}}}{C_{\mathrm{gs}} + C_{\mathrm{gd}}} \\
w_{3\mathrm{dB}} &= \frac{1}{R_{\mathrm{s}}(C_{\mathrm{gs}} + C_{\mathrm{gd}}(1 + g_{\mathrm{m}} R'_{\mathrm{out}})) + R'_{\mathrm{out}} C_{\mathrm{gd}}}
\end{aligned}
$$

f_{T} is intrinsic to the device, while with $w_{3\mathrm{dB}}$ we have the effect of $R_{\mathrm{s}}, R'_{\mathrm{out}}, \& A_{\mathrm{v,LF}}$. Do not need more gain than really needed.

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