Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.011: Signals, Systems and Inference

QUIZ 2

QUESTION & ANSWER BOOKLET

Your Full Name:	
Recitation Time :	o'clock

This quiz is **closed book**, but **three** sheets of notes (both sides) are allowed. Calculators and other electronic aids will not be necessary and are not allowed.

Check that this QUESTION & ANSWER BOOKLET has pages numbered up to 8. The booklet contains spaces for all relevant work and reasoning.

Neat work and clear explanations count; show all relevant work and reasoning! You may want to first work things through on scratch paper and then neatly transfer to this booklet the work you would like us to look at. Let us know if you need additional scratch paper. **Only** this booklet will be considered in the grading; **no additional answer or solution written elsewhere will be considered.** Absolutely no exceptions!

There are **3 problems, each carrying the indicated number of points**, for a total of 30 points. (The points assigned to subparts may be changed slightly when we get to grading.)

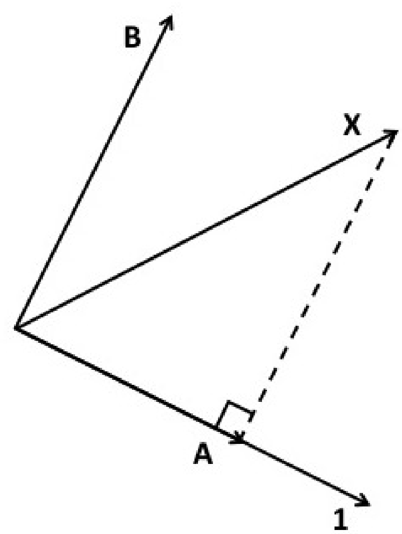
Problem	**Your Score**
1 (6 points)	
2 (6 points)	
3 (18 points)	
Total (30 points)	

Some of the questions on the following pages ask whether a given statement is **True** or **False**. In each such case, circle your answer and give a sufficiently detailed and convincing explanation of this answer. (For a False statement, a clear counter-example might suffice.)

You will get **no points at all** for simply marking True or False, with no supporting reasoning!

Problem 1 (6 points)

The random variable X has mean μ_X and variance σ_X^2. The figure shows a vector representation of this random variable and a vector representation of the "random variable" 1. Also shown are the components A and B of X, respectively along 1 and orthogonal to 1. What are the lengths of X, A and B, expressed in terms of μ_X and σ_X?



Length of $X =$

Length of $A =$

Length of $B =$

Problem 2 (6 points)

(a) **Claim**: If X and Y are uncorrelated, then $E[X^2 Y] = E[X^2]E[Y]$.

TRUE or **FALSE** ? **Explanation**:

(b) **Claim**: If X and Y are uncorrelated, then $E[Y|X] = E[Y]$.

TRUE or **FALSE** ? **Explanation**:

Problem 3 (18 points)

All parts of this question refer to a WSS random process $x[n]$ with mean value $\mu_x = 3$ and autocovariance function

$$C_{xx}[m] = -0.6\,\delta[m+2] + 1.2\,\delta[m] - 0.6\,\delta[m-2]\ .$$

As usual, we will denote the fluctuation from the mean by $\widetilde{x}[n] = x[n] - \mu_x$.

3(a) (2 points) Make a fully labeled sketch of $C_{xx}[m]$ as a function of m:

3(b) (1 point) **Claim**: The (weak) law of large numbers would tell us that $\lim_{L\to\infty} \frac{1}{2L+1} \sum_{n=-L}^{L} x[n] = \mu_x$ if the values of $x[\cdot]$ at different times were uncorrelated, but this identity holds true in this case as well, even though the uncorrelatedness condition does not hold.

TRUE or **FALSE** ?

Explanation:

3(c) (4 points) Write down the LMMSE estimator $\widehat{x}_L[9]$ of $x[9]$ that uses a measurement of $x[7]$, and determine its mean square error (MSE).

$\widehat{x}_L[9] =$

MSE $=$

3(d) (4 points) The fluctuation spectral density of the process $x[\cdot]$, i.e., the PSD of $\widetilde{x}[\cdot]$, can be written in the form

$$D_{xx}(e^{j\Omega}) = a + b\cos(c\,\Omega)\ .$$

Determine the constants a, b, c and (on the next page) sketch $D_{xx}(e^{j\Omega})$ for $|\Omega| \leq \pi$. (Recall that $\cos(\theta) = \frac{1}{2}e^{j\theta} + \frac{1}{2}e^{-j\theta}$.)

$a =$ $b =$ $c =$

Sketch of $D_{xx}(e^{j\Omega})$:

3(e) (1 point) Use your sketch in 3(d) to determine if the expected instantaneous power of the fluctuations from the mean is concentrated at

LOW or **INTERMEDIATE** or **HIGH** frequencies?

(No explanation needed.)

3(f) We know from class that the LMMSE estimator of $x[n+2]$ using measurements of $x[n]$, $x[n-1]$ and $x[n-2]$ has the form

$$\widehat{x}_L[n+2] = \mu_x + d\,\widetilde{x}[n] + e\,\widetilde{x}[n-1] + f\,\widetilde{x}[n-2]$$

for some constants d, e, f. The corresponding **estimation error** is

$$\widetilde{x}[n+2] - d\,\widetilde{x}[n] - e\,\widetilde{x}[n-1] - f\,\widetilde{x}[n-2]\ .$$

(i) (2 points) **Claim**: $e = 0$, i.e., this LMMSE estimator of $x[n+2]$ does not actually make use of the measurement of $x[n-1]$.

TRUE or **FALSE** ?

(Hint: Check orthogonality of the error to $\widetilde{x}[n-1]$.)
Explanation:

(**Continued on other side** $\Longrightarrow$)

(ii) (4 points) **Claim**: $f = 0$, i.e., this LMMSE estimator of $x[n+2]$ does not actually make use of the measurement of $x[n-2]$.

TRUE or **FALSE** ?

(Hint: Don't solve by intuition, do some calculation!)
Explanation: