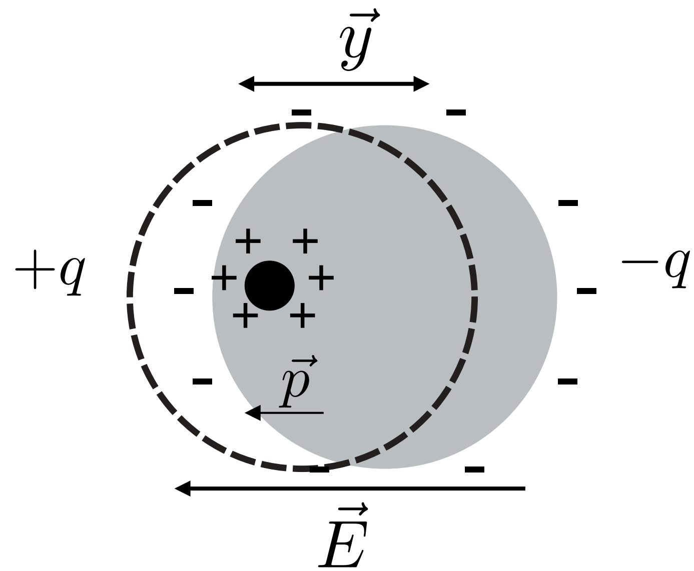
Interaction of Atoms and Electromagnetic Waves

Outline

- Review: Polarization and Dipoles
- Lorentz Oscillator Model of an Atom
- Dielectric constant and Refractive index

True or False?

1. The dipole moment of this atom is $\vec{p} = q\vec{y}$, and points in the same direction as the polarizing electric field.

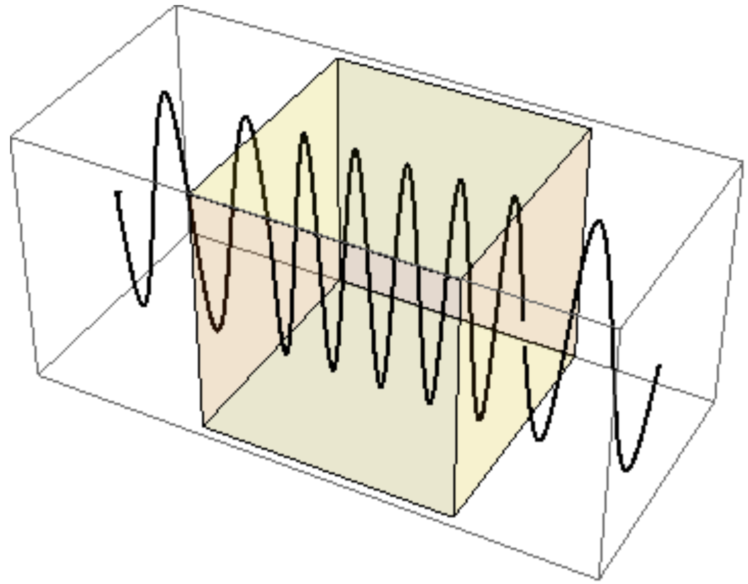


2. The susceptibility relates the electric field to the polarization in this form: $\vec{P} = \epsilon_o \chi_e \vec{E}$

3. The refractive index can be written $n = \sqrt{\dfrac{\epsilon_o \mu_o}{\epsilon \mu}}$

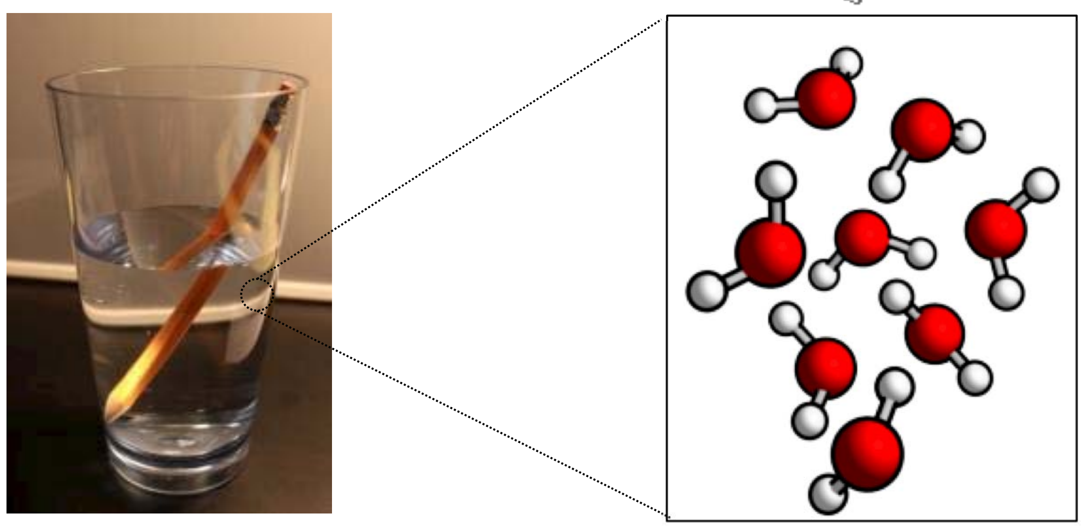
Refractive Index: Waves in Materials

$$v_p = \frac{1}{\sqrt{\mu\epsilon}}$$



Index of refraction

$$n \equiv \frac{c}{v_p}$$



How do we get from molecules/charges and fields to index of refraction ?

Index of Refraction

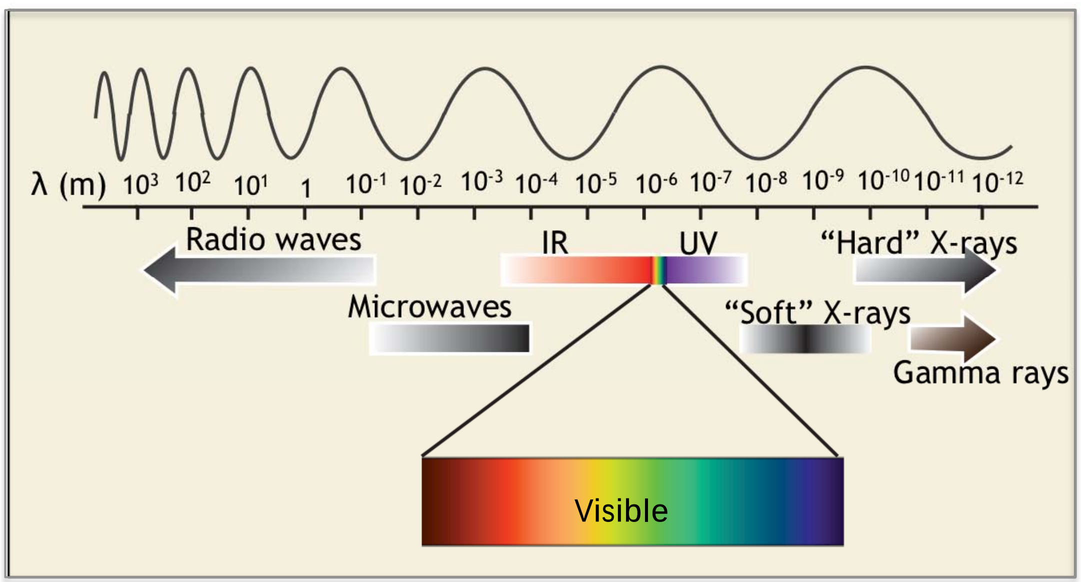
$$\nu\lambda = v_p = c/n$$

frequency (ν), wavelength (λ)

When propagating in a material,

$$c \to c/n$$
$$\lambda \to \lambda_o/n$$
$$k \to k_o/n$$

Material	n
Vacuum	1
Air	1.000277
Water liquid	1.3330
Water ice	1.31
Diamond	2.419
Silicon	3.96
at 5 x 10^{14} Hz	

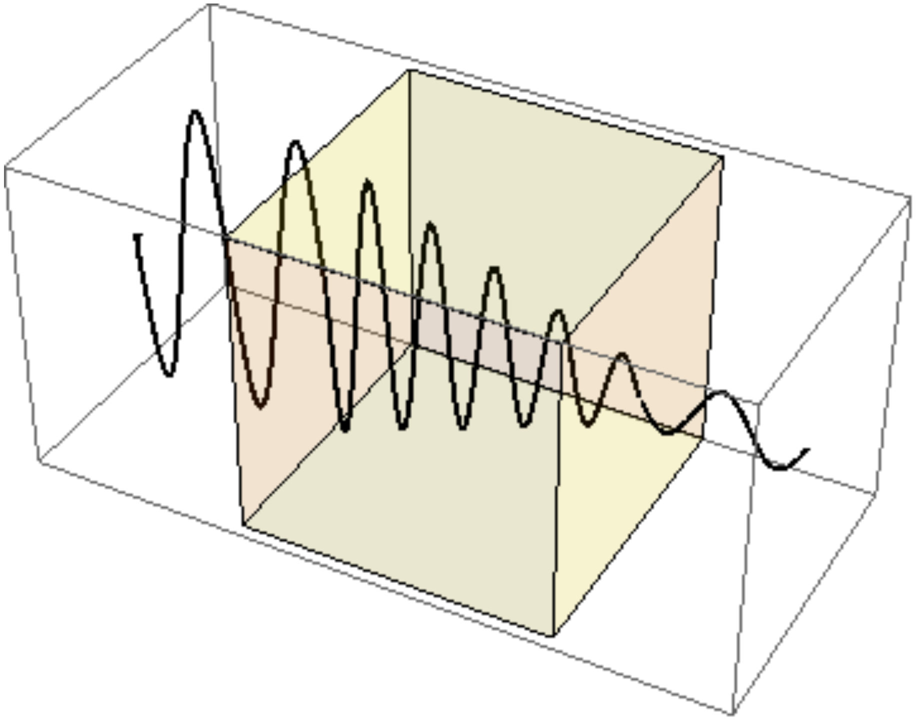


$$E(t,z) = Re\{\tilde{E}_0 e^{j(\omega t - k_0 n z)}\}$$

$$E(t,z) = Re\{\tilde{E}_0 e^{j(\omega t - k\ z)}\}$$

Absorption

Photograph by Hey Paul on Flickr.



Why are these stained glass different colors?

$$E(t,z) = Re\{\tilde{E}_0 e^{-\alpha z/2} e^{j(\omega t - k_0 n z)}\}$$

Absorption coefficient (α)

Refractive index (n)

Tomorrow: lump refractive index and absorption into a complex refractive index $\tilde{n}$

Incident Solar Radiation

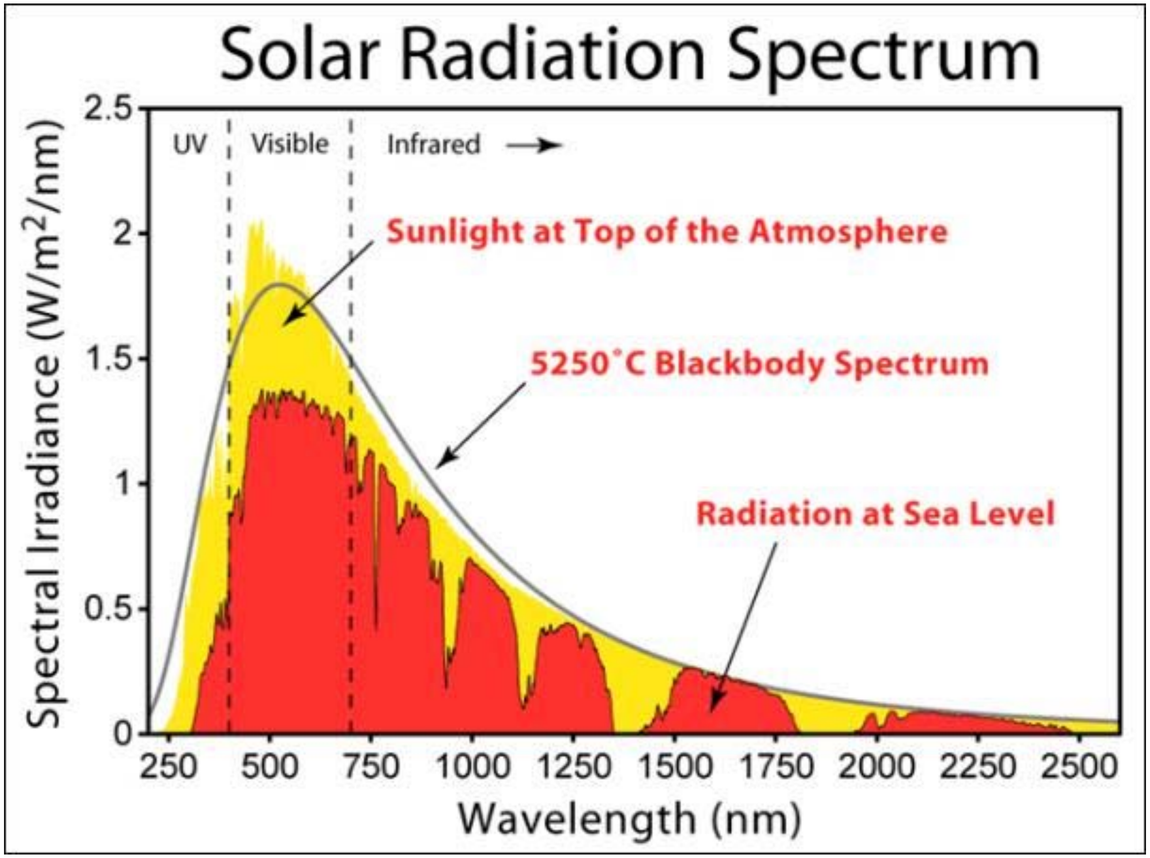
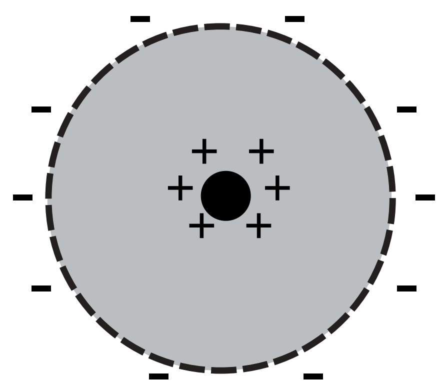


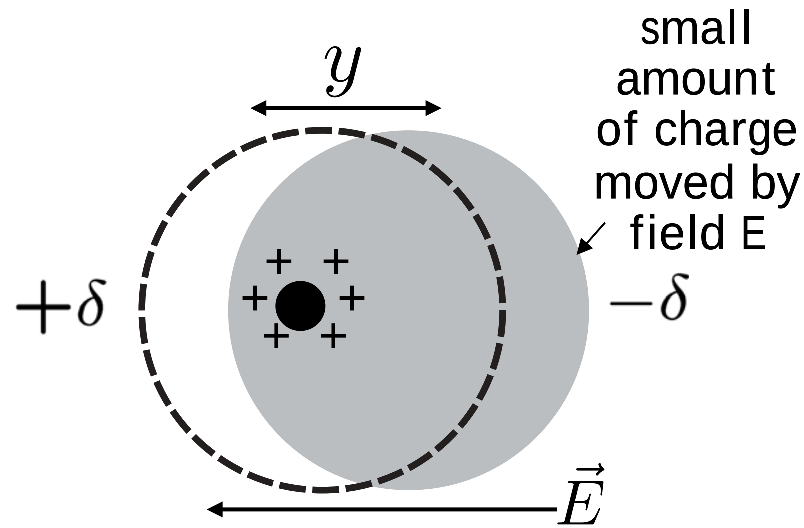
Image created by Robert A. Rohde / Global Warming Art. Used with permission.

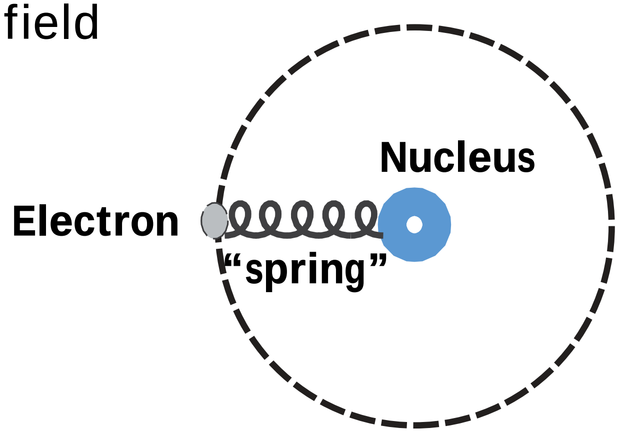
How do we introduce propagation through a medium (atmosphere) into Maxwell's Equations?

Microscopic Description of Dielectric Constant



NO external $\vec{E}$ field





Density of dipoles...

$$\vec{P} = N\delta\vec{x}$$

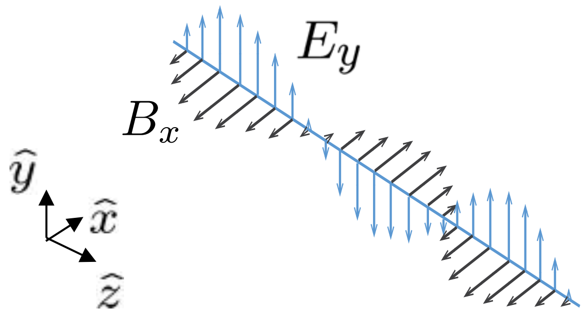
... equivalent to ...

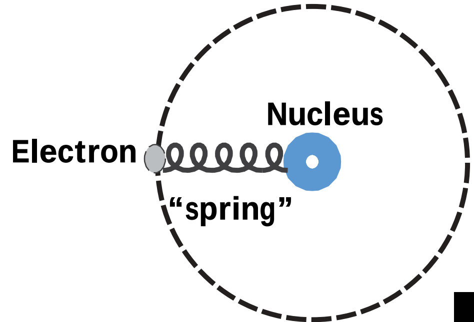
Electric field polarizes molecules...

$$\vec{P} = \epsilon_o \chi_e \vec{E}$$

Lorentz Oscillator

Lorentz was a late nineteenth century physicist, and quantum mechanics had not yet been discovered. However, he did understand the results of classical mechanics and electromagnetic theory. Therefore, he described the problem of atom-field interactions in these terms. Lorentz thought of an atom as a mass (the nucleus) connected to another smaller mass (the electron) by a spring. The spring would be set into motion by an electric field interacting with the charge of the electron. The field would either repel or attract the electron which would result in either compressing or stretching the spring.





Hendrik Lorentz (1853-1928)
Nobel in 1902 for Zeeman Effect

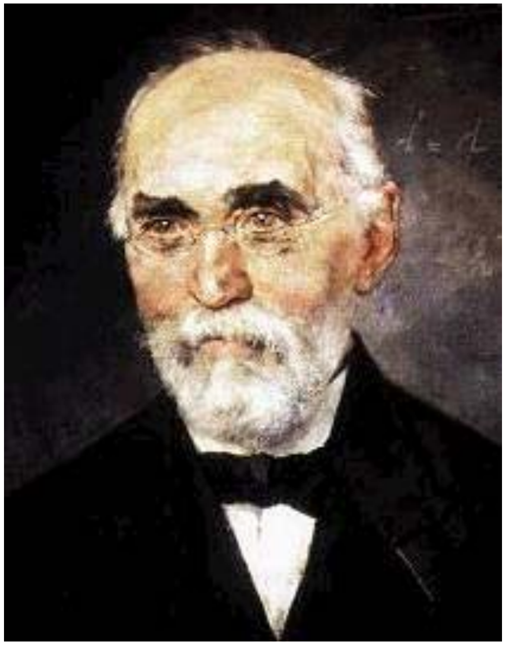


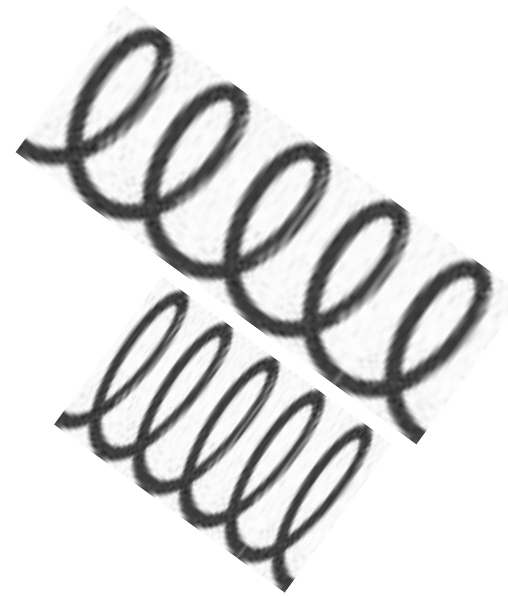
Image in public domain

Lorentz was not positing the existence of a physical spring connecting the electron to an atom; however, he did postulate that the force binding the two could be described by Hooke's Law:

$$F(y) = -k_s y$$

where y is the displacement from equilibrium. If Lorentz's system comes into contact with an electric field, then the electron will simply be displaced from equilibrium. The oscillating electric field of the electromagnetic wave will set the electron into harmonic motion. The effect of the magnetic field can be omitted because it is miniscule compared to the electric field.

Springs have a resonant frequency



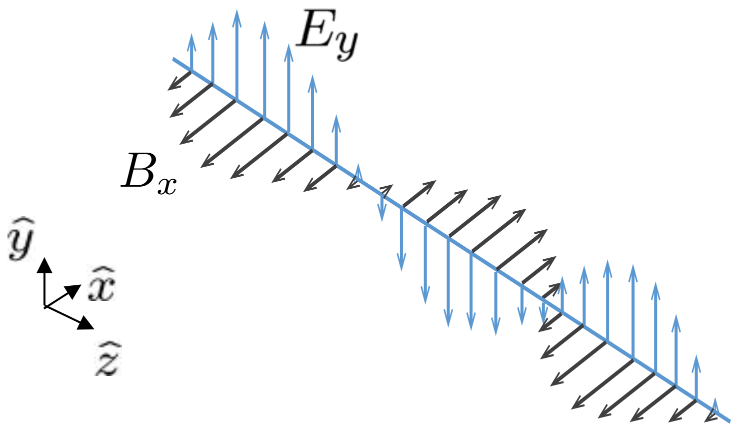
Hooke's Law $$m\frac{d^2y}{dt^2} = -ky$$

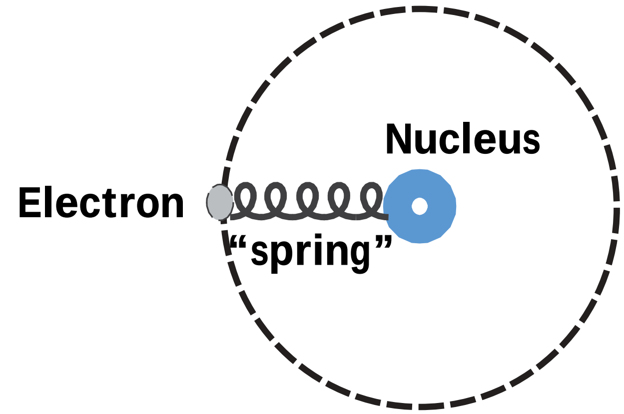
$$\frac{d^2y}{dt^2} = -\frac{k}{m}y$$

Solution $$y(t) = Asin(t\underbrace{\sqrt{\frac{k}{m}}}_{\omega_0}) + Bcos(t\underbrace{\sqrt{\frac{k}{m}}}_{\omega_0})$$

So we can write: $$\omega_o^2 = \frac{k}{m} \quad \Rightarrow \quad k = m\omega_o^2$$

Microscopic Description of Dielectric Constant





$$m\frac{d^2y}{dt^2} = -m\omega_o^2 y + qE_y - m\gamma\frac{dy}{dt}$$

Electron mass

Restoring force (binding electron & nucleus)

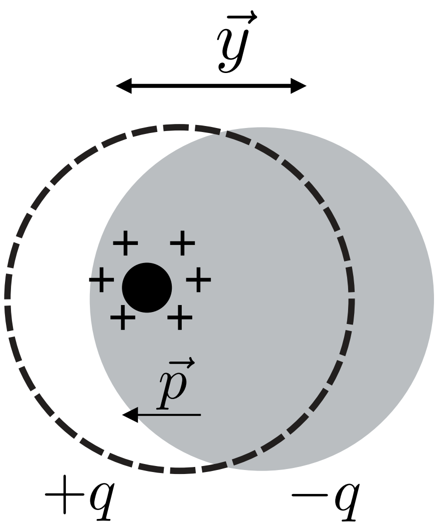
$\vec{E}$ field force

Damping

Solution using complex variables

Lets plug-in the expressions for E_y and y
into the differential equation from slide 9:

Natural resonance



$$\frac{d^2}{dt^2}y(t) + \gamma\frac{d}{dt}y(t) + \omega_o^2 y(t) = \frac{q}{m}E_y(t)$$

$$E_y(t) = Re\{E_y e^{j\omega t}\} \qquad y(t) = Re\{y e^{j\omega t}\}$$

$$\omega^2 y + j\omega\gamma y + \omega_o^2 y = \frac{q}{m}E_y$$

$$y = \frac{q}{m}\frac{1}{(\omega_o^2 - \omega^2) + j\omega\gamma}E_y$$

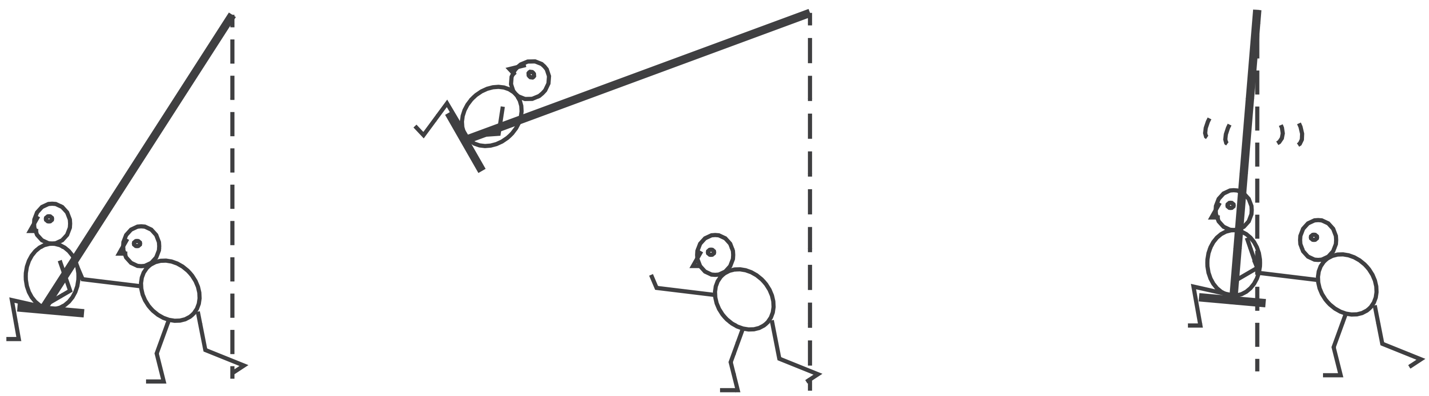
Oscillator Resonance

$$y = \frac{q}{m}\frac{1}{(\omega_0^2 - \omega^2) + j\omega\gamma}E_y$$

$$E_y(t) = Re\{E_y e^{j\omega t}\}$$

$$y(t) = Re\{y e^{j\omega t}\}$$

Driven harmonic oscillator: **Amplitude** and **Phase** depend on frequency



Low frequency

medium amplitude

Displacement, y
in phase with E_y

At resonance

large amplitude

Displacement, y
90º out of phase with E_y

High frequency

vanishing amplitude

Displacement y and E_y
in antiphase

Polarization

Since charge displacement, y, is directly related to polarization, P, of our material we can then rewrite the differential equation:

$$\vec{D} = \epsilon_o \vec{E} + \vec{P}$$

$$P_y = Nqy$$

For linear polarization in $\hat{y}$ direction

$$(\frac{d}{dt^2} + \gamma \frac{d}{dt} + \omega_o^2) P_y(t) = \frac{Nq^2}{m} E_y(t) = \epsilon_o \omega_p^2 E_y(t)$$

$$\omega_p^2 = \frac{Nq^2}{\epsilon_o m}$$

$$P_y(t) = Re\{P_y e^{j\omega t}\}$$

$$P_y = \frac{\omega_p^2}{(\omega_o^2 - \omega^2) + j\gamma\omega} \epsilon_o E_y$$

Dielectric Constant from the Lorentz Model

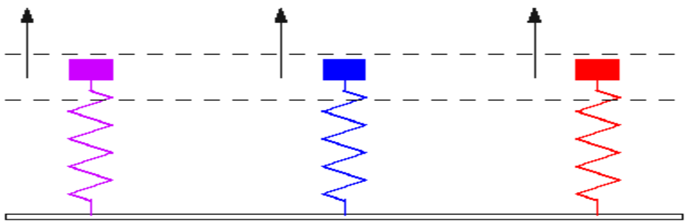
$$\vec{D} = \epsilon_o \vec{E} + \vec{P}$$

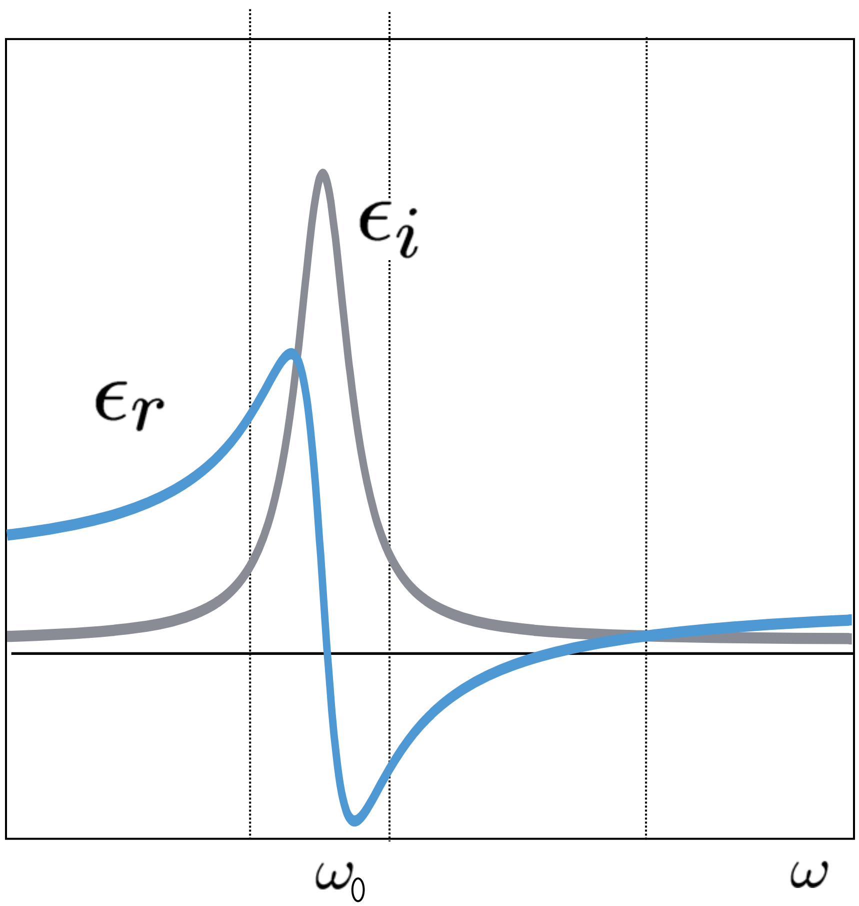
$$\vec{P} = \frac{\omega_p^2}{(\omega_o^2 - \omega^2) + j\gamma\omega} \epsilon_o \vec{E}$$

$$\vec{D} = \epsilon_o \left[1 + \frac{\omega_p^2}{(\omega_o^2 - \omega^2) + j\gamma\omega} \right] \vec{E}$$

$$\boxed{\epsilon = \epsilon_o \left[1 + \frac{\omega_p^2}{(\omega_o^2 - \omega^2) + j\gamma\omega} \right]}$$

Microscopic Lorentz Oscillator Model





$$\vec{P} = \frac{\omega_p^2}{(\omega_o^2 - \omega^2) + j\gamma\omega}\epsilon_o \vec{E}$$

$$\omega_p^2 = \frac{Nq^2}{\epsilon_o m}$$

$$\epsilon = \epsilon_r - j\epsilon_i$$

Real and imaginary parts

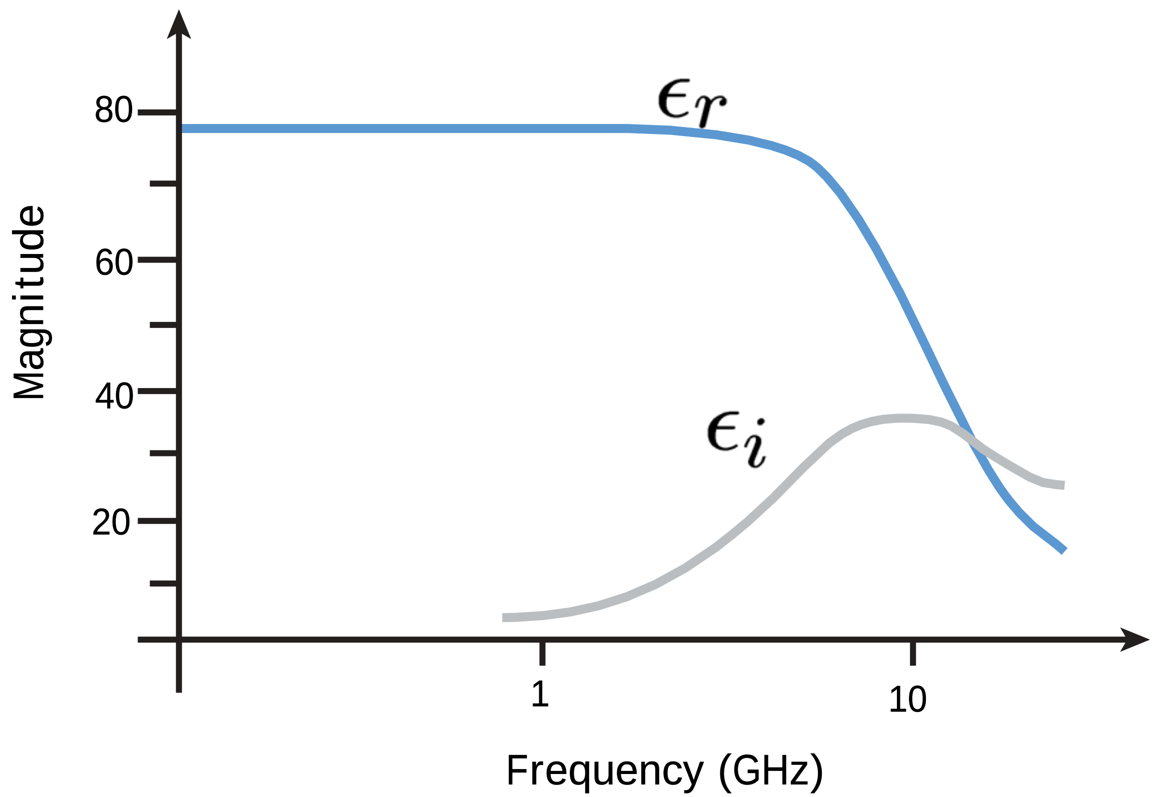
$$\epsilon = \frac{\omega_p^2}{(\omega_o^2 - \omega^2) + j\omega\gamma}$$

$$= \frac{\omega_p^2(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2)^2 + \omega^2\gamma^2} - j\,\frac{\omega_p^2\omega\gamma}{(\omega_o^2 - \omega^2)^2 + \omega^2\gamma^2}$$

$$\epsilon = \epsilon_r - j\epsilon_i$$

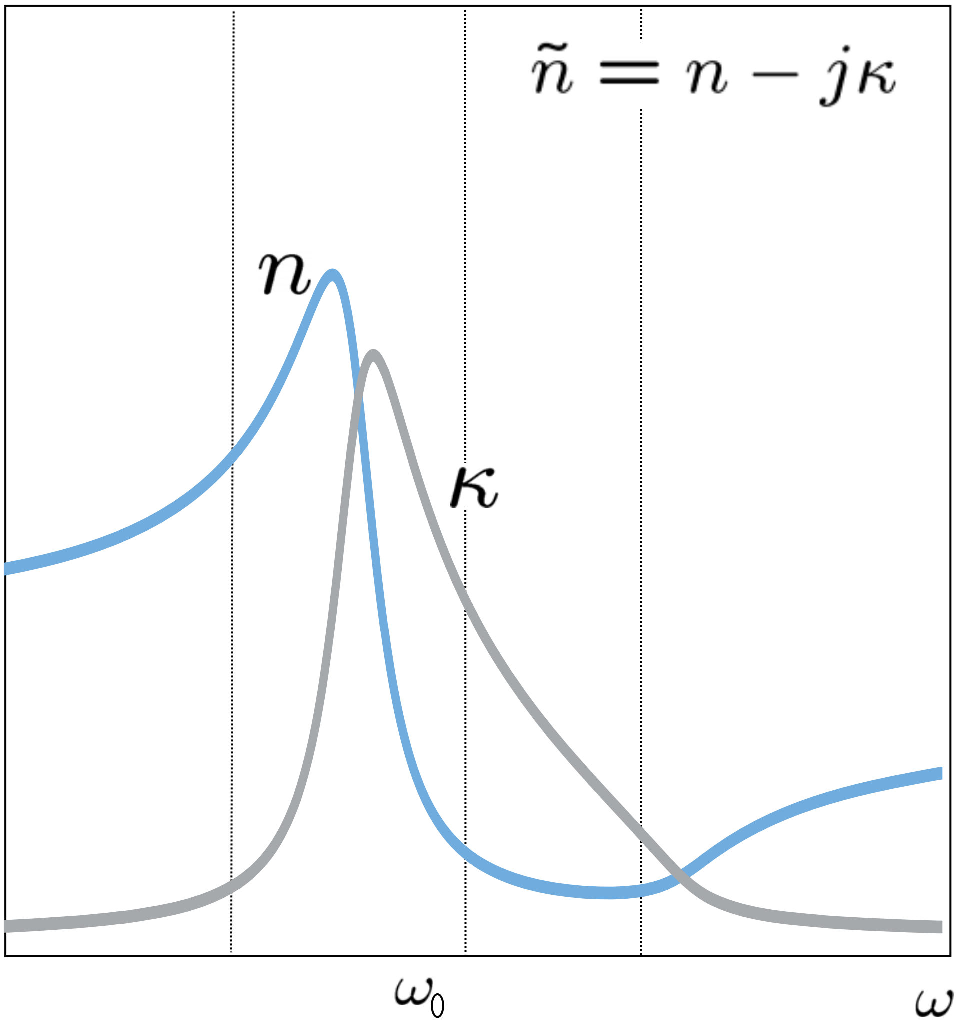
$$\epsilon_r = \frac{\omega_p^2(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2)^2 + \omega^2\gamma^2} \qquad \epsilon_i = \frac{\omega_p^2(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2)^2 + \omega^2\gamma^2}$$

Dielectric constant of water



Microwave ovens usually operate at 2.45 GHz

Complex Refractive Index

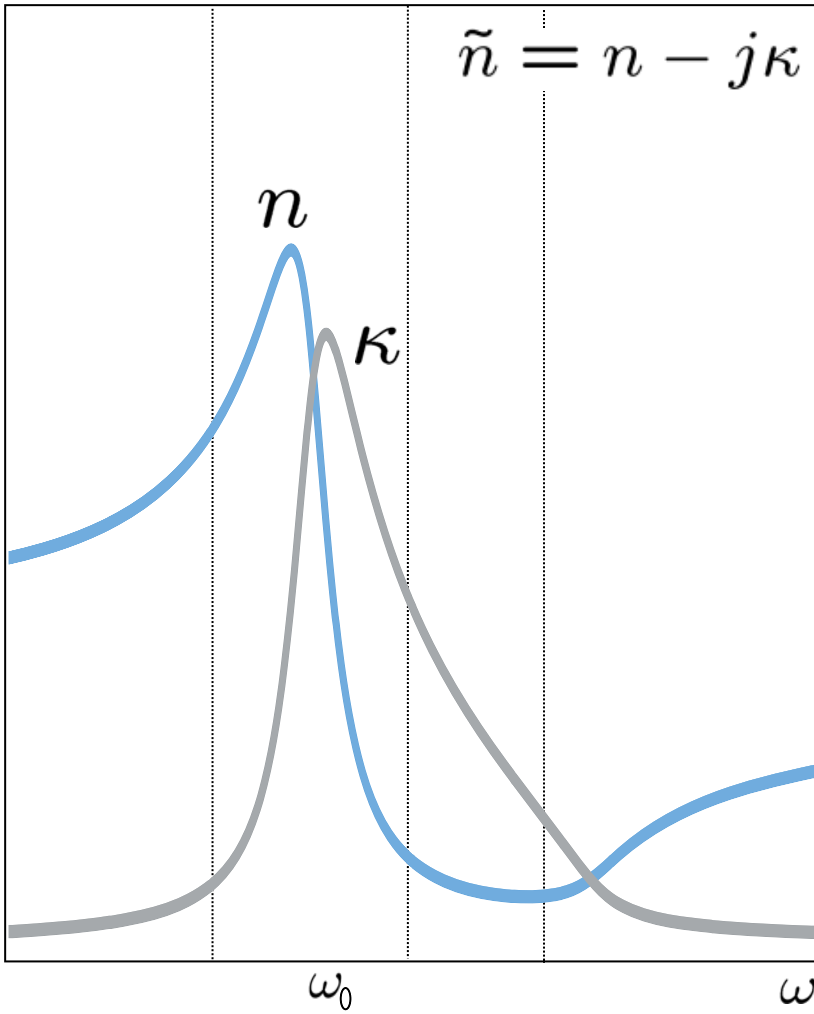


$$\tilde{n} = n - j\kappa$$

$$\tilde{\epsilon} = \tilde{n}^2$$
$$= (n - j\kappa)^2$$
$$= n^2 - \kappa^2 - 2jn\kappa$$

$$\epsilon_r = n^2 - \kappa^2$$
$$\epsilon_i = 2n_r n_i$$

Absorption Coefficient



$$n = \frac{1}{\sqrt{2}}\sqrt{\epsilon_r + \sqrt{\epsilon_r^2 + \epsilon_i^2}}$$

$$\kappa = \frac{1}{\sqrt{2}}\sqrt{-\epsilon_r + \sqrt{\epsilon_r^2 + \epsilon_i^2}}$$

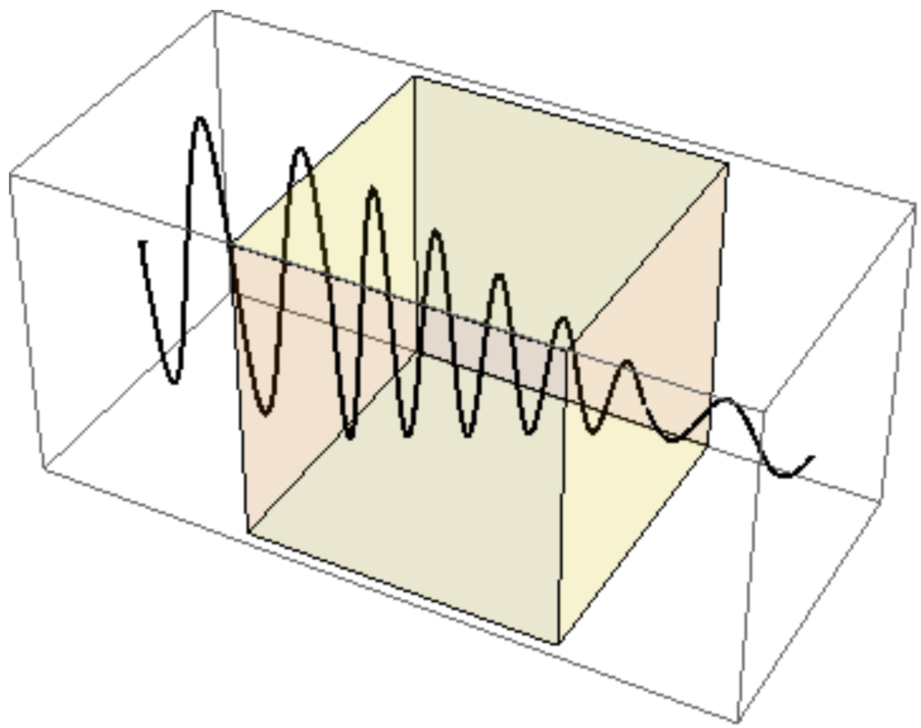
$$E(t,z) = Re\{\tilde{E}_0 e^{-\alpha z/2} e^{j(\omega t - k_0 n z)}\}$$

Absorption

Refractive index

$$\alpha = 2k_o\kappa = 2\frac{2\pi}{\lambda_o}\kappa \quad [\text{cm}^{-1}]$$

Absorption



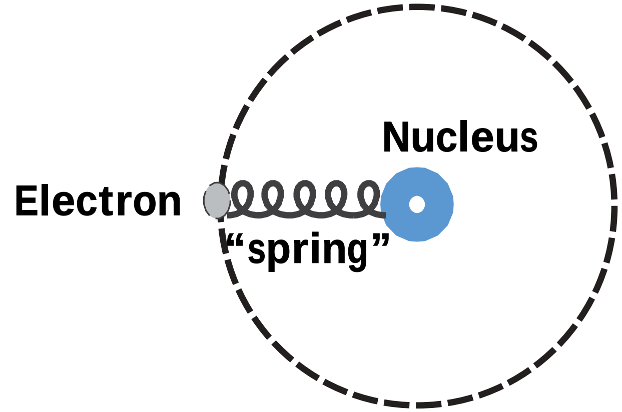
$$E(t,z) = Re\{\tilde{E}_0 e^{-\alpha z/2} e^{j(\omega t - k_0 n z)}\}$$

Absorption coefficient

Refractive index

$$I(z) = I_o e^{-\alpha z}$$ Beer-Lambert Law or Beer's Law

Key Takeaways



Lorentz Oscillator Model

$$m\frac{d^2y}{dt^2} = -m\omega_o^2 y + qE_y - m\gamma\frac{dy}{dt}$$

$$\vec{P} = \frac{\omega_p^2}{(\omega_o^2 - \omega^2) + j\gamma\omega}\epsilon_o\vec{E}$$

$$\Rightarrow \tilde{n} = n - j\kappa \qquad \alpha = 2k_o\kappa$$

$$\Rightarrow \epsilon = \epsilon_r - j\epsilon_i$$

Decay

$$E(t,z) = Re\{\tilde{E}_0 e^{-\alpha z/2} e^{j(\omega t - k_0 n z)}\}$$

Absorption coefficient

Refractive index

$I(z) = I_o e^{-\alpha z}$ Beer’s Law