Lecture 12 - Sub-threshold MOSFET Operation - Outline

- **Announcement**

  Hour exam two: in 2 weeks, Thursday, Nov. 5, 7:30-9:30 pm

  ALSO: sign up for an iLab account!!

- **Review**

  MOSFET model: gradual channel approximation **(Example: n-MOS)**

$$i_D \approx \begin{cases} 0 & \text{for } (v_{GS} - V_T)/\alpha \le 0 \le v_{DS} \quad \textbf{(cutoff)} \\ K(v_{GS} - V_T)^2/2 & \text{for } 0 \le (v_{GS} - V_T)/\alpha \le v_{DS} \quad \textbf{(saturation)} \\ K(v_{GS} - V_T - \alpha v_{DS}/2)\alpha v_{DS} & \text{for } 0 \le v_{DS} \le (v_{GS} - V_T)/\alpha \quad \textbf{(linear)} \end{cases}$$

with $K \equiv (W/\alpha L)\mu_e C_{ox}^*$, $V_T = V_{FB} - 2\phi_{p\text{-}Si} + [2\varepsilon_{Si} qN_A(|2\phi_{p\text{-}Si}| - v_{BS})]^{1/2}/C_{ox}^*$

and $\alpha = 1 + [(\varepsilon_{Si} qN_A/2(|2\phi_{p\text{-}Si}| - v_{BS})]^{1/2}/C_{ox}$ **(frequently $\alpha \approx 1$)**

  The factor α: what it means physically

- **Sub-threshold operation - qualitative explanation**

  Looking back at Lecture 10 **(Sub-threshold electron charge)**

  Operating an n-channel MOSFET as a lateral npn BJT

  The sub-threshold MOSFET gate-controlled lateral BJT

  Why we care and need to quantify these observations

- **Quantitative sub-threshold modeling**

  $i_{D,sub\text{-}threshold}(\phi(0))$, then $i_{D,s\text{-}t}(v_{GS}, v_{DS})$ [with $v_{BS} = 0$]

  Stepping back and looking at the equations

Final comments on α

The Gradual Channel result ignoring α and valid for $v_{BS} \le 0$, and $v_{DS} \ge 0$ is:

$$i_G(v_{GS},v_{DS},v_{BS}) = 0, \quad i_B(v_{GS},v_{DS},v_{BS}) = 0, \quad \text{and}$$

$$i_D(v_{GS},v_{DS},v_{BS}) = \begin{cases} 0 & \text{for} \quad [v_{GS} - V_T(v_{BS})] < 0 < v_{DS} \\ \dfrac{K}{2}[v_{GS} - V_T(v_{BS})]^2 & \text{for} \quad 0 < [v_{GS} - V_T(v_{BS})] < v_{DS} \\ K\left\{v_{GS} - V_T(v_{BS}) - \dfrac{v_{DS}}{2}\right\} v_{DS} & \text{for} \quad 0 < v_{DS} < [v_{GS} - V_T(v_{BS})] \end{cases}$$

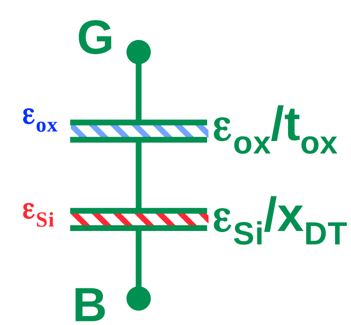
$$\text{with } K \equiv \frac{W}{L}\mu_e C_{ox}^* \quad \text{and} \quad C_{ox}^* \equiv \varepsilon_{ox}/t_{ox}$$

We noted last lecture that these simple expressions without α are easy to remember, and refining them to include α involves easy to remember substitutions: $\quad v_{DS} \Rightarrow \alpha v_{DS} \qquad L \Rightarrow \alpha L \qquad K \Rightarrow K/\alpha$

What we haven't done yet is to look at α itself, and ask what it means. What is it physically?

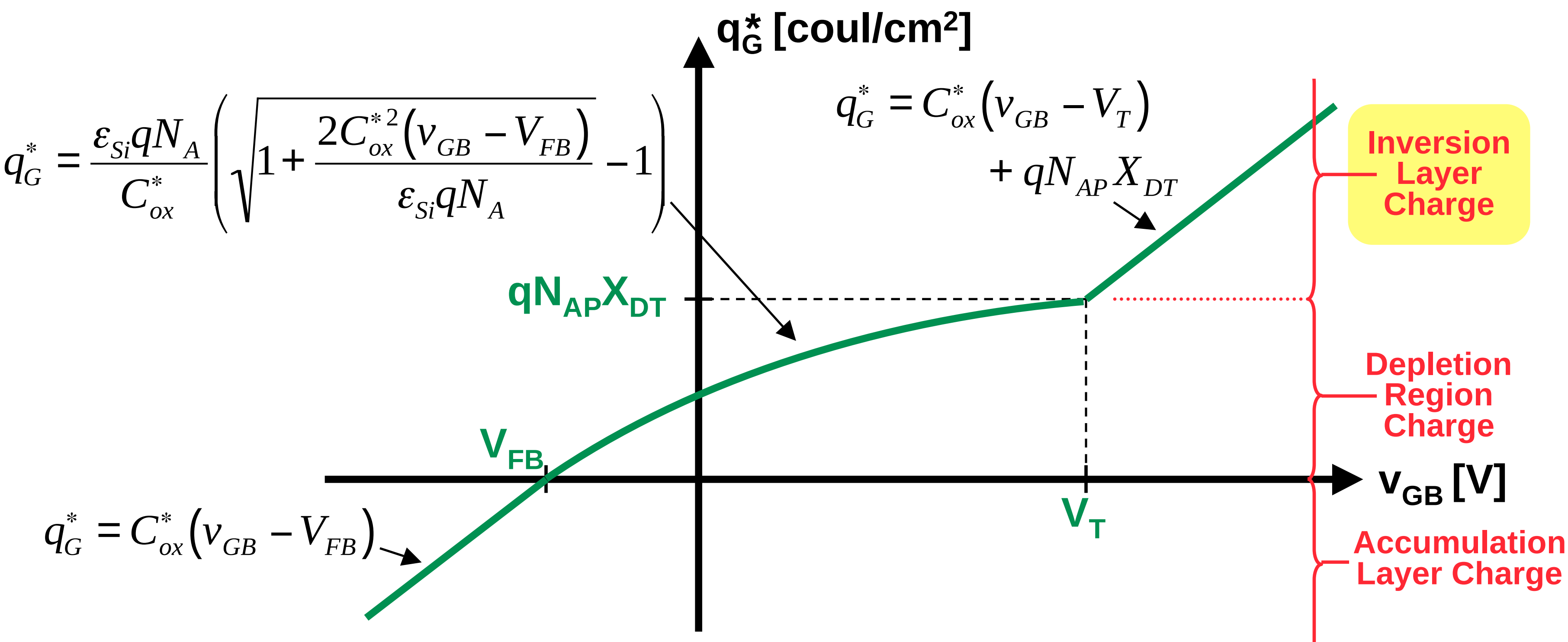
$$\alpha \equiv 1 + \frac{1}{C_{ox}^*}\sqrt{\frac{\varepsilon_{Si} q N_A}{2\left[\left|2\phi_{p-Si}\right| - v_{BS}\right]}} = \frac{C_{ox}^* + \varepsilon_{Si}\overbrace{\sqrt{qN_A / 2\varepsilon_{Si}\left[\left|2\phi_{p-Si}\right| - v_{BS}\right]}}^{1/x_{DT}(V_{BS})}}{C_{ox}^*}$$

$$= 1 + \frac{\varepsilon_{Si}/x_{DT}}{\varepsilon_{ox}/t_{ox}} = \boxed{1 + \frac{\varepsilon_{Si}}{\varepsilon_{ox}}\frac{t_{ox}}{x_{DT}}} = 1 + \frac{C_{DT}^*}{C_{ox}^*} = \frac{C_{DT}^*}{C_{GB}^*}$$



Look back at Lec. 10.

MOS Capacitors: the gate charge as v_{GB} is varied



$$C_{ox}^* \equiv \frac{\varepsilon_{ox}}{t_{ox}}$$

The charge expressions:

$$q_G^*(v_{GB}) = \begin{cases} C_{ox}^*(v_{GB}-V_{FB}) & \text{for} \quad v_{GB} \le V_{FB} \\ \dfrac{\varepsilon_{Si} q N_A}{C_{ox}^*}\left(\sqrt{1+\dfrac{2C_{ox}^{*2}(v_{GB}-V_{FB})}{\varepsilon_{Si} q N_A}}-1\right) & \text{for} \quad V_{FB} \le v_{GB} \le V_T \\ C_{ox}^*(v_{GB}-V_T) + qN_A X_{DT} & \text{for} \quad V_T \le v_{GB} \end{cases}$$

MOS Capacitors: How good is all this modeling? How can we know?

Poisson's Equation in MOS

As we argued when starting, J_h and J_e are zero in steady state so the carrier populations are in equilibrium with the potential barriers, $\phi(x)$, as they are in thermal equilibrium, and we have:

$$n(x) = n_i e^{q\phi(x)/kT} \quad \text{and} \quad p(x) = n_i e^{-q\phi(x)/kT}$$

Once again this means we can find $\phi(x)$, and then n(x) and p(x), by solving Poisson's equation:

$$\frac{d^2\phi(x)}{dx^2} = -\frac{q}{\varepsilon}\left[n_i\left(e^{-q\phi(x)/kT} - e^{q\phi(x)/kT}\right) + N_d(x) - N_a(x)\right]$$

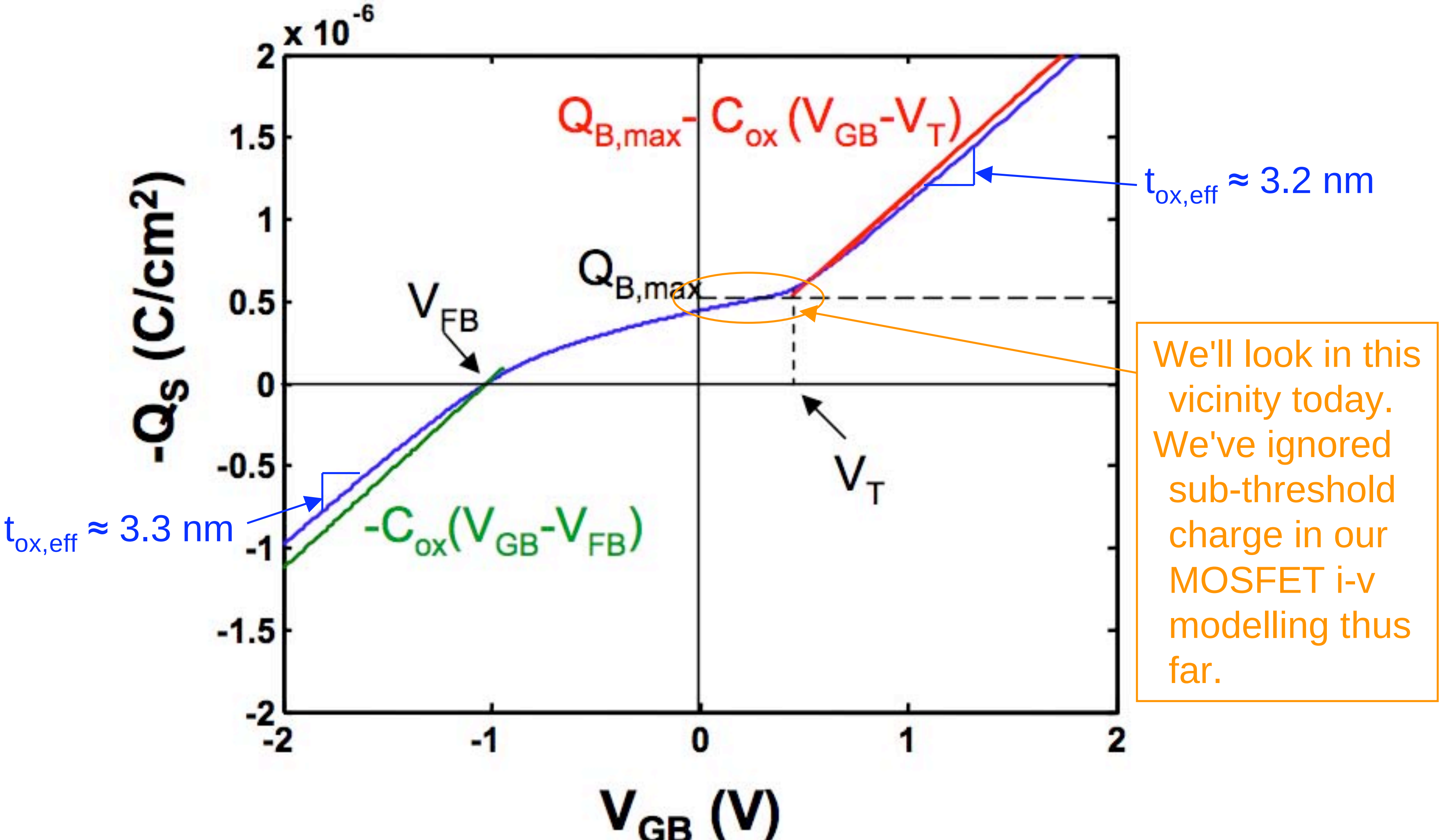
This version is only valid, however, when $|\phi(x)| \leq -\phi_p$.

When $|\phi(x)| > -\phi_p$ we have accumulation and inversion layers, and we assume them to be infinitely thin sheets of charge, i.e. we model them as delta functions.

Foil 9 from Lecture 10

Poisson's Equation calculation of gate charge

Calculation compared with depletion approximation model for t_{ox} = 3 nm and $N_A = 10^{18}$ cm^{-3}:



Plot courtesy of Prof. Antoniadis

MOS Capacitors: Sub-threshold charge
Assessing how much we are neglecting

Sheet density of electrons below threshold in weak inversion

In the depletion approximation for the MOS we say that the charge due to the electrons is negligible before we reach threshold and the strong inversion layer builds up:

$$q_{N(inversion)}(v_{GB}) = -C_{ox}^{*}(v_{GB} - V_T)$$

But how good an approximation is this? To see, we calculate the electron charge below threshold (weak inversion):

$$q_{N(sub-threshold)}(v_{GB}) \ = \ -q \int_{x_d(v_{GB})}^{0} n_i e^{q\phi(x)/kT} dx$$

ϕ(x) is a non-linear function of x, making the integral difficult,

$$\phi(x) \ = \ \phi_p + \frac{qN_A}{2\varepsilon_{Si}}(x - x_d)^2$$

but if we use a linear approximation for ϕ(x) near x = 0, where the term in the integral is largest, we can get a very good approximate analytical expression for the integral.

Sub-threshold electron charge, cont.

We begin by saying

$$\phi(x) \approx \phi(0) + ax \quad \text{where} \quad a \equiv \left.\frac{d\phi(x)}{dx}\right|_{x=0} = -\sqrt{\frac{2qN_A\left[\phi(0)-\phi_p\right]}{\varepsilon_{Si}}}$$

With this linear approximation to $\phi(x)$ we can do the integral and find

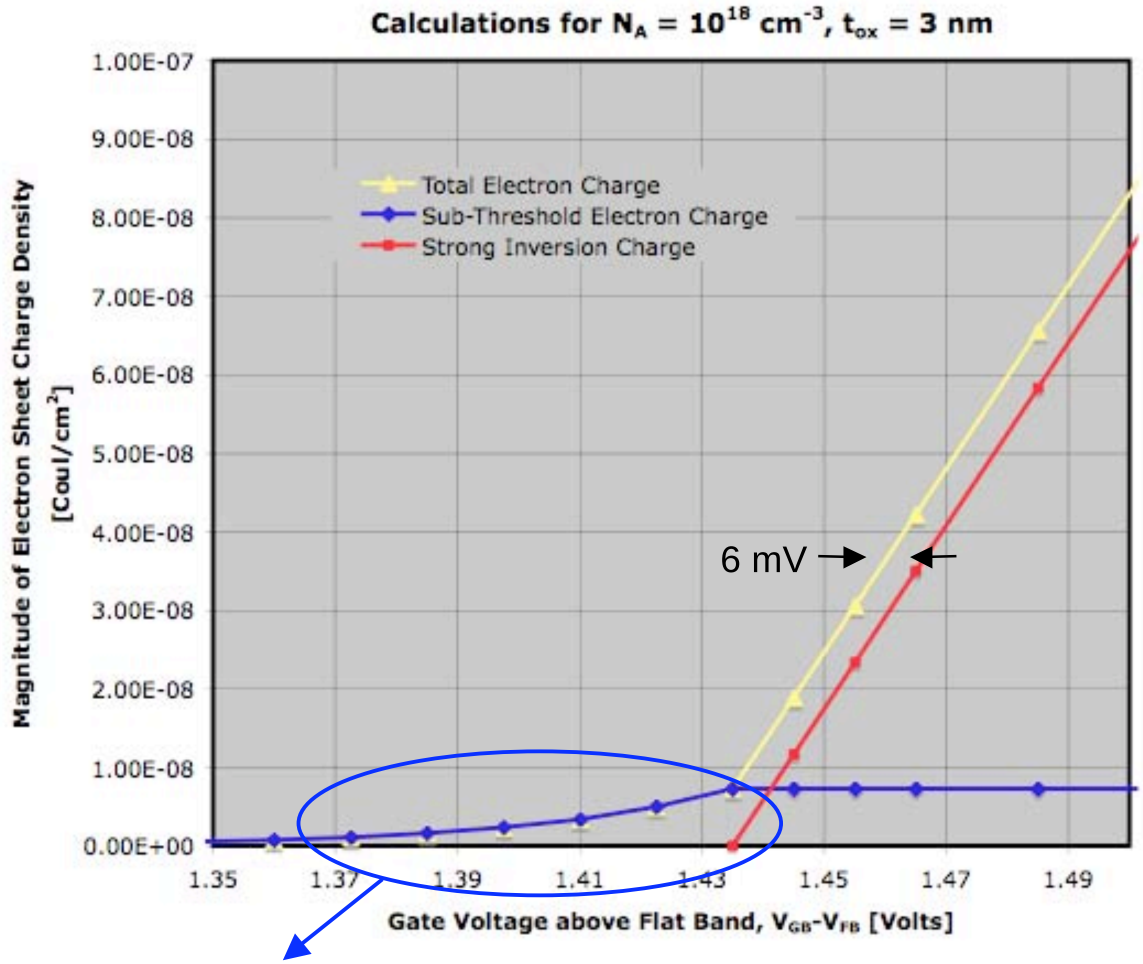
$$q_{N(sub-threshold)}(v_{GB}) \approx q\frac{kT}{q}\frac{n(0)}{a} = -q\frac{kT}{q}\sqrt{\frac{\varepsilon_{Si}}{2qN_A\left[\phi(0)-\phi_p\right]}}\, n_i e^{q\phi(0)/kT}$$

To proceed it is easiest to evaluate this expression for various values of $\phi(0)$ below threshold (when its value is $-\phi_p$), and to also find the corresponding value of v_{GB}, from

$$v_{GB} - V_{FB} = \phi(0) - \phi_p + \frac{t_{ox}}{\varepsilon_{ox}}\sqrt{2\varepsilon_{Si}qN_A\left[\phi(0)-\phi_p\right]}$$

This has been done and is plotted along with the strong inversion layer charge above threshold on the following foil.

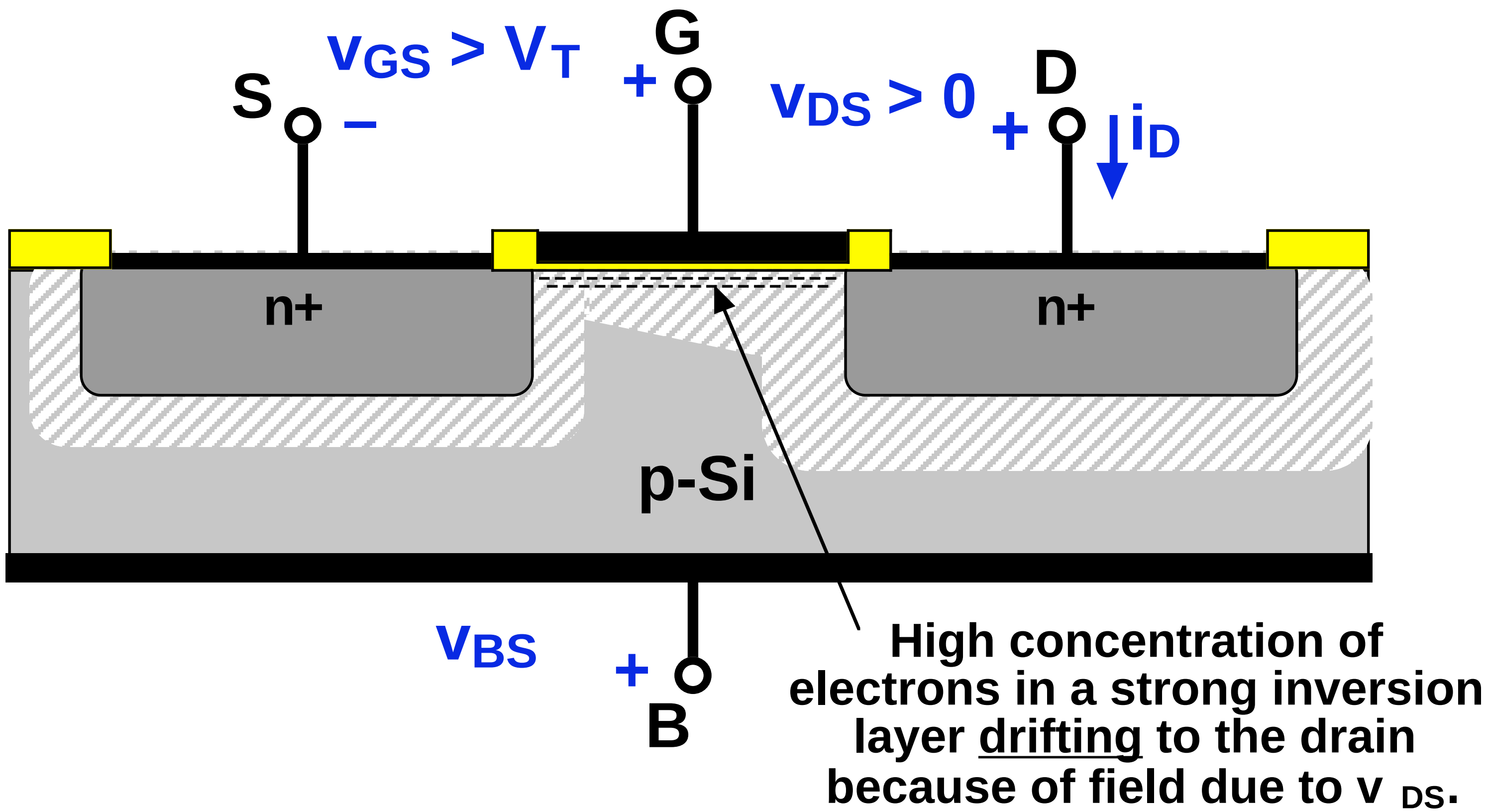
Sub-threshold electron charge, cont.



Neglecting this charge in the electrostatics calculation resulted in only a 6 mV error in our estimate of the threshold voltage value.

Today we will look at its impact on the sub-threshold drain current.

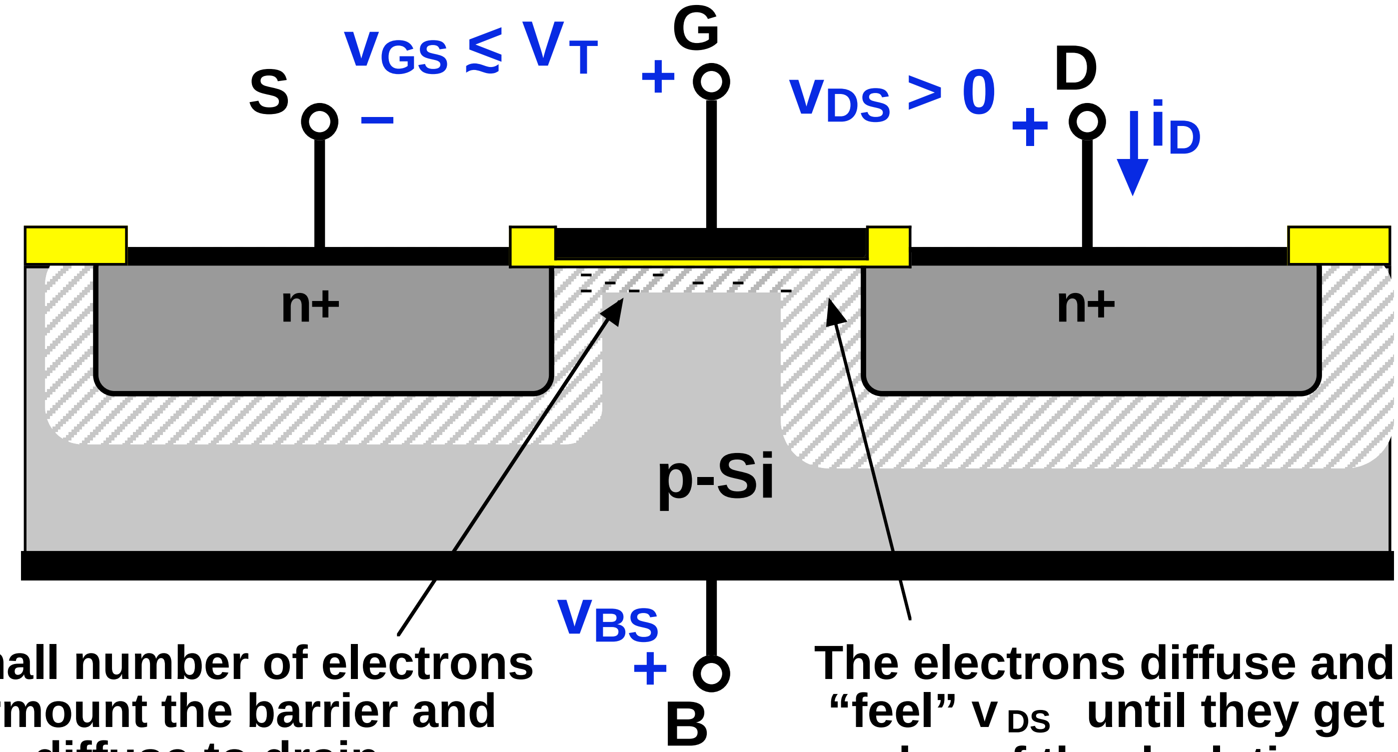
MOSFETs: Conventional strong inversion operation, $V_{GS} > V_T$



n-type surface channel; drift flux from source to drain

In our gradual channel approximation modeling we have assume a high conductivity n-type channel has been induced under the gate.

MOSFETs: Sub-threshold operation, $V_{GS} \lesssim V_T$



A small number of electrons surmount the barrier and diffuse to drain.

The electrons diffuse and do not "feel" v_{DS} until they get to the edge of the depletion region.

No surface channel; diffusion flux from source to drain when $v_{DS} > 0$

For any $v_{GB} > V_{FB}$ some electrons in the source can surmount the barrier and diffuse to the drain. Though always small, this flux can become consequential as v_{GS} approaches V_T.

MOSFETs: Sub-threshold operation, $V_{GS} \lesssim V_T$

What do we mean by "consequential"?

When is this current big enough to matter?
There are at least three places where it matters:

1. It can limit the gain of a MOSFET linear amplifier.

In Lecture 21 we will learn that we achieve maximum gain from MOSFETs operating in strong inversion when we bias as close to threshold as possible. This current limits how close we can get.

2. It is a major source of power dissipation and heating in modern VLSI digital ICs.

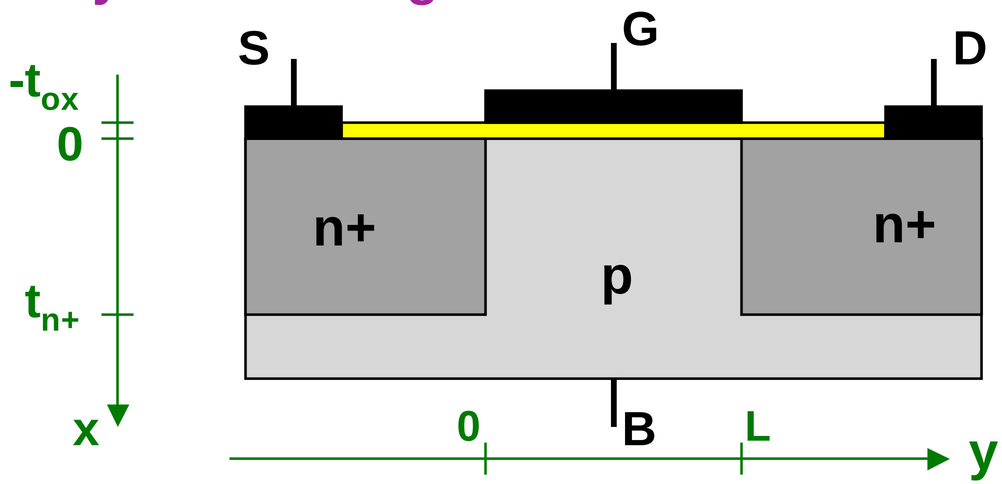
When you have millions of MOSFETs on an IC chip, even a little bit of current through the half that are supposed to be "off" can add up to a lot of power dissipation. We'll see this in Lecture 16.

3. It can be used to make very low voltage, ultra-low power integrated circuits.

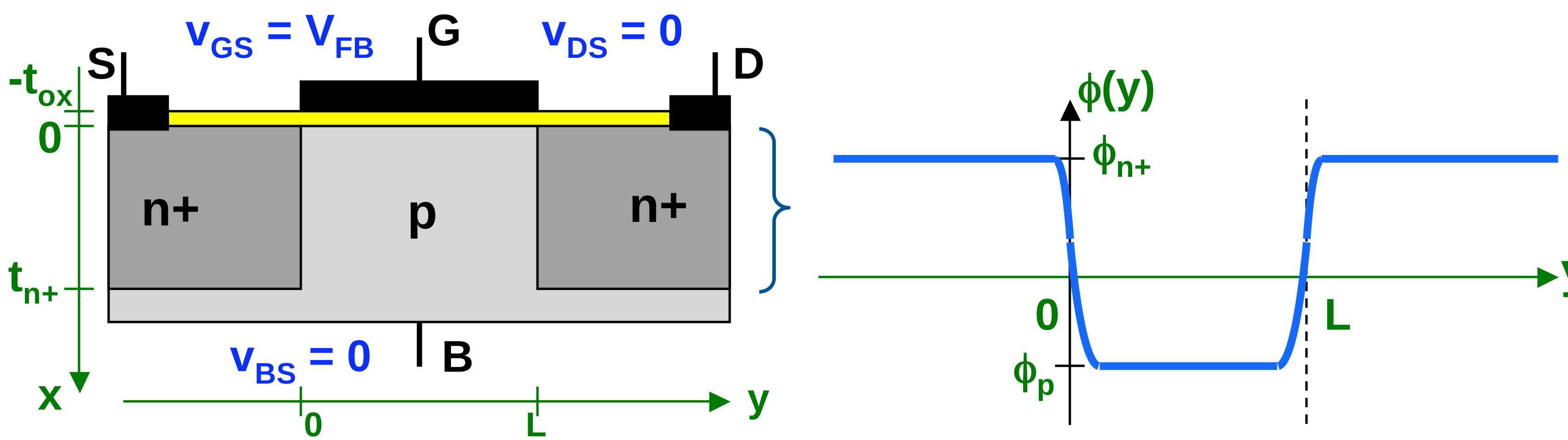
In Lecture 25 we'll talk about MIT/TI research on sub-threshold circuits with 0.3 V supplies and using μW's of power.

Sub-threshold Operation of MOSFETs: finding i_D

Begin by considering the device illustrated below:

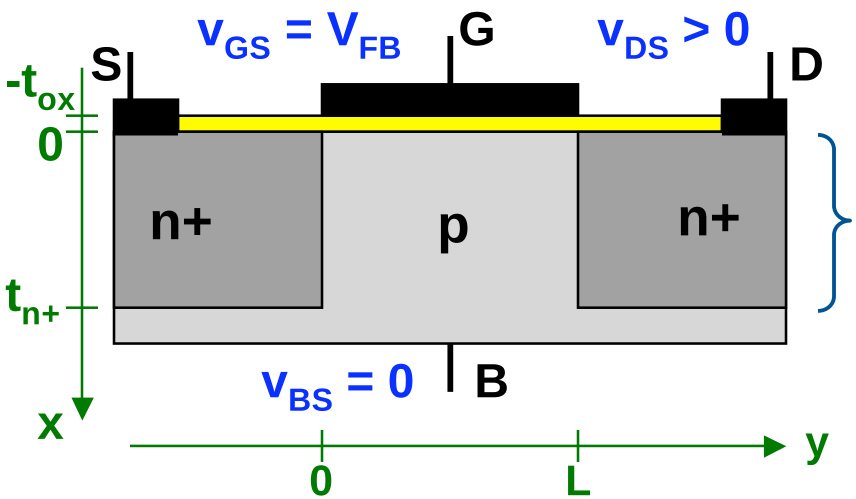


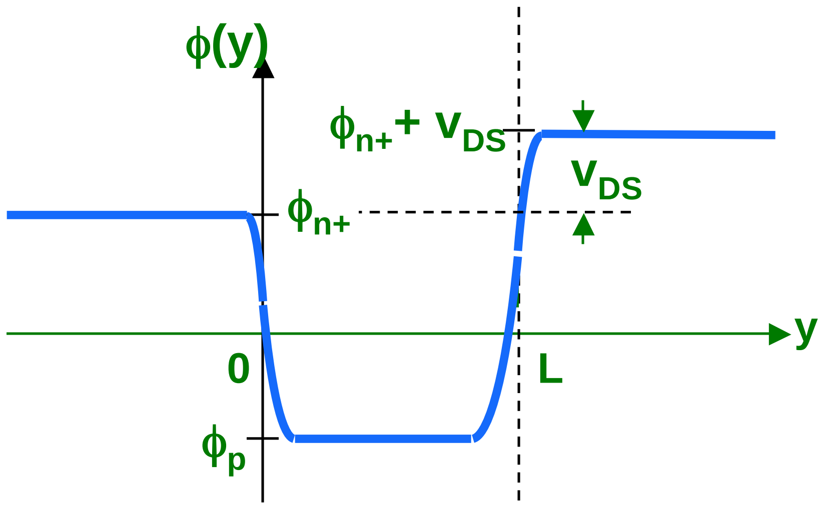
- Set $v_{GS} = V_{FB}$, and $v_{DS} = v_{BS} = 0$.
- The potential profile vs. y, $\phi(y)$ at <u>any x</u> between 0 and t_{n+} is then:



Sub-threshold Operation of MOSFETs, cont.

- Now consider $\phi(y)$ when $v_{GS} = V_{FB}$, $v_{BS} = 0$. and $v_{DS} > 0$:

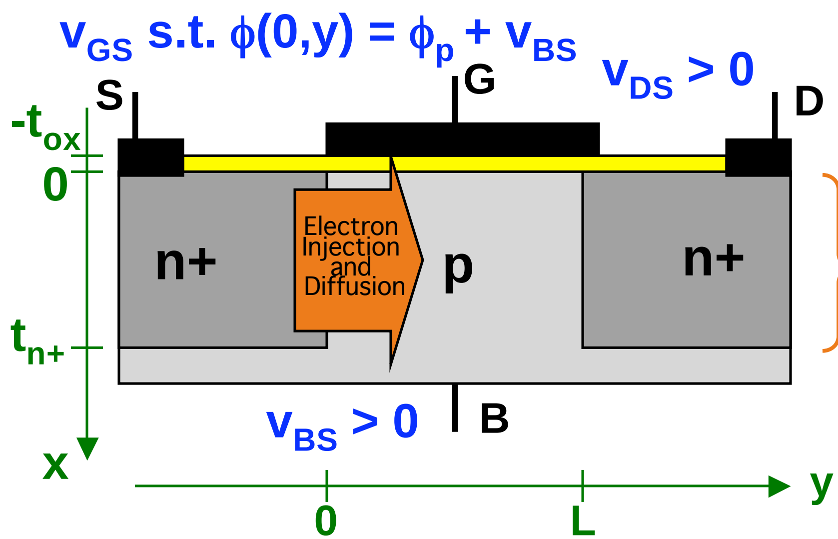


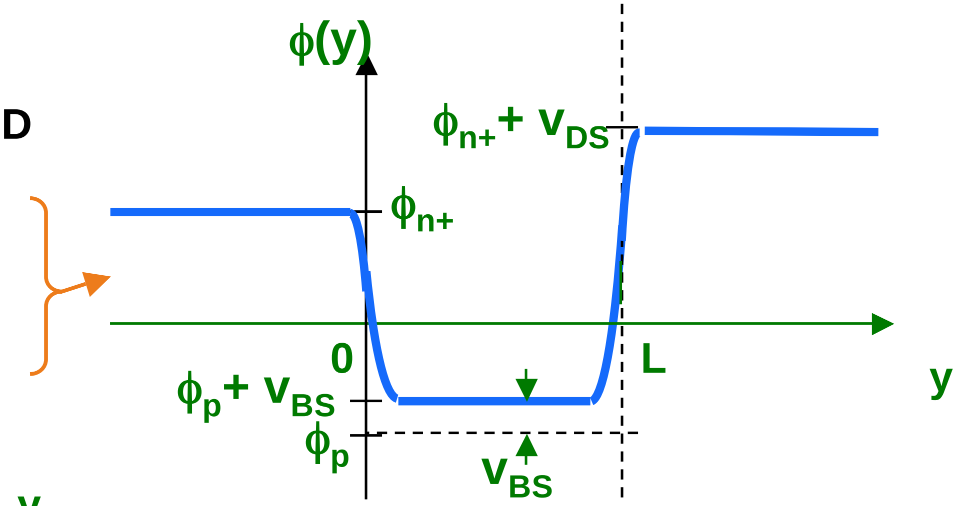


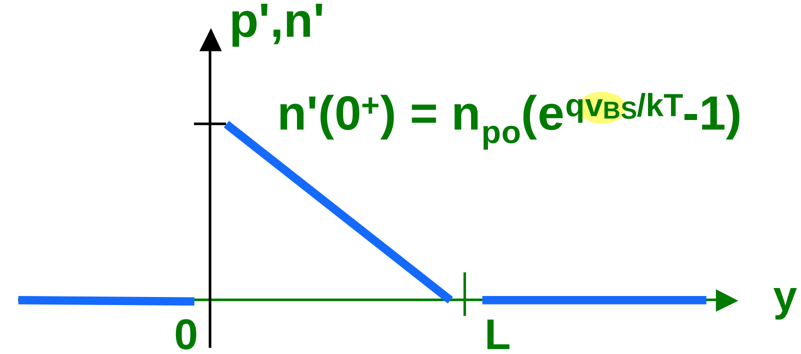
- So far this is standard MOSFET operating procedure. We could apply a positive voltage to the gate and when it was larger than V_T we would see the normal drain current that we modeled earlier. Rather than do this, however, consider forward biasing the substrate-source diode junction, I.e, $v_{BS} > 0$…

Sub-threshold Operation of MOSFETs, cont.

- Apply $v_{BS} > 0$, keep the same $v_{DS} > 0$, and adjust v_{GS} such that the potential at the oxide-Si interface, $\phi(0,y)$, equals $\phi_p + v_{BS}$.
- Now consider $\phi(x,y)$:







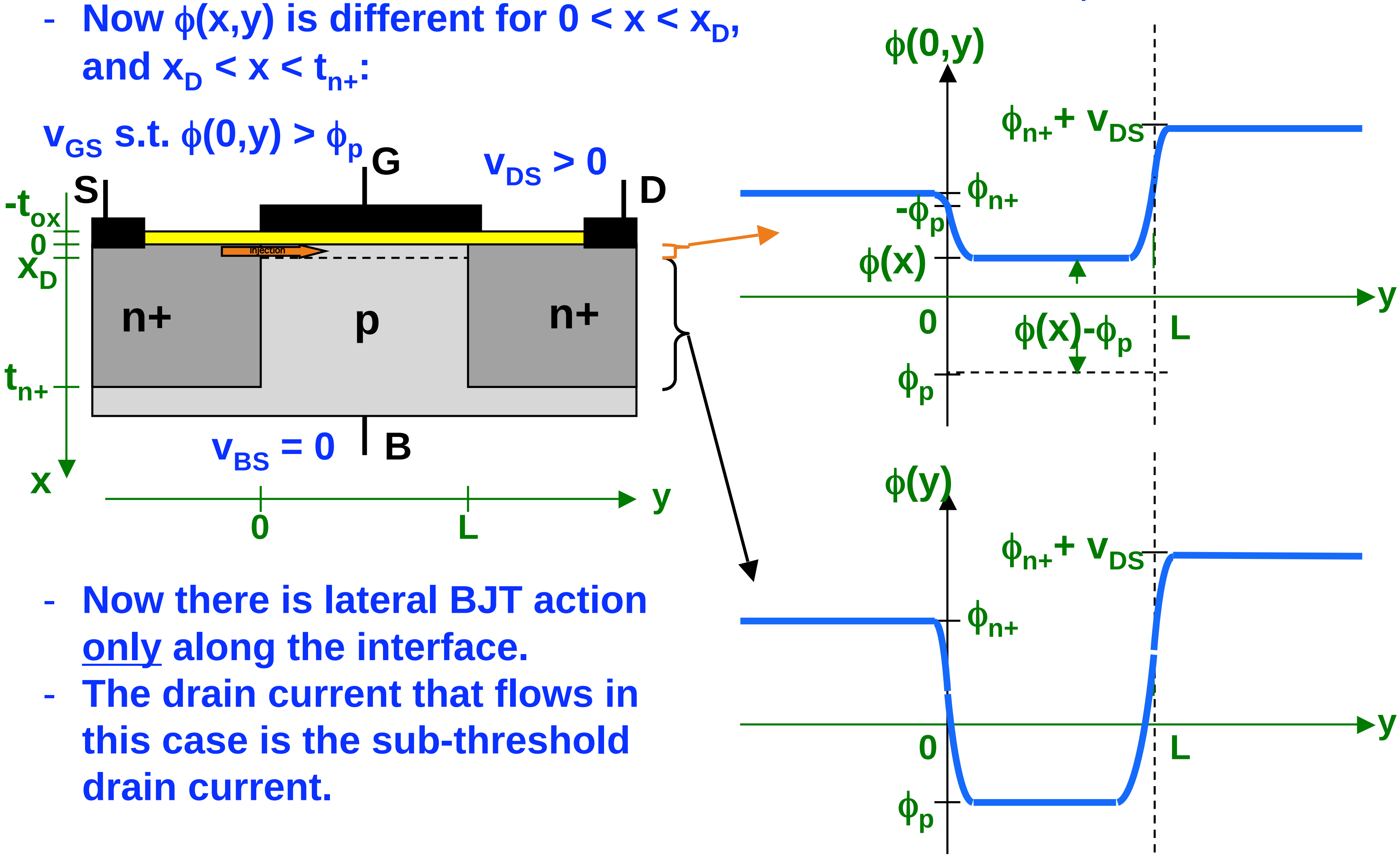
- With this biasing the structure is being operated as a lateral BJT! The drain/collector current is:

$$i_{D/C} \approx W\, t_{n+} q n_i^2 \frac{D_e}{N_{Ap} L_{eff}} \left(e^{q v_{BS}/kT} - 1 \right)$$

- This is not sub-threshold operation yet.

Sub-threshold Operation of MOSFETs, cont.

- Now again make v_{BS} = 0, but keep the same v_{DS} and v_{GS} so that the potential at the oxide-Si interface, $\phi(0,y)$, is still > ϕ_p.
- Now $\phi(x,y)$ is different for $0 < x < x_D$, and $x_D < x < t_{n+}$:

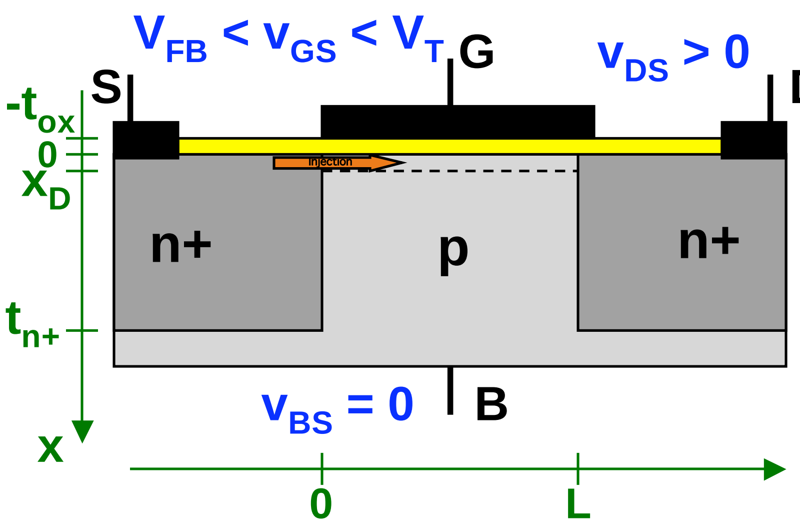


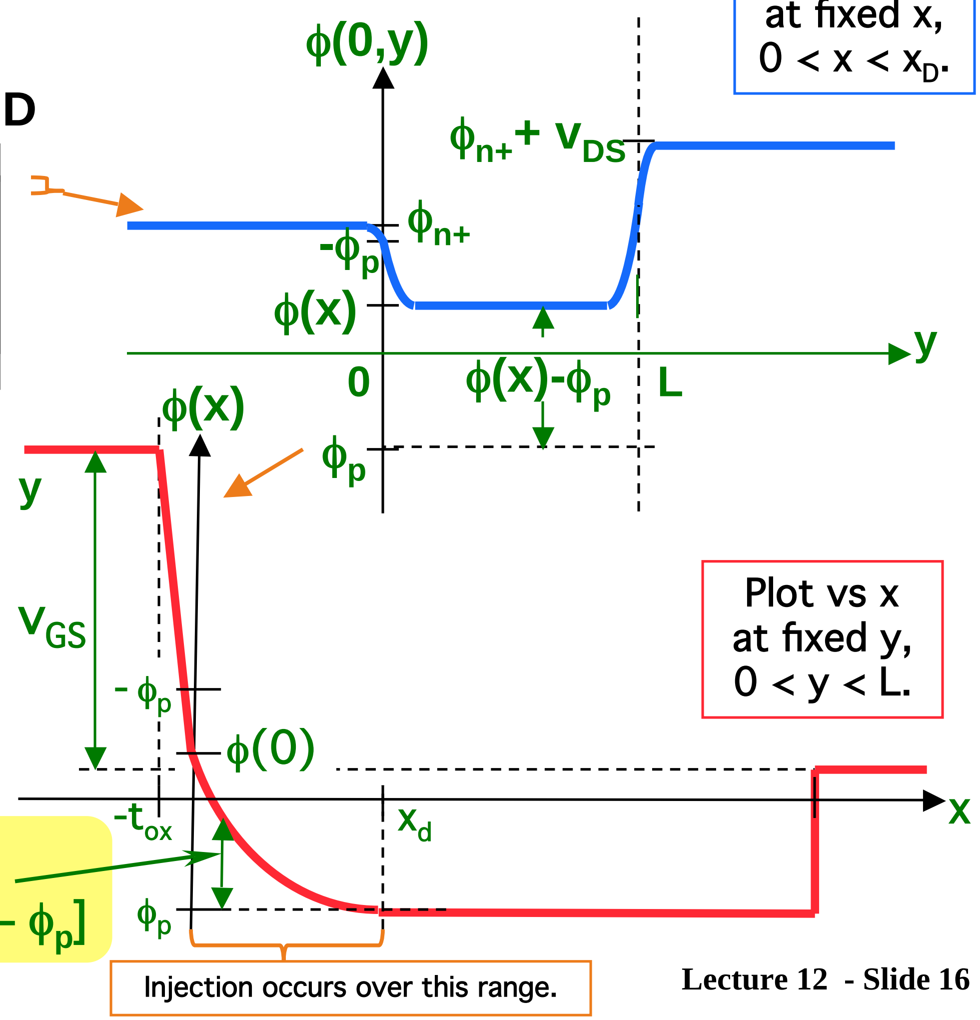
- Now there is lateral BJT action only along the interface.
- The drain current that flows in this case is the sub-threshold drain current.

- This is sub-threshold operation!

Sub-threshold Operation of MOSFETs, cont.

- The barrier at the n⁺-p junction is lowered near the oxide-Si interface for any $v_{GS} > V_{FB}$.
- The barrier is lowered by $\phi(x) - \phi_p$ for $0 < x < x_D$.
  (This is the effective v_{BE} on the lateral BJT between x and x + dx.)



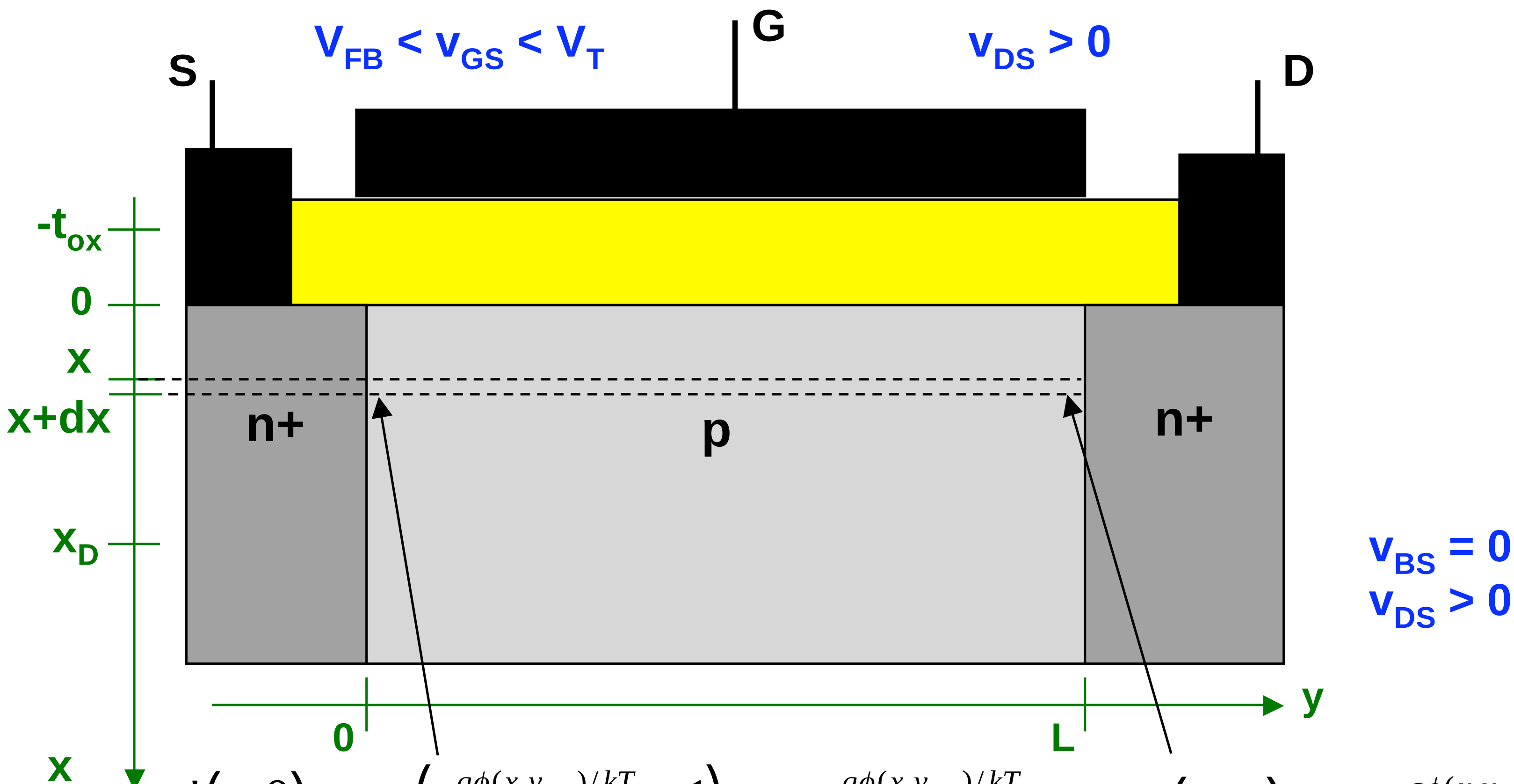


- The barrier lowering (effective forward bias) (1) is controlled by v_{GS}, and (2) decreases quickly with x.

$$v_{BE,eff}(x) = [\phi(x) - \phi_p]$$

Sub-threshold Operation of MOSFETs, cont.

- To calculate i_D, we first find the current in each dx thick slab:



$$n'(x,0) = n_i\left(e^{q\phi(x,v_{GS})/kT} - 1\right) \approx n_i e^{q\phi(x,v_{GS})/kT} \qquad n'(x,L) \approx n_i e^{q\phi(x,v_{GD})/kT}$$

$$di_D(x) = q\,D_e\,\frac{n'(x,0) - n'(x,L)}{L}\,W\,dx \approx \frac{W}{L} D_e\,q\,n_i e^{q\phi(x,v_{GS})/kT}\left(1 - e^{-qv_{DS}/kT}\right)dx$$

Sub-threshold Operation of MOSFETs, cont.

- **Then we add up all the contributions to get i_D:**

$$i_D = \frac{W}{L} D_e \left[\underbrace{\int_{x_d}^{0} q\, n_i e^{q\phi(x,v_{GS})/kT} dx}_{q_{N(sub\text{-}threshold)}} \right] \left(1 - e^{-qv_{DS}/kT}\right)$$

- **This is what we called $q_{N(sub\text{-}threshold)}$ in Lecture 9 and today on Foil 7. Substituting the expression we found for this (see Foil 7), we have:**

$$i_{D(sub-threshold)} = \frac{W}{L} D_e \left[q \frac{kT}{q} \sqrt{\frac{\varepsilon_{Si}}{2qN_A\left[\phi(0,v_{GS}) - \phi_p\right]}}\; n_i e^{q\,\phi(0,v_{GS})/kT} \right] \left(1 - e^{-qv_{DS}/kT}\right)$$

- **Using the Einstein relation and replacing n_i with $N_A e^{q\phi_p/kT}$, we obtain:**

$$i_{D(sub-th)} = \frac{W}{L} \mu_e\, C_{ox}^{*} \left(\frac{kT}{q}\right)^2 \frac{1}{2C_{ox}^{*}} \sqrt{\frac{2q\varepsilon_{Si} N_A}{\left[\phi(0,v_{GS}) - \phi_p\right]}}\; e^{q\left\{\phi(0,v_{GS}) - \left[-\phi_p\right]\right\}/kT} \left(1 - e^{-qv_{DS}/kT}\right)$$

- **To finish (we are almost done) we need to replace $\phi(0,v_{GS})$ with v_{GS} since we want the drain current's dependence on the terminal voltage.**

Sub-threshold Operation of MOSFETs, cont.

- The relationship relating $\phi(0,v_{GS})$ and v_{GS} is:

$$v_{GS} = V_{FB} + \left[\phi(0) - \phi_p\right] + \frac{1}{C_{ox}^*}\sqrt{2\varepsilon_{Si} q N_A \left[\phi(0) - \phi_p\right]}$$

- From this we can relate a change in v_{GS} to a change in $\phi(0)$, which is what we really need. To first order the two are linearly related:

$$\Delta v_{GS} \approx \frac{dv_{GS}}{d\phi(0)}\,\Delta\phi(0) = \underbrace{\left\{1 + \frac{1}{2\,C_{ox}^*}\sqrt{\frac{2\varepsilon_{Si} q N_A}{\left[\phi(0) - \phi_p\right]}}\right\}}_{\equiv n}\Delta\phi(0) \equiv n\,\Delta\phi(0)$$

- In the current equation we have the quantity $\{\phi(0,v_{GS}) - [-\phi_p]\}$. $-\phi_p$ is simply $\phi(0,V_T)$, the potential at x = 0 when the gate voltage is V_T, so

$$\left\{\phi(0,v_{GS}) - \left[-\phi_p\right]\right\} = \left\{\phi(0,v_{GS}) - \phi(0,V_T)\right\} = \left\{v_{GS} - V_T\right\}/n$$

- Using this and the definition for n, we arrive at:

$$\boxed{i_{D(sub-threshold)} \approx \frac{W}{L}\mu_e\, C_{ox}^* \left(\frac{kT}{q}\right)^2 (n-1)\; e^{q\{v_{GS}-V_T\}/nkT}\left(1 - e^{-qv_{DS}/kT}\right)}$$

Sub-threshold Operation of MOSFETs, cont.

- To fully complete our modeling, we must add two more points:

1. The dependences on v_{BS} and v_{DS}:

   v_{BS}: The threshold voltage depends on v_{BS}. $\phi(0,V_T)$ does also, i.e. $\phi(0,V_T) = -\phi_p - v_{BS}$, and so do the junction barriers. Taking this all into account we find that the only change we need to make is to acknowledge that n and V_T both depend on v_{BS}.

   v_{DS}: The drain to source voltage introduced a factor $(1 - e^{-qv_{DS}/kT}) \approx 1$. This is discussed in the handout posted on Stellar.

   The complete expression for i_D is:

$$i_{D,s-t}(v_{GS},v_{DS},v_{BS}) \approx \frac{W}{L}\mu_e\, C_{ox}^*\left(\frac{kT}{q}\right)^2\left[n(v_{BS})-1\right]\, e^{q\{v_{GS}-V_T(v_{BS})\}/nkT}\left(1-e^{-qv_{DS}/kT}\right)$$

2. The factor n:

   The value of n depends on $\phi(0,v_{GS})$. Notice, however, that the sub-threshold current is largest as $\phi(0,v_{GS})$ approaches $-\phi_p - v_{BS}$, so it makes sense to evaluate it there and take that as its value for all v_{GS}:

$$n \equiv \left\{1+\frac{1}{2C_{ox}^*}\sqrt{\frac{2\varepsilon_{Si}qN_A}{\left[\phi(0)-\phi_p\right]}}\right\} \approx \left\{1+\frac{1}{C_{ox}^*}\sqrt{\frac{\varepsilon_{Si}qN_A}{2\left[-2\phi_p - v_{BS}\right]}}\right\}$$

** Notice that this is exactly the same expression as that for α!

Sub-threshold Operation of MOSFETs, cont.

- Comparing current levels above and below threshold:

  The ranges of the two models do not overlap, but is it still interesting to compare the largest possible value of the sub-threshold drain current model (v_{GS} - V_T = 0 V),* with the strong inversion model at v_{GS} - V_T = 0.06 V, 0.1 V, and 0.2 V:

$$\frac{i_{D(sub-threshold)}}{K} \approx \left(\frac{kT}{q}\right)^2 (n-1)\; e^{q\{v_{GS}-V_T\}/nkT}$$

$v_{BS} = 0$

$(0.025)^2$ 0.25 1 $= 1.56 \times 10^{-4}\ V^2$

$$\frac{i_{D(strong\ inversion)}}{K} \approx \frac{1}{2\alpha}\left(v_{GS}-V_T\right)^2$$

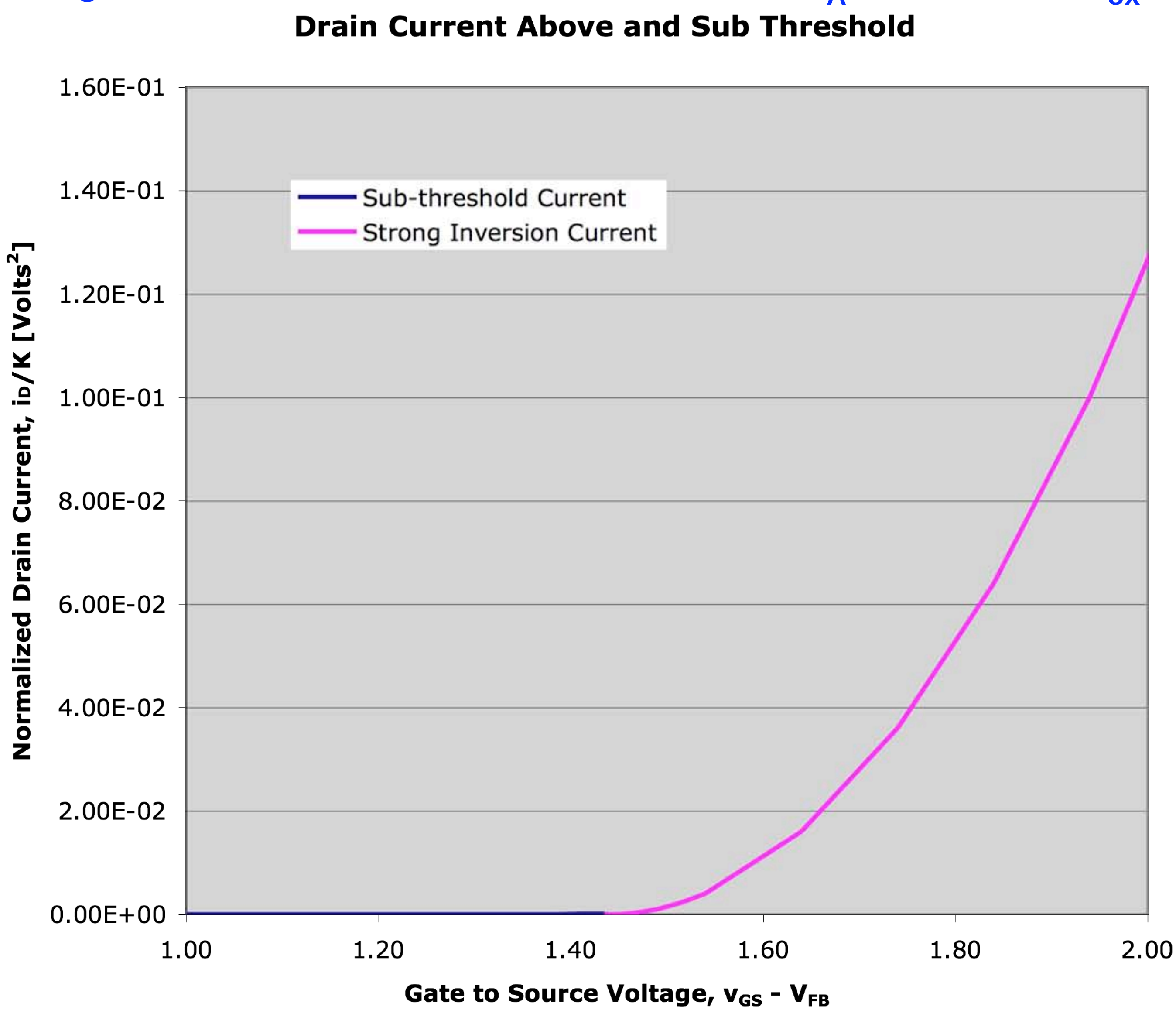
0.4	$(0.06)^2$	$= 1.5 \times 10^{-3}\ V^2$
	$(0.1)^2$	$= 4 \times 10^{-3}\ V^2$
	$(0.2)^2$	$= 1.6 \times 10^{-2}\ V^2$

We see that the current in strong inversion drift current quickly becomes much larger, although only grows quadratically.

* This is pushing the model, particularly with regard to the diffusion current model, beyond it's range of strict validity, and is probably somewhat of an over-estimate.

Sub-threshold Operation of MOSFETs, cont.

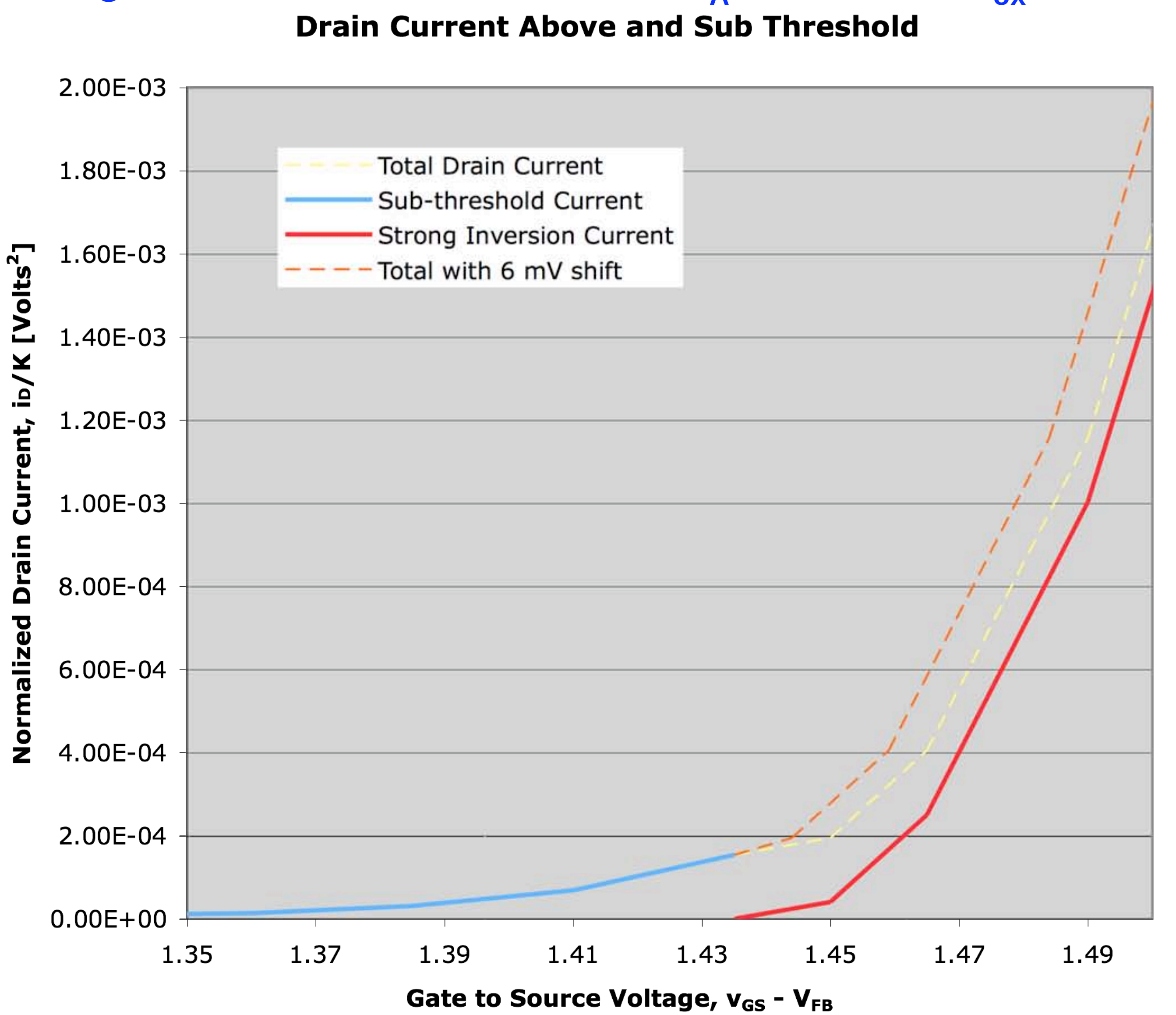
- Plotting our models for the earlier device: $N_A = 10^{18}$ cm^{-3}, $t_{ox} = 3$ nm:



$v_{BS} = 0$

Sub-threshold Operation of MOSFETs, cont.

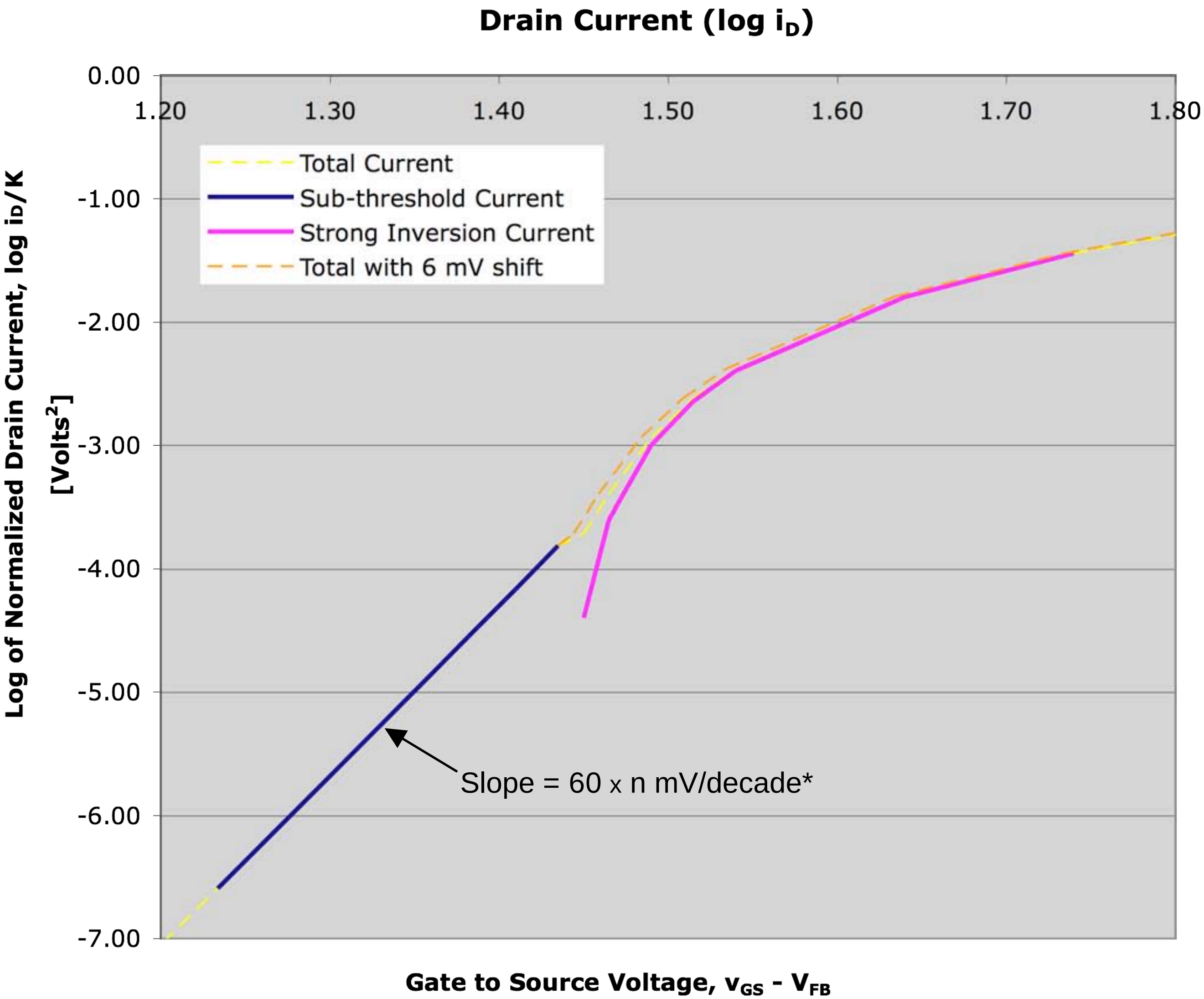
- Zooming into a lower current scale: $N_A = 10^{18}\ cm^{-3}$, $t_{ox} = 3$ nm:



$v_{BS} = 0$

Sub-threshold Operation of MOSFETs, cont.

- Repeating the plot with a log current scale: $N_A = 10^{18}$ cm^{-3}, $t_{ox} = 3$ nm:



$v_{BS} = 0$

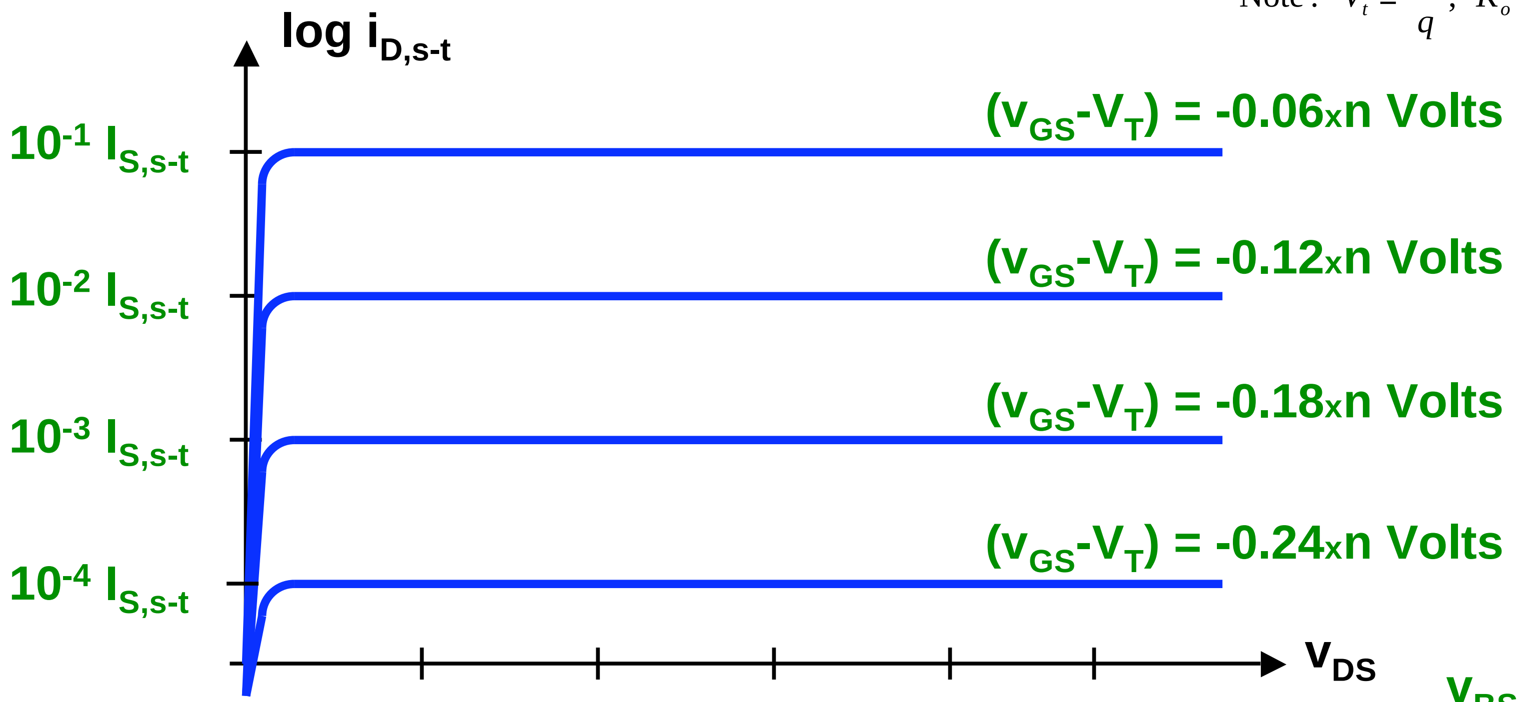
* n = 1.25 here so 75 mV/decade

Sub-threshold Output Characteristic

- We plot a family of i_D vs v_{DS} curves with $(v_{GS} - V_T)$ as the family variable, after first defining the sub-threshold diode saturation current, $I_{S,s\text{-}t}$:

$$I_{S,s-t} \equiv \frac{W}{L}\mu_e\, C_{ox}^* \left(\frac{kT}{q}\right)^2 [n-1] \;=\; K_o V_t^2 [n-1]$$

$$\text{Note}: \quad V_t \equiv \frac{kT}{q}, \;\; K_o \equiv \frac{W}{L}\mu_e\, C_{ox}^*$$



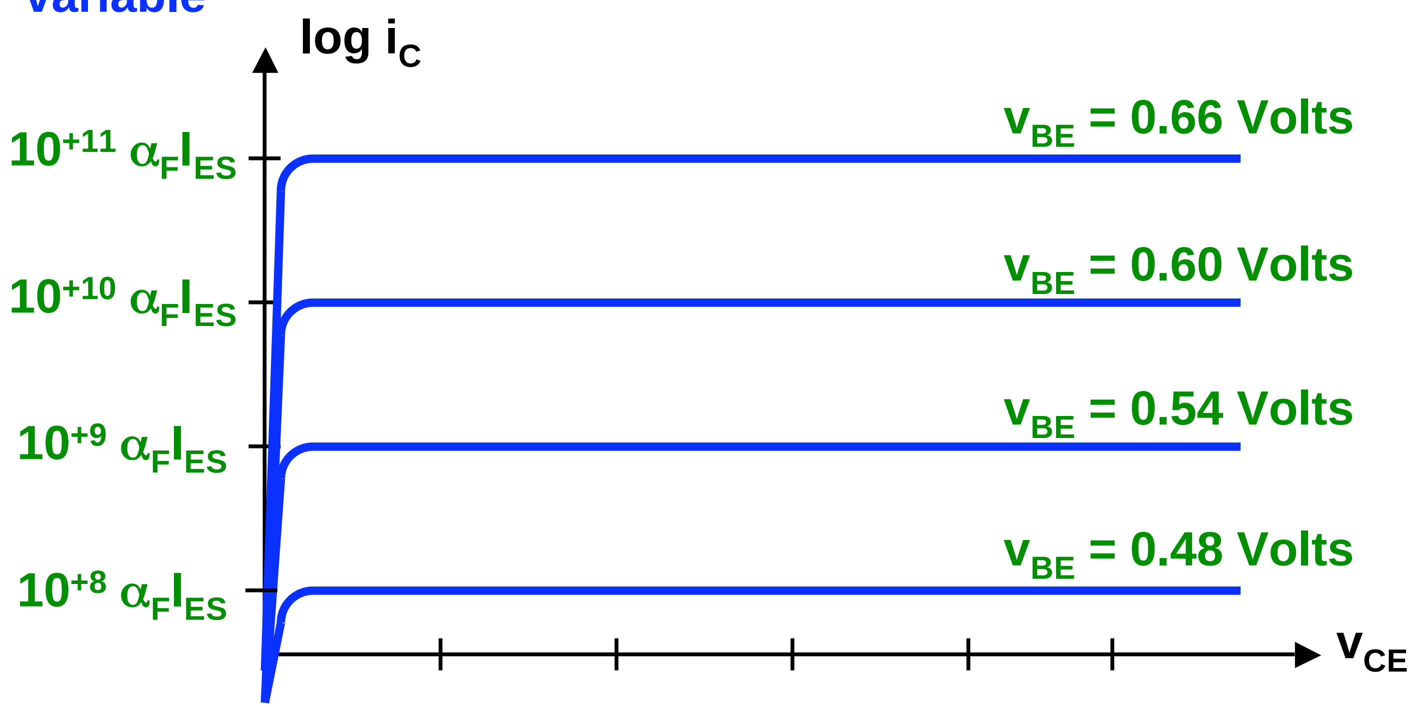
$v_{BS} = 0$

$$i_{D,s-t}(v_{GS}, v_{DS}) \approx I_{S,s-t}\, e^{q\{v_{GS}-V_T\}/nkT}\left(1 - e^{-qv_{DS}/kT}\right)$$

Note: The device we modeled had n = 1.25, so it follows a "75 mV rule" [i.e. 60 x n = 75].

Sub-threshold Output Characteristic, cont.

- To compare this with something we've already seen, consider the BJT and plot a family of i_C vs v_{CE} curves with v_{BE} as the family variable



$$i_C(v_{BE}, v_{CE}) \approx \alpha_F\, I_{ES} e^{q v_{BE}/kT}\left(1 - e^{-q v_{CE}/kT}\right)$$

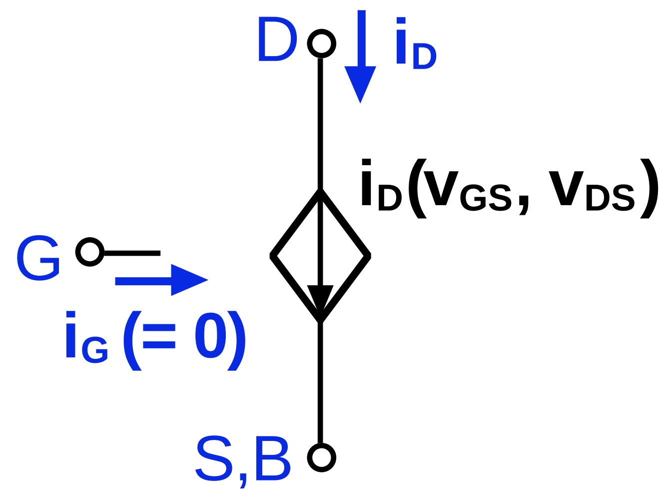
- The two biggest differences are (1) the magnitudes of the I_S's, and (2) the factor of "n" in the MOSFET case. The totality of v_{BE} reduces the barrier, whereas only a fraction 1/n of v_{GS} does.
- A third difference is that a BJT has a base current.*

* This is the price paid for having n = 1 in a BJT.

Large Signal Model for MOSFET Operating Sub-threshold

- The large signal model for a MOSFET operating in the weak inversion or sub-threshold region looks the same model as that for a device operating in strong inversion ($v_{GS} > V_T$) EXCEPT there is a different equation relating i_D to v_{GS}, v_{DS}, and v_{BS}:

  We will limit our model to $v_{GS} \le V_T, v_{DS} > 3kT/q$ and $v_{BS} = 0.$



$$i_{G,s-t}(v_{GS}, v_{DS}, v_{BS}) = 0$$

$$i_{D,s-t}(v_{GS}, v_{DS}, 0) \approx I_{S,s-t} \underbrace{\left(1 - \lambda v_{DS}\right)}_{\text{Early effect}} e^{q\{v_{GS} - V_{To}\}/nkT} \underbrace{\cancel{\left(1 - e^{-qv_{DS}/kT}\right)}}_{\approx 1 \text{ for } v_{DS} > 3kT/q}$$

Lecture 12 - Sub-threshold MOSFET Operation - Summary

Sub-threshold operation - qualitative explanation

Look back at Lecture 10 (Sub-threshold electron charge)

BJT action in depletion/weak inversion layer along oxide the interface

MOSFET gate-controlled lateral BJT

Important in/for

1. power dissipation in normally-off logic gates
2. limiting the gain of strong inversion linear amplifiers
3. realizing ultra-low power, very low voltage electronics

Quantitative sub-threshold modeling

This gives us a precise description of the voltage dependence

It also gives us the information on $I_{S,s\text{-}t}$ and n we need for device design

$$i_{D,s-t}(v_{GS},v_{DS},v_{BS}) \approx I_{S,s-t}e^{q\{v_{GS}-V_T(v_{BS})\}/nkT}\left(1-e^{-qv_{DS}/kT}\right)$$

with:

$$I_{S,s-t} \equiv \frac{W}{L}\mu_e\, C_{ox}^*\left(\frac{kT}{q}\right)^2[n-1] \quad \text{and} \quad n \approx \left\{1+\frac{1}{C_{ox}^*}\sqrt{\frac{\varepsilon_{Si}qN_A}{2[-2\phi_p - v_{BS}]}}\right\} = \alpha$$