“Solenoids” In Magnetostatics

<u>Outline</u>

Solenoids
Solenoidal Inductors
Toroids & Applications
Permanent Magnets

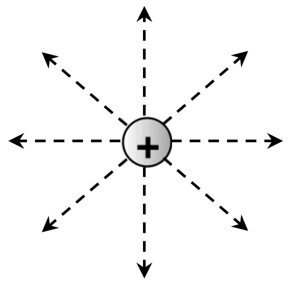
TRUE or FALSE?

1. The normal electric field is always continuous at a surface. ____

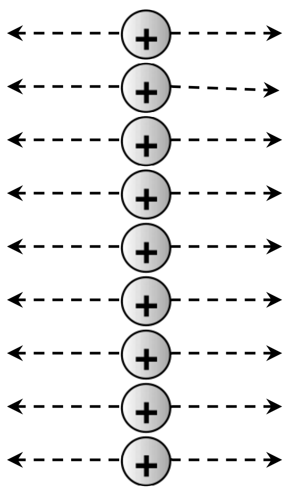
2. The normal magnetic field is always continuous at a surface. ____

3. There is a current-carrying wire coming out of the board. If we integrate along the path, we will find that the magnetic field is zero along the path. ____

Electrostatