Microscopic Ohm's Law

Outline

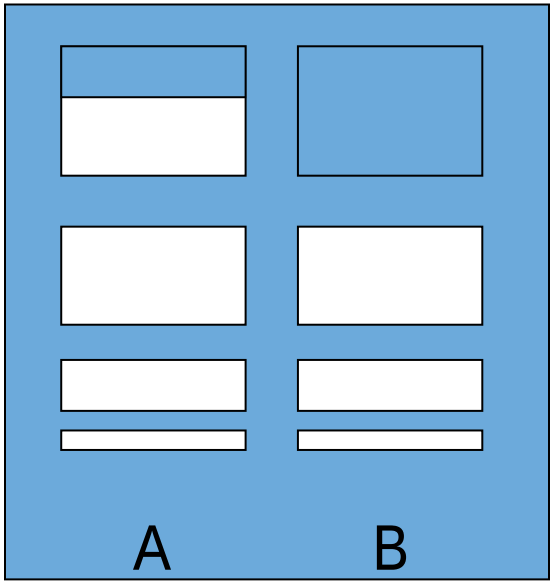
Semiconductor Review

Electron Scattering and Effective Mass

Microscopic Derivation of Ohm's Law

TRUE / FALSE

1. Judging from the filled bands, material A is an insulator.

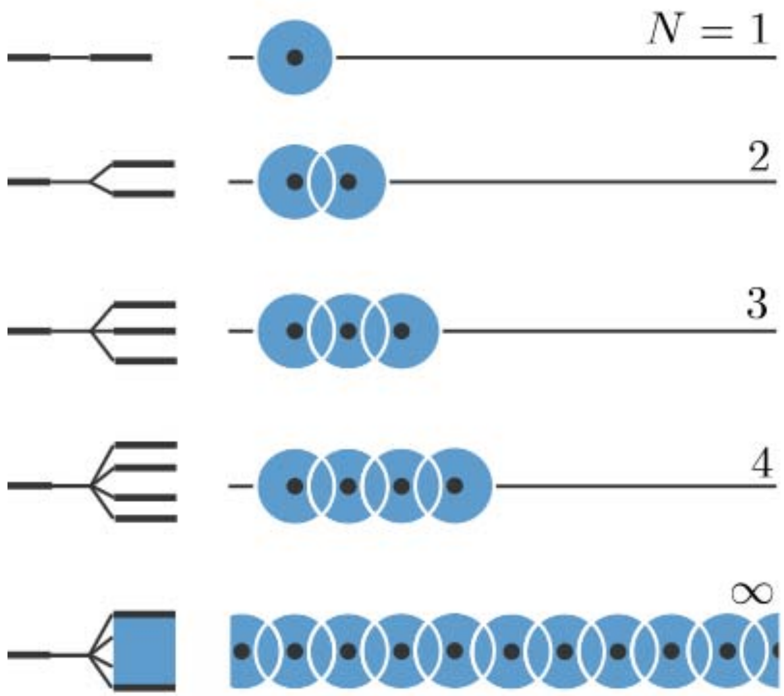


2. Shining light on a semiconductor should decrease its resistance.

3. The band gap is a certain location in a semiconductor that electrons are forbidden to enter.

1-D Lattice of Atoms

Single orbital, single atom basis



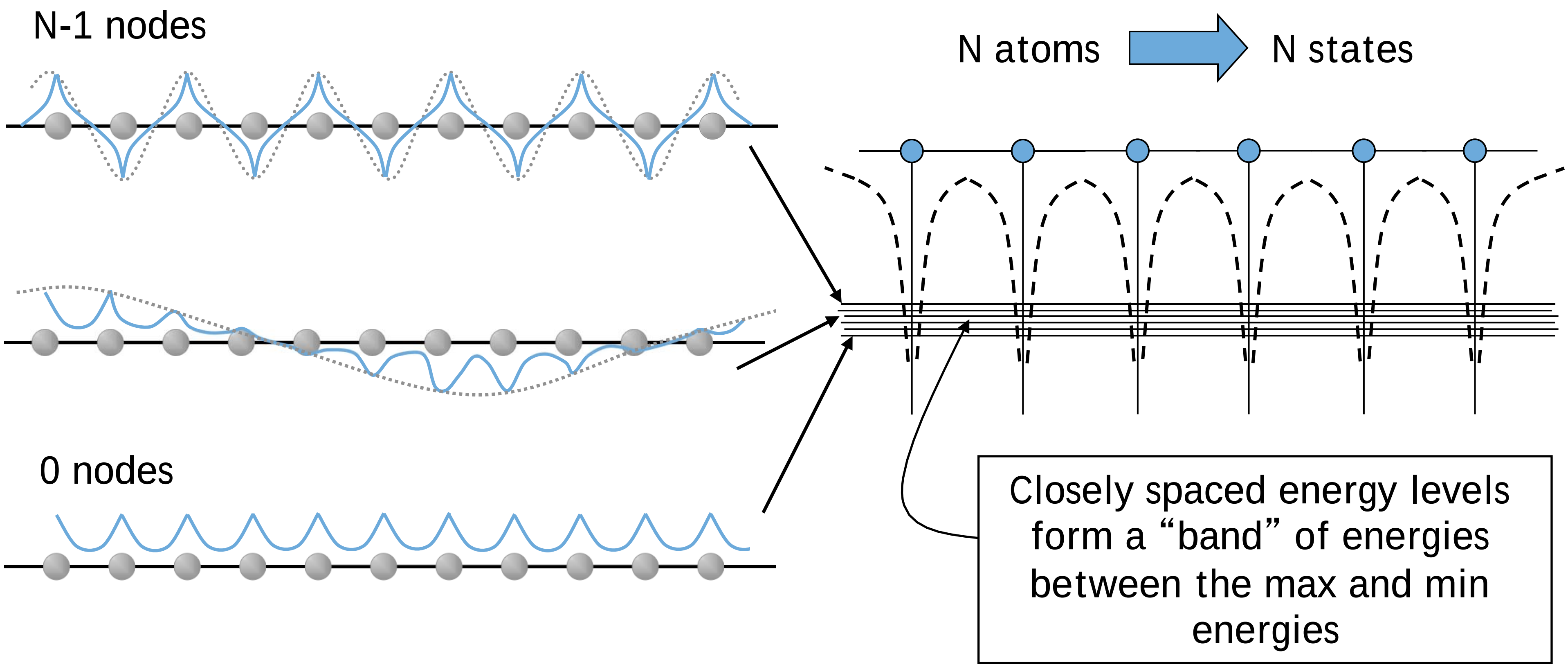
Adding atoms…

- reduces curvature of lowest energy state (incrementally)
- increases number of states (nodes)
- beyond ~10 atoms the bandwidth does not change with crystal size

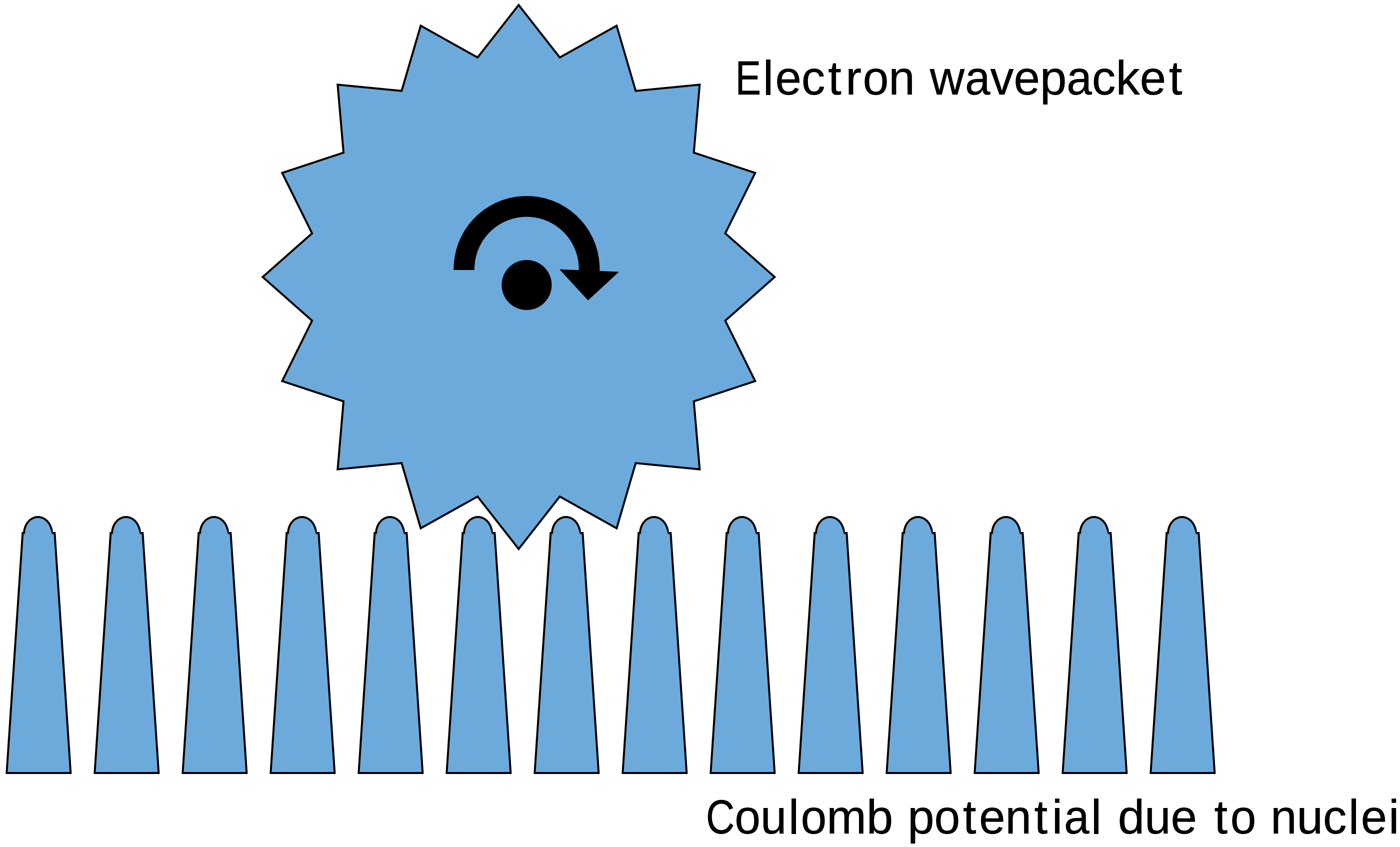
Decreasing distance between atoms (lattice constant) …

- increases bandwidth

From Molecules to Solids



Electron Wavepacket in Periodic Potential

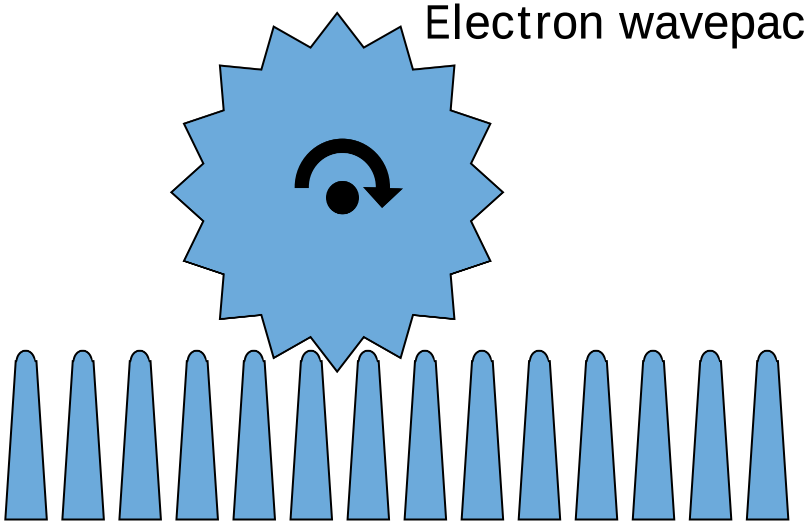


For smooth motion

- wavepacket width >> atomic spacing
- any change in lattice periodicity ‘scatters’ wavepacket
    - vibrations
    - impurities (dopants)

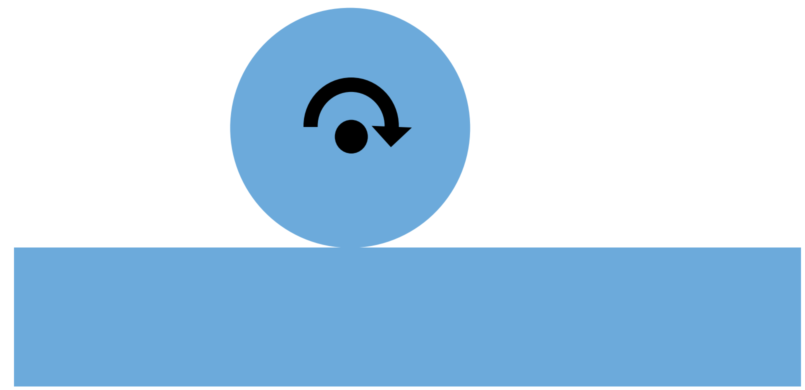
Equivalent Free Particle

Electron wavepacket



Coulomb potential due to nuclei

Effective ‘free’ electron wavepacket



Wavepacket moves as if it had an effective mass…

$$F_{ext} = m * a$$

Electron responds to external force as if it had an effective mass

Surprise: Effective Mass for Semiconductors

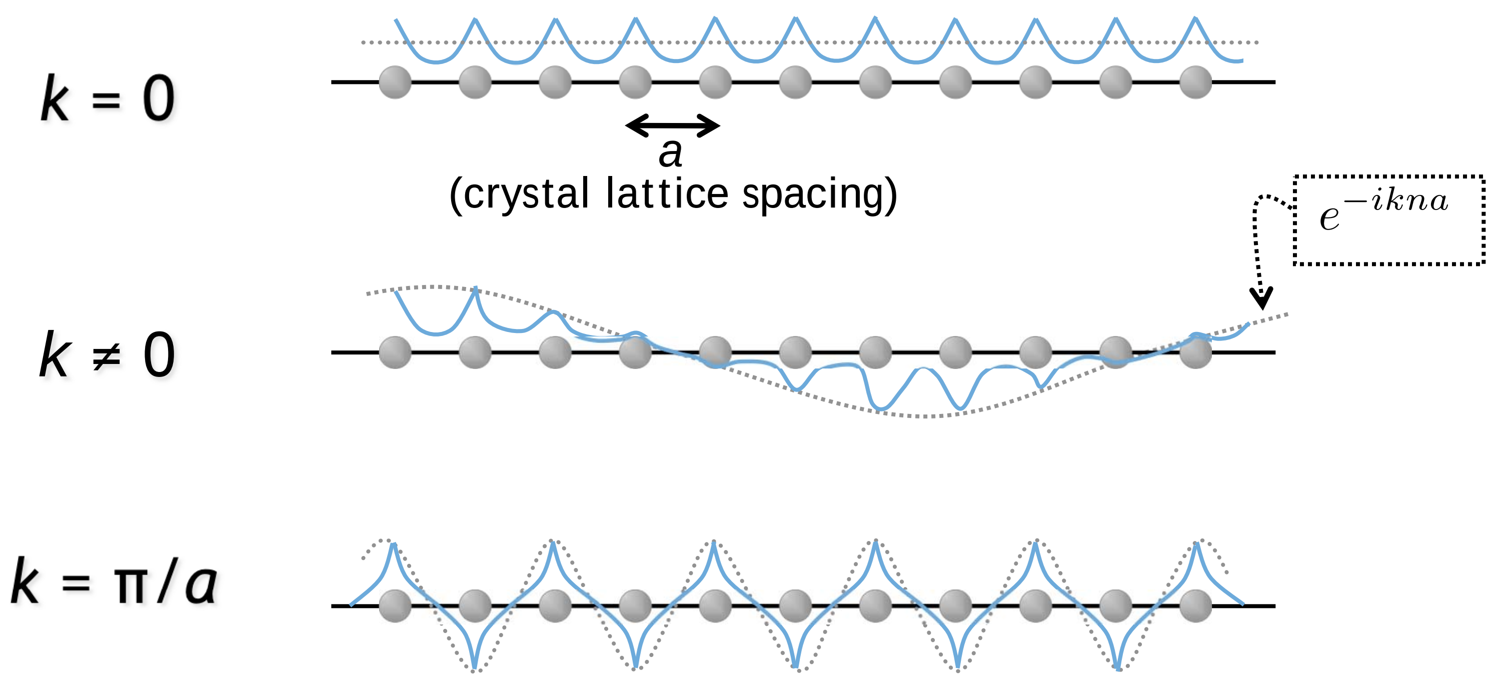
Electrons wavepackets
often have effective mass smaller than free electrons !

Name	Symbol	Germanium	Silicon	Gallium Arsenide
Smallest energy bandgap at 300 K	E_g (eV)	0.66	1.12	1.424
Effective mass for conductivity calculations				
Electrons	$m_{e}^{*}{}_{,cond}/m_0$	0.12	0.26	0.067
Holes	$m_{h}^{*}{}_{,cond}/m_0$	0.21	0.36	0.34

Which material will make faster transistors ?

Approximate Wavefunction for 1-D Lattice

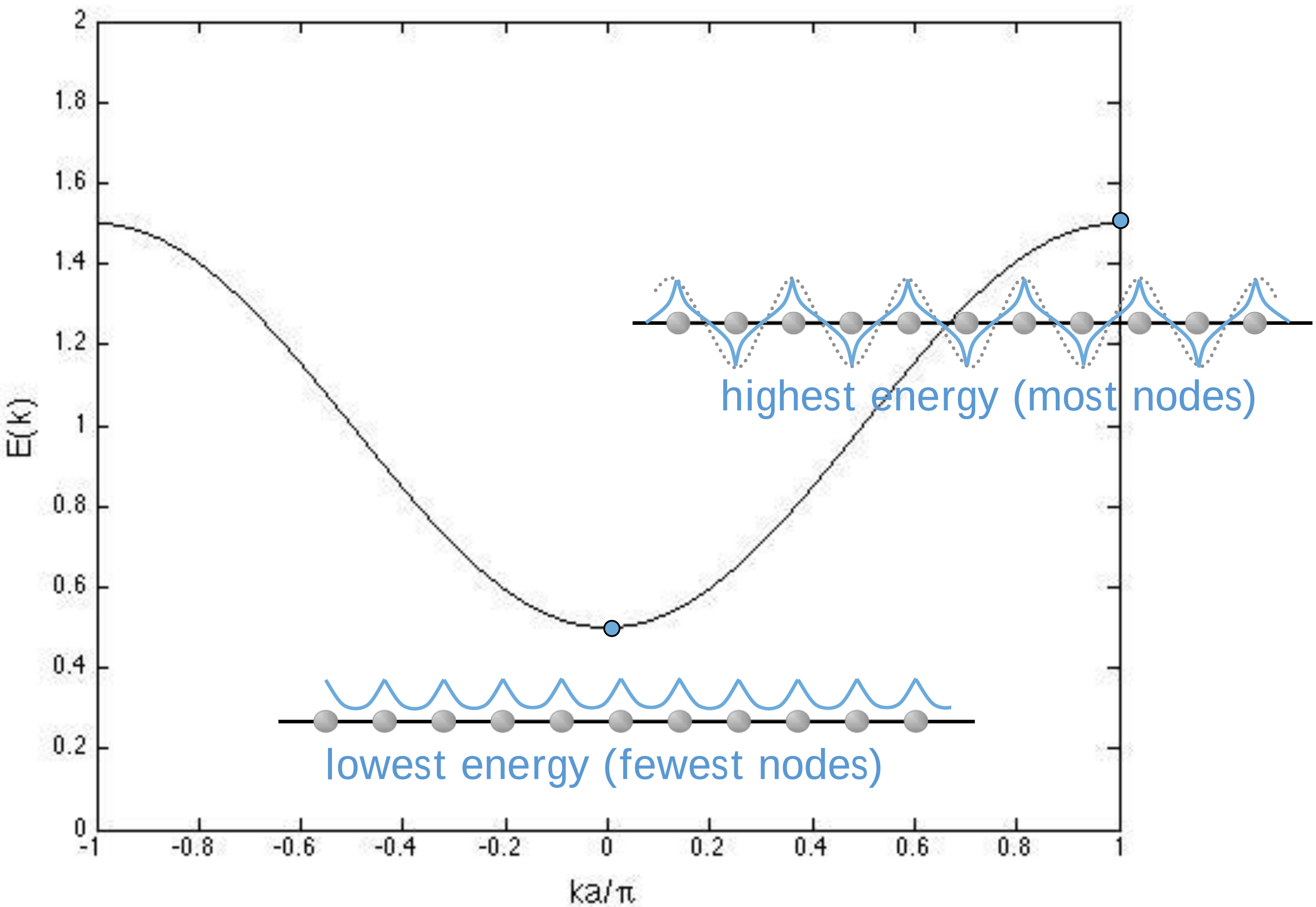
Single orbital, single atom basis



k is a convenient way to enumerate the different energy levels
(count the nodes)

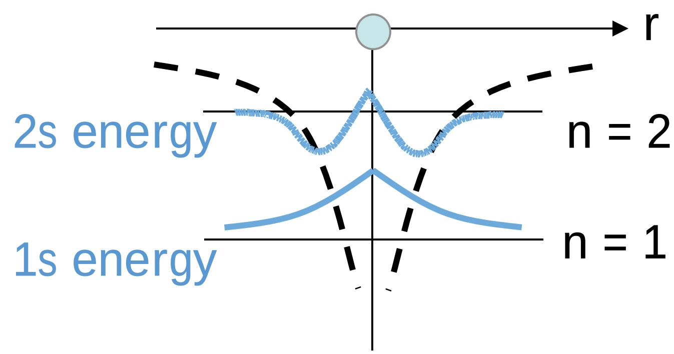
Bloch Functions: $\psi_{n,k}(r) = u_{n,k}(r)e^{ikr}$ $\quad u_{n,k}(r) \approx \text{orbitals}$

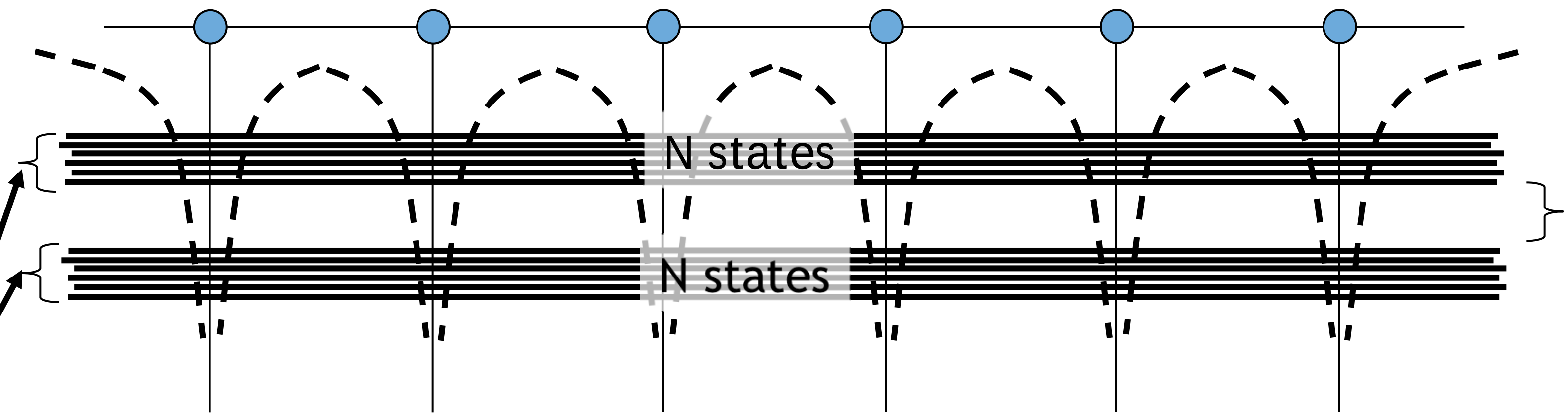
Energy Band for 1-D Lattice
Single orbital, single atom basis



- Number of states in band = number of atoms
- Number of electrons to fill band = number of atoms x 2 (spin)

From Molecules to Solids



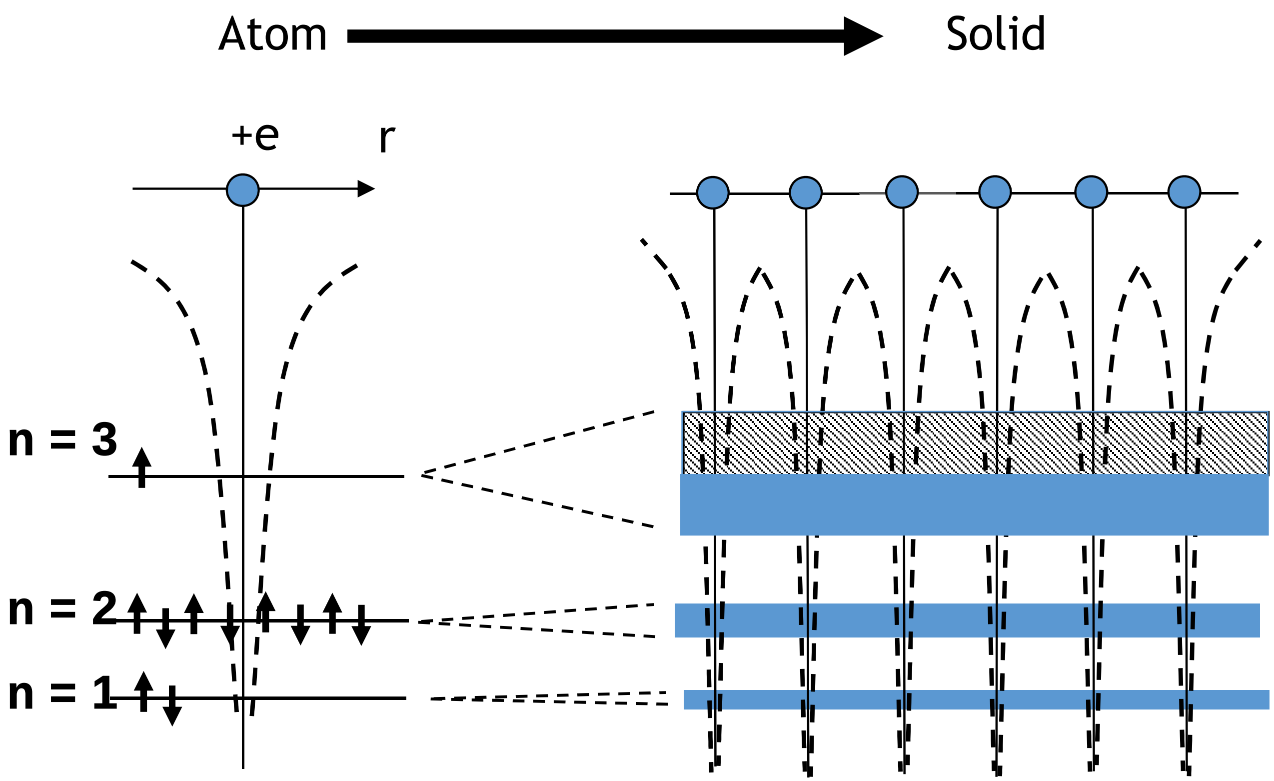


Bands of “allowed” energies for electrons

Bands Gap – range of energy where there are no “allowed states”

The total number of states = (number of atoms) x (number of orbitals in each atom)

Bands from Multiple Orbitals



Example of Na

Z = 11 $1s^2 2s^2 2p^6 3s^1$

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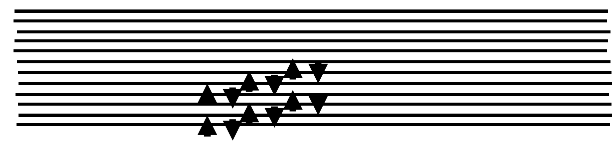


These two facts are the basis for our understanding of metals, semiconductors, and insulators !!!

- Each atomic state → a band of states in the crystal

  These are the "allowed" states for electrons in the crystal
  → Fill according to Pauli Exclusion Principle

- There may be gaps between the bands

  These are "forbidden" energies where there are no states for electrons

What do you expect to be a metal ?

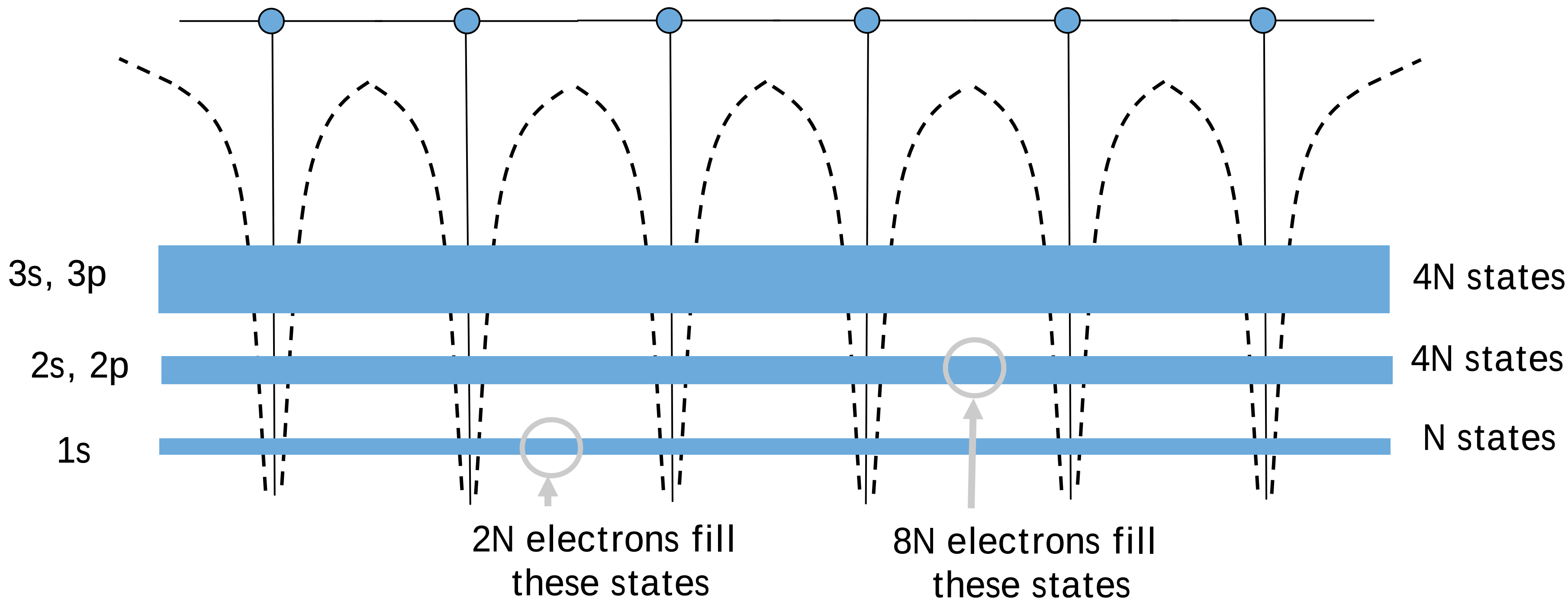
Na? Mg? Al? Si? P?

What about semiconductors like silicon?

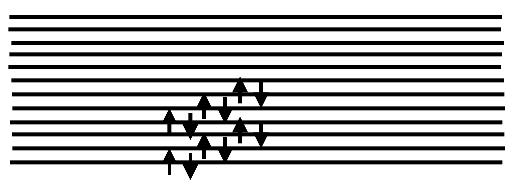
Fill the Bloch states according to Pauli Principle

Z = 14	$1s^2 2s^2 2p^6 3s^2 3p^2$

Total # atoms = N
Total # electrons = 14N



It appears that, like Na, Si will also have a half filled band: The 3s3p band has 4N orbital states and 4N electrons.



By this analysis, Si should be a good metal, just like Na.

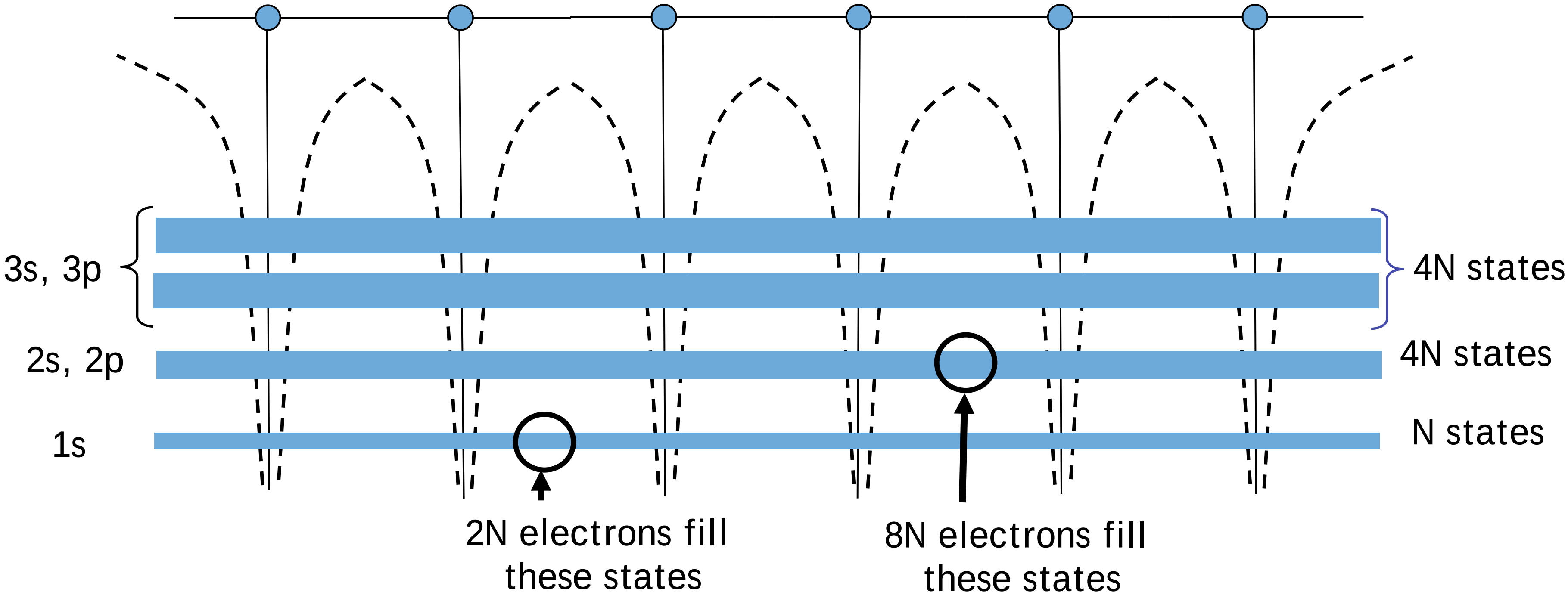
But something special happens for Group IV elements.

Silicon Bandgap

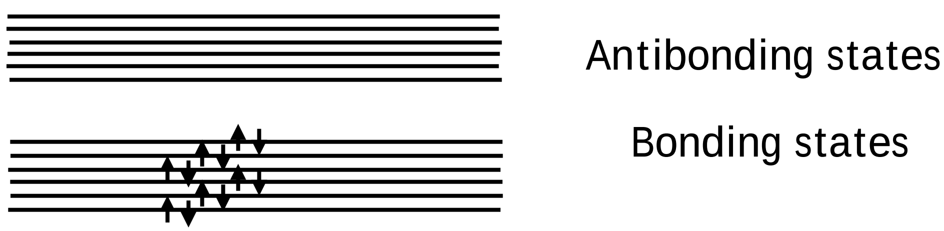
Fill the Bloch states according to Pauli Principle

Z = 14 $1s^2 2s^2 2p^6 3s^2 3p^2$

Total # atoms = N
Total # electrons = 14N

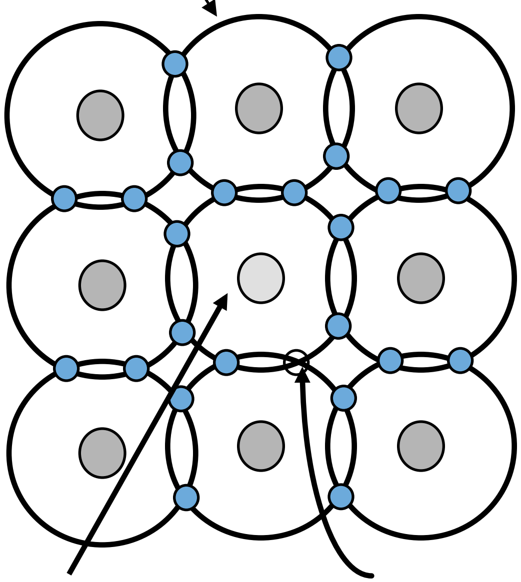


The 3s-3p band splits into two:



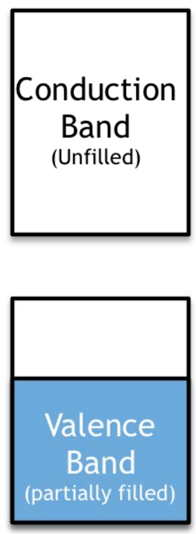
Controlling Conductivity: Doping Solids

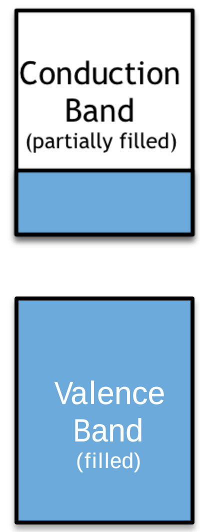
Silicon crystal



Boron atom (5) hole

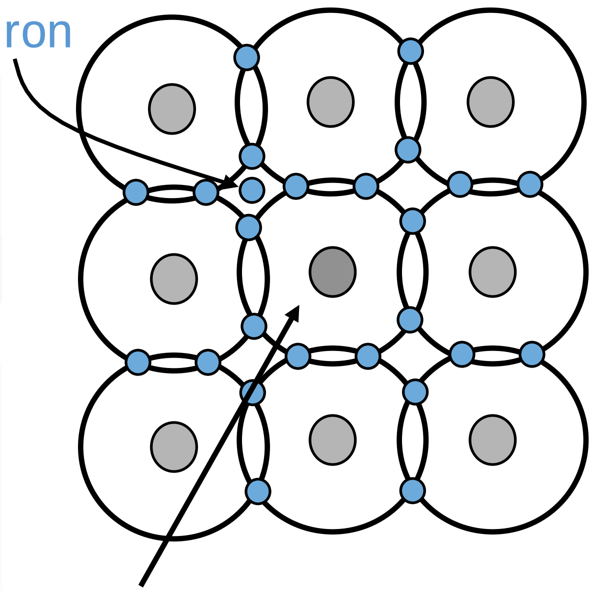
ACCEPTOR DOPING:
P-type Semiconductor
Dopants: B, Al





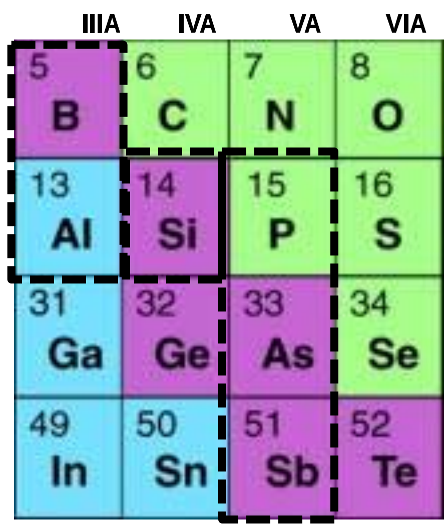
Silicon crystal

Extra electron



Arsenic atom (33)

DONOR DOPING
N-type Semiconductor
Dopants: As, P, Sb



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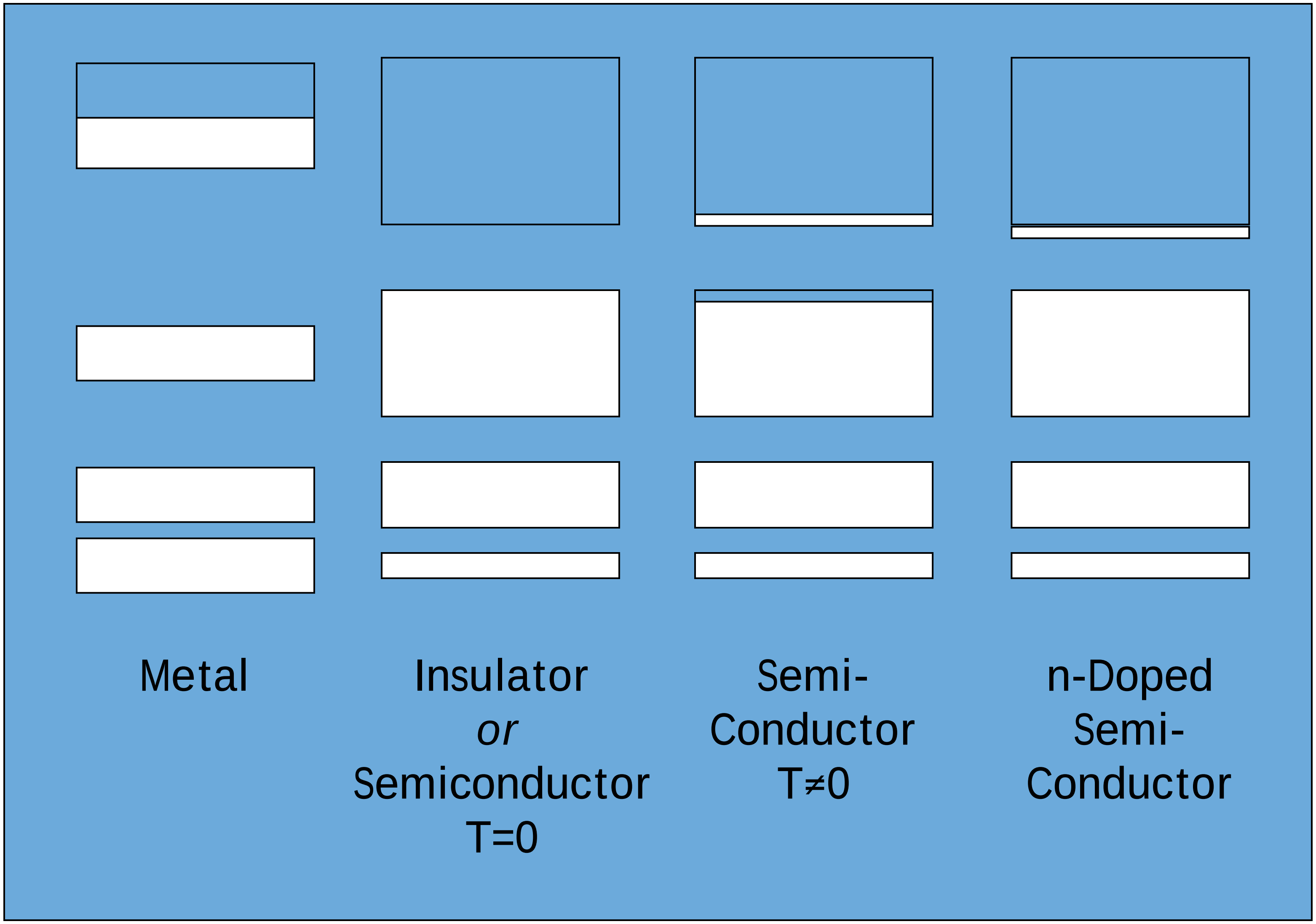
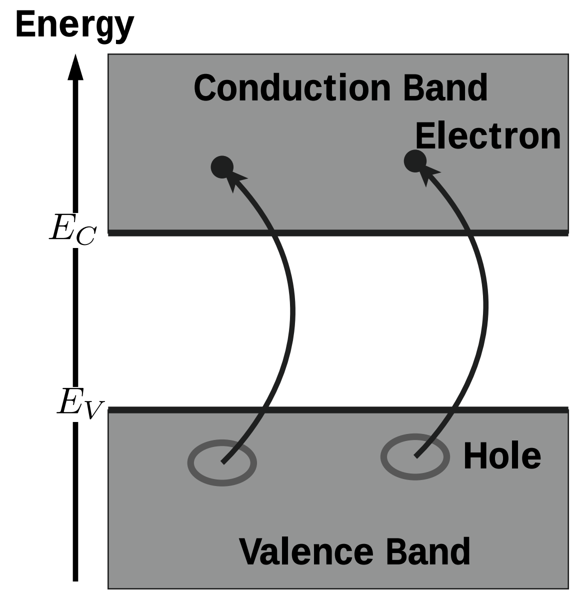
Making Silicon Conduct



