MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.012 MICROELECTRONIC DEVICES AND CIRCUITS

Answers to Exam 1 - Spring 2006

Problem 1: Graded by Prof. Hoyt

a) i) N_a = B concentration = $2 \times 10^{16}\ \text{cm}^{-3}$.

ii) $N_A = N_a - N_d$ = B concentration - As concentration = $2 \times 10^{16} - 1 \times 10^{16} = 10^{16}\ \text{cm}^{-3}$.

iii) $p_o \approx N_A = 10^{16}\ \text{cm}^{-3}$.

iv) $n_o = n_i^2/p_o = (10^{20}\ \text{cm}^{-6})/(10^{16}\ \text{cm}^{-6}) = 10^4\ \text{cm}^{-3}$.

v) $\sigma_o = qn_o\mu_e + qp_o\mu_h \approx qp_o\mu_h = (1.6 \times 10^{19}\ \text{coul})(10^{16}\ \text{cm}^{-3})(600\ \text{cm}^2/\text{V-s}) = 0.96\ \text{S/cm}$.

vi) $\phi_p = -(kT/q) \ln (p_o/n_i) \approx -60\ \text{mV} \times 6 = -360\ \text{mV} = -0.36\ \text{V}$.

b) i) $R = \rho\, L/w\, t = L/\sigma\, w\, t = (10^{-1}\ \text{cm})/(0.96\ \text{S/cm})(10^{-3}\ \text{cm})(10^{-4}\ \text{cm}) = 1.04 \times 10^6\ \Omega$.

ii) $E = V_{AB}/L = 2\ \text{V}/(10^{-1}\ \text{cm}) = 20\ \text{V/cm}$.

iii) $J_{dr} = \sigma E = 0.96\ \text{S/cm} \times 20\ \text{V/cm} = 19.2\ \text{A/cm2}$

c) i) $L_{min} = (D_{min}\tau_{min})^{1/2} = [(40\ \text{cm/s})(10^{-7}\ \text{s})]^{1/2} = 2 \times 10^{-3}\ \text{cm}$.

ii) Far from the ohmic contacts $n' \approx p' \approx G_L\tau_{min} = (10^{23}\ \text{cm}^{-2}\ \text{s}^{-1})(10^{-7}\ \text{s}) = 10^{16}\ \text{cm}^{-3}$. (Note this is no longer strictly low level injection, but we did not take off points for this because we had intended it to be LLI; it did initially but then someone whose initials are CF changed the lifetime at the last minute.)

$$\sigma \approx q\mu_h p_o + q\mu_e n' + q\mu_h p' = \sigma_o + q(\mu_e + q\mu_h)n'$$
$$= 0.96\ \text{S/cm} + (1.6 \times 10^{-19}\ \text{coul})(600\ \text{cm}^2/\text{V-s} + 1600\ \text{cm}^2/\text{V-s})(10^{16}\ \text{cm}^{-3})$$
$$= 0.96 + 3.52 = 4.48\ \text{S/cm}$$

Problem 2: Graded by Prof. Fonstad

a) i) $dn'/dx\,|_{x=0} = 0$

ii) $n'(L) = 0$

b) $L_{min}^2 = D_{min}\tau_{min}$ so $\tau_{min} = L_{min}^2/D_{min}$. $D_{min} = D_e = \mu_e/40 = 40\ \text{cm}^2/\text{s}$, and $L_{min} = 100\ \mu\text{m} = 10^{-2}\ \text{cm}$. Thus $\tau_{min} = (10^{-2})^2/40 = 2.5 \times 10^{-6}\ \text{s} = 2.5\ \mu\text{s}$.

c) i) D, because the minority carrier diffusion flux must be zero at x = 0, and the flux builds up (increases) linearly because there is uniform optical creation of hole-electron pairs and no appreciable recombination. Alternatively one could say that the integral of the generation function (a constant in this case) yields the gradient in the excess population, and the flux proportional to this. The integral of a constant is a line of constant slope.

ii) A, because it is consistent with the answer in (i), because it is the only profile that fits the boundary conditions, and/or it is what one gets by integrating a constant twice.

d) With no recombination the total flux density into the ohmic contact must be the the total generation within the sample, which is the integral of $g_L(x)$ from x = 0 to x = L. In this case $g_L(x)$ is a constant, call it G_L, so the integral is $G_L L$. The current density is the charge per carrier, -q, times the flux: $J_{min}(x) = -q\, G_L L = -1.6 \times 10^{-19} \times 10^{19} \times 10^{-3} = -1.6 \times 10^{-3}$ A/cm^2 = -1.6 mA/cm^2.

e) i) The recombination at any point is $n'(x)/\tau_{min}$, so the total recombination per unit area is $\int_{0\to L}[\, n'(x)/\tau_{min}]dx$. If one finds n'(x) and does this integral you get $G_L L^3/3\tau_{min}D_{min} = G_L L^3/3L_{min}^2$. The numerical answer is 3.2×10^{13} cm^{-2}s^{-1}.

ii) Much greater than, because even though there is non-zero recombination in the bulk, it is still much less than occurs at the ends. If one compares the answers in Parts (d) and (e-i), one sees that the fraction recombining in the bulk is $L^2/3L_{min}^2$. In this case this is 1/300, so 300 times as much recombination occurs at the contact as within the bulk.

The important concept upon which Part (e) of this problem is based is that one can find the excess carrier and flux profiles in a short base situation (i.e., long diffusion length) assuming that the lifetime in infinite, and then turn around and find out how much recombination occurs (and this how good the initial assumption was) by using the true minority carrier lifetime and the profile calculated assuming infinite lifetime to estimate the amount of recombination. Comparing this to the total minority carrier generation and/or injection, or to the recombination fluxes at the contacts.

Problem 3: Graded by Prof. Antoniadis

a) i) 0.84 V using the 60 mV rule; 0.81 V if use kT = 0.025 V and evaluate the exponential.

ii) 1 μm

iii) $J_e/J_h = (D_e/D_h)(N_D/N_{A1})[w_n/(w_p-x_p)] = 3 \times (1/3) \times$

b) i) Same as (a-i).

ii) $x_p \approx [2\varepsilon_{Si}(\phi_b - V_1)/qN_{A1})]^{1/2}$ so $V_1 = -(qN_{A1}x_p^{\,2}/2\varepsilon_{Si}) + \phi_b = -2.4$ V

iii) The small signal linear equivalent capacitance is the same as a parallel plate capacitor with the same width as the depletion region and the dielectric constant of silicon:

$$C_{ab}(V_1) = e_{Si}/w(V_1) \approx \varepsilon_{Si}/x_p(V_1) = 10^{-12}/2 \times 10^{-4} = 5 \times 10^{-9}\ \text{F/cm}^2.$$

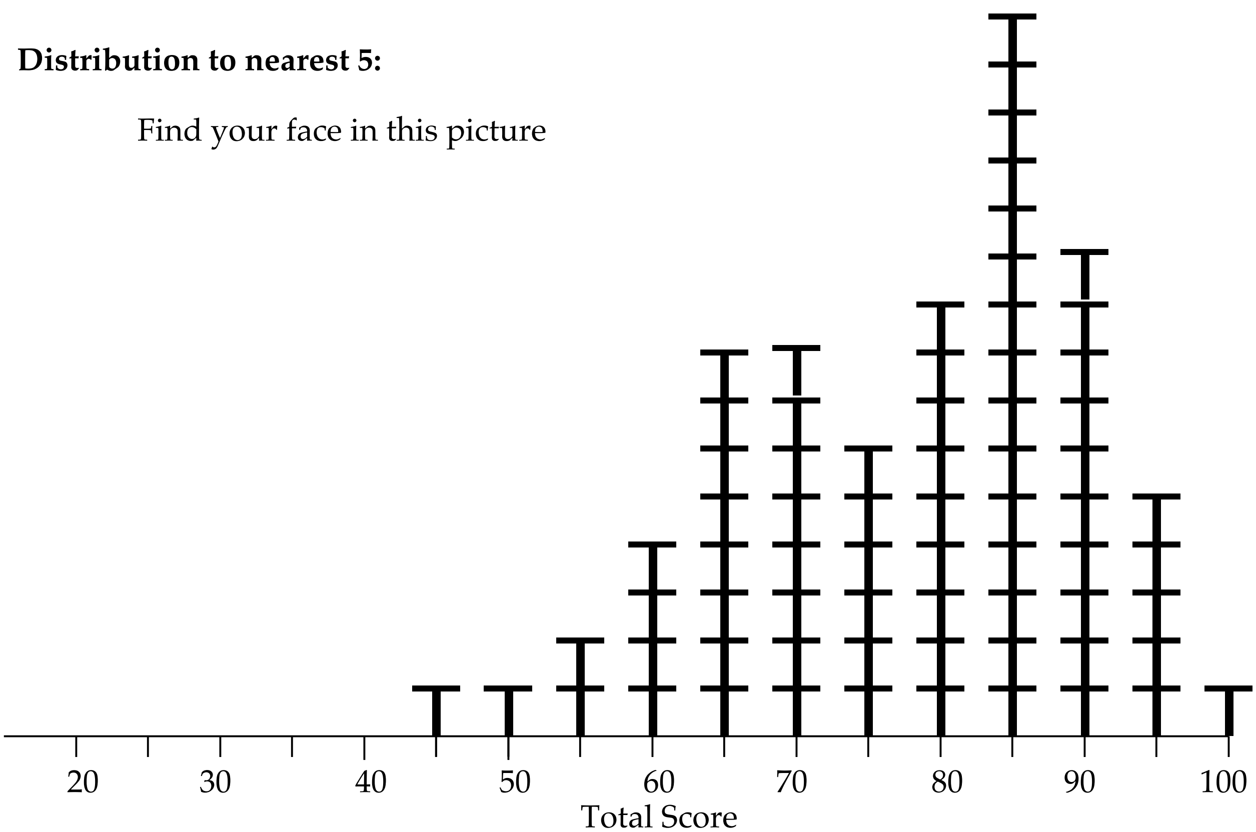
iv) Figure b because x_p will increase only marginally for $v_{AB} > V_1$ due to $N_{A2} >> N_{A1}$. Consequently, the depletion region width, w, which is essentially the same as x_p, will not increase either. The capacitance per unit area, C_{ab}, is ε_{Si}/w, so it will not decrease appreciably for $v_{AB} > V_1$.

Average/Standard deviation:

Problem 1	29.4	4.5
Problem 2	24.4	5.3
Problem 3	23.9	5.3
Total	77.8	11.9

Distribution to nearest 5:

Find your face in this picture



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