Today's Culture Moment

The bandgap in Si is 1.12 eV at room temperature. What is "reddest" color (the longest wavelength) that you could use to excite an electron to the conduction band?



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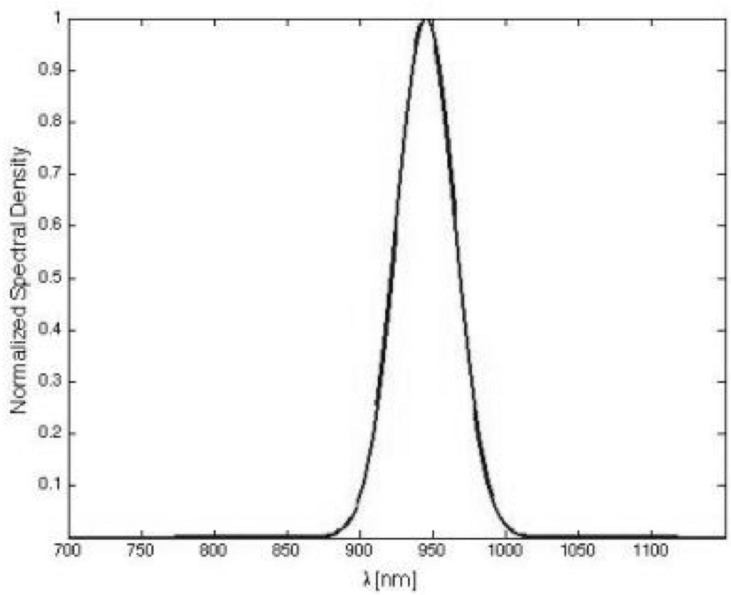

Typical IR remote control

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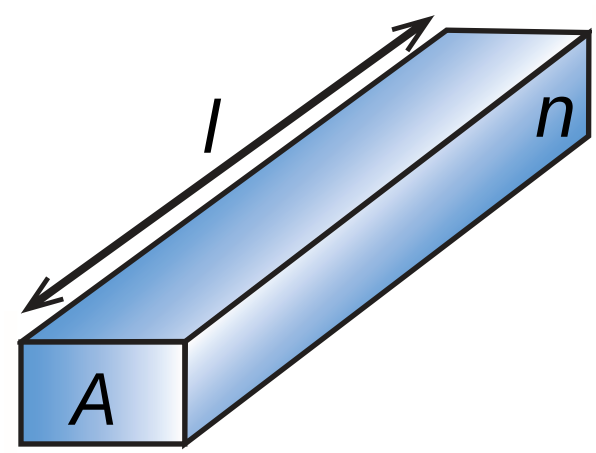
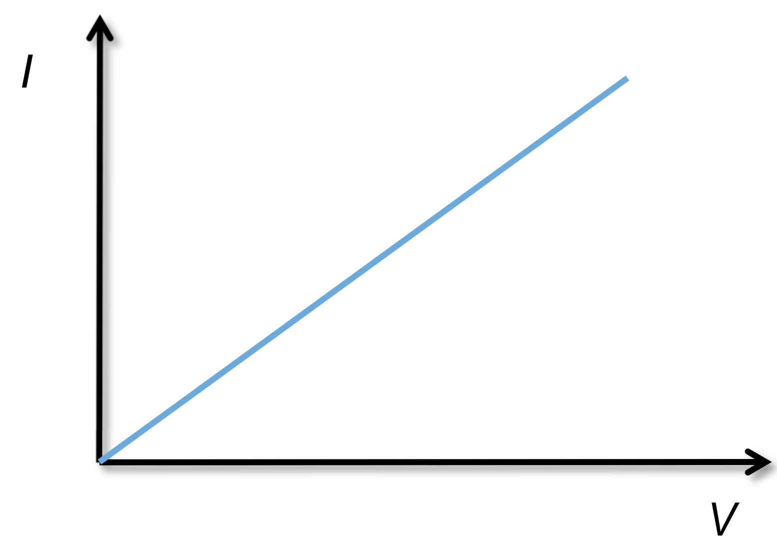
IR detector

Semiconductor Resistor

Given that you are applying a constant E-field (Voltage) why do you get a fixed velocity (Current) ? In other words why is the Force proportional to Velocity ?

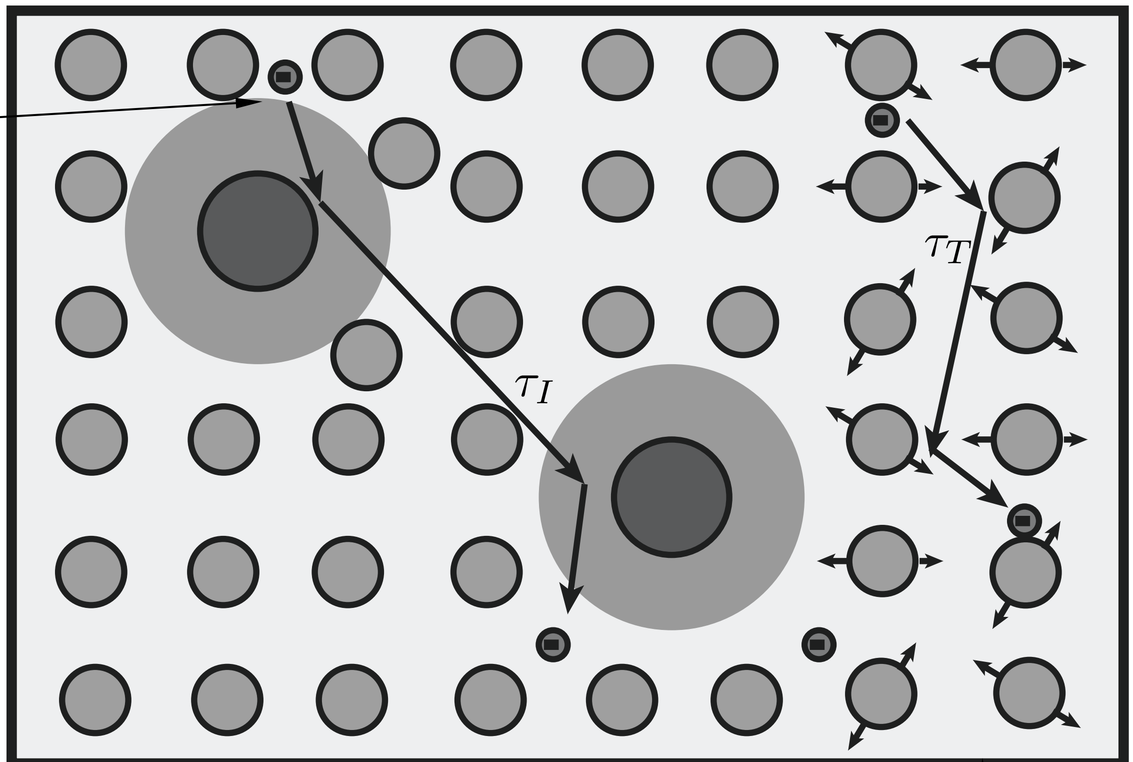




How does the resistance depend on geometry ?

Microscopic Scattering

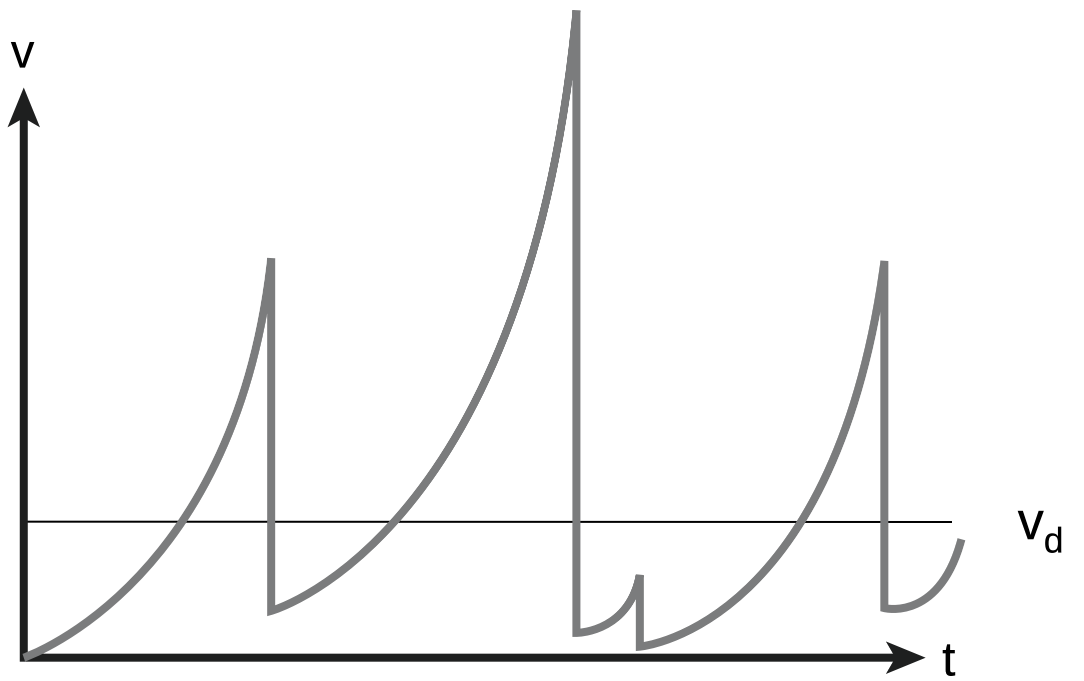
A local, unexpected change in V(x) of electron as it approaches the impurity



Strained region
by impurity exerts
a scattering force

Scattering from thermal vibrations

Microscopic Transport



Balance equation for forces on electrons:

$$m\frac{d\mathbf{v}(r,t)}{dt} = \underbrace{-m\frac{\mathbf{v}(r,t)}{\tau}}_{\text{Drag Force}} - \underbrace{e\left[\mathbf{E}(r,t) + \mathbf{v}(r,t) \times \mathbf{B}(r,t)\right]}_{\text{Lorentz Force}}$$

Microscopic Variables for Electrical Transport

Drude Theory

Balance equation for forces on electrons:

$$m\frac{d\mathbf{v}(r,t)}{dt} = \underbrace{-m\frac{\mathbf{v}(r,t)}{\tau}}_{\text{Drag Force}} - \underbrace{e\left[\mathbf{E}(r,t) + \mathbf{v}(r,t) \times \mathbf{B}(r,t)\right]}_{\text{Lorentz Force}}$$

In steady-state when **B**=0:

Note: Inside a semiconductor m = m (effective mass of the electron)*

$$\vec{v} = -\frac{e\tau}{m^*}\vec{E}_{DC}$$

$$\vec{J} = -ne\vec{v} = \frac{ne^2\tau}{m^*}\vec{E}_{DC}$$

$$\vec{J} = \sigma\vec{E}_{DC} \quad \text{and} \quad \sigma = \frac{ne^2\tau}{m^*}$$

Semiconductor Resistor

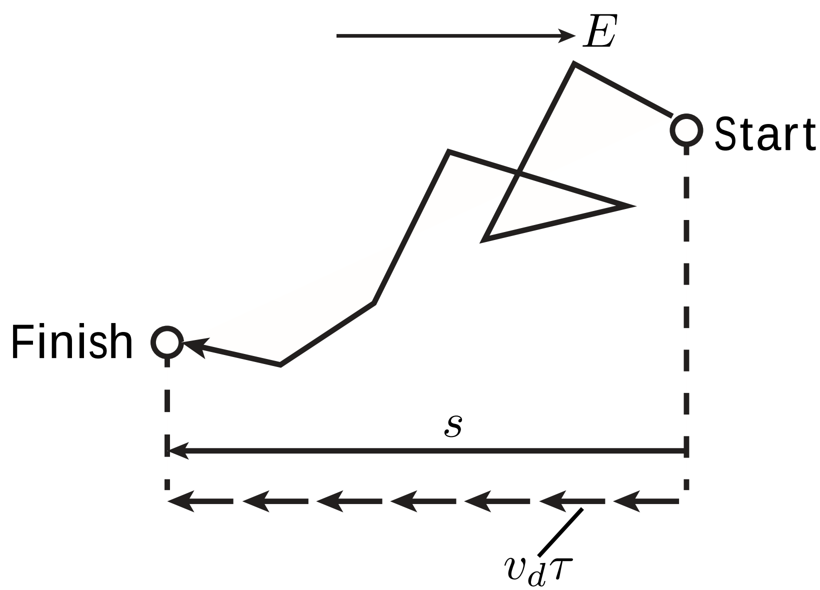
$$\vec{J} = \sigma \vec{E}_{DC} \quad \text{and} \quad \sigma = \frac{ne^2\tau}{m^*}$$

Recovering macroscopic variables:

$$I = \int \vec{J} \cdot d\vec{A} = \sigma \int \vec{E} \cdot d\vec{A} = \sigma \frac{V}{l} A$$

$$V = I \frac{l}{\sigma A} = I \frac{\rho l}{A} = IR$$

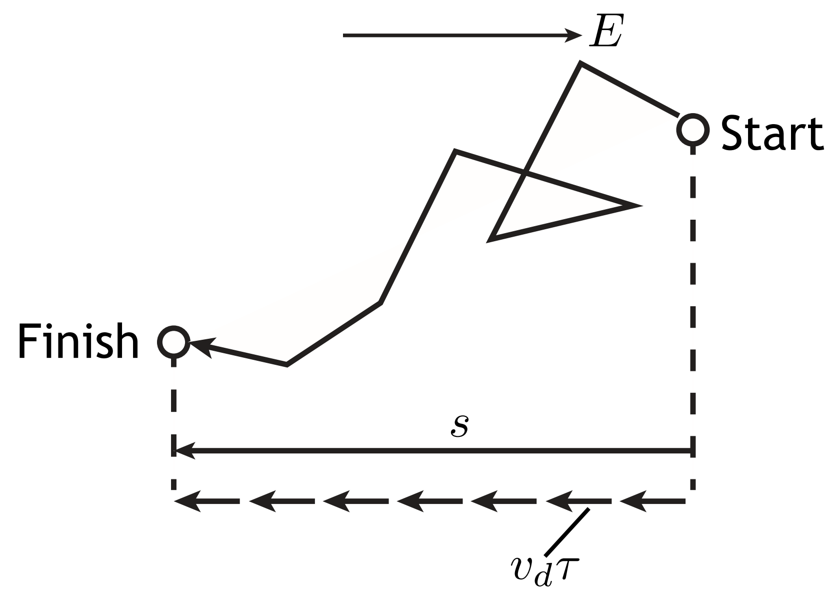
OHM's LAW



Microscopic Variables for Electrical Transport

$$\vec{J} = \sigma \vec{E}_{DC} \quad \text{and} \quad \sigma = \frac{ne^2\tau}{m^*}$$

$$\tau = \frac{m^*\sigma}{ne^2}$$



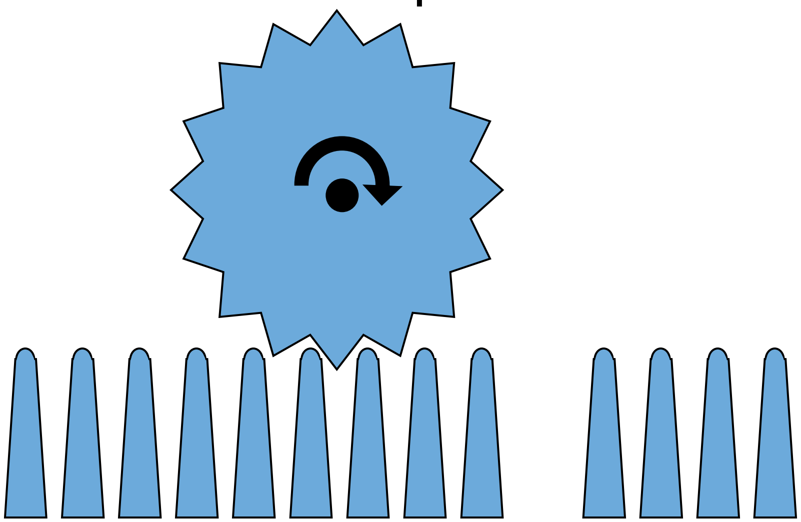
For silicon crystal doped at n = 10^{17} cm^{-3} :
σ = 11.2 $(\Omega\ cm)^{-1}$, μ = 700 cm^2/(Vs)and m* = 0.26 m_o

$$\tau = \frac{(0.26)(9.1 \times 10^{-31}\ \text{kg})(11.200\ \text{m}^{-1}\Omega^{-1})}{10^{23}\ \text{m}^{-3}(1.6 \times 10^{-19}\ \text{C})^2} = 10^{-13}\ \text{s} = 100\ \text{fs}$$

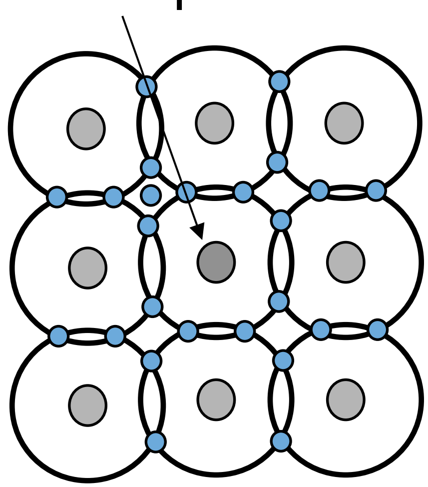
At electric fields of E = 10^6 V/m = 10^4 V/cm,
v = μE = 700 cm^2/(Vs) * 10^4 V/cm = 7 x 10^6 cm/s = 7 x 10^4 m/s
scattering event every 7 nm ~ 25 atomic sites

Electron Mobility

Electron wavepacket



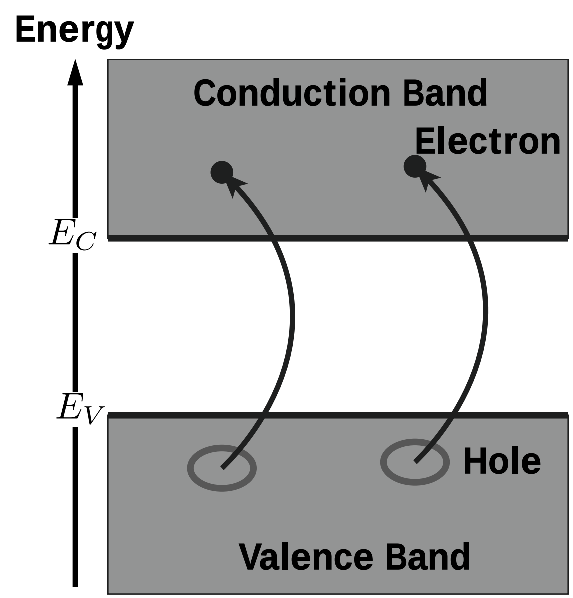
Change in periodic potential



$$\vec{J} = \sigma \vec{E} = ne\vec{v} = ne\mu\vec{E}$$

$$\vec{v} = \mu\vec{E}$$

Electron velocity for a fixed applied E-field

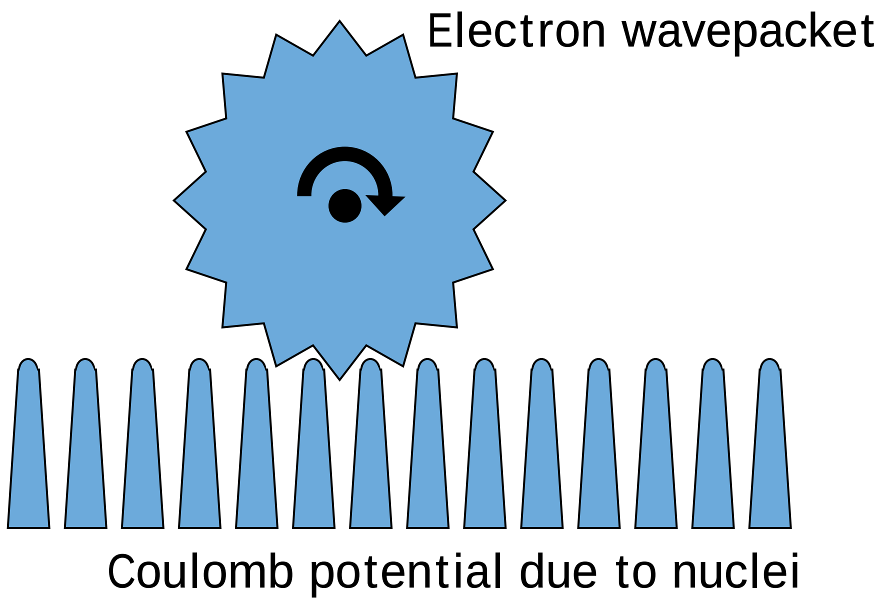


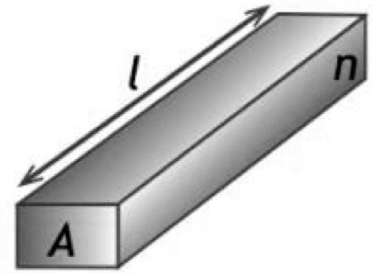
Electron Mobility

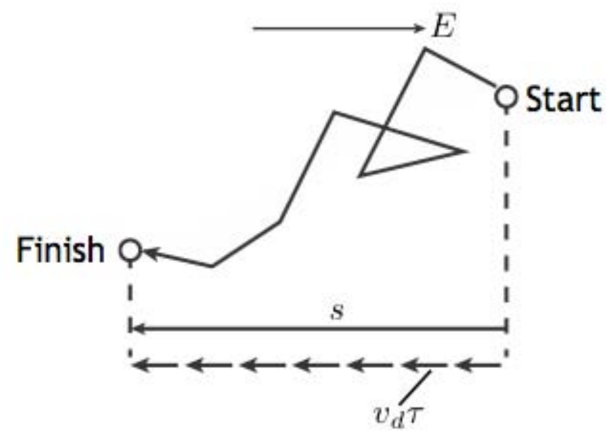
$$\sigma_e = n\left|e\right|\mu_e = 1/\rho$$

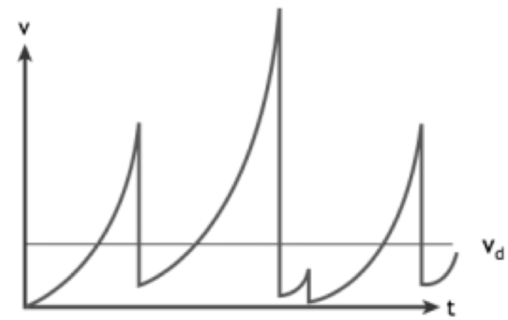
- **Intrinsic Semiconductors** (no dopants)
  - Dominated by number of carriers, which increases exponentially with increasing temperature due to increased probability of electrons jumping across the band gap
  - At high enough temperatures phonon scattering dominates → velocity saturation
- **Metals**
  - Dominated by mobility, which decreases with increasing temperature

Key Takeaways









Wavepacket moves as if it had
an effective mass...

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$$\vec{J} = -ne\vec{v} = \frac{ne^2\tau}{m^*}\vec{E}_{DC}$$

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Spring 2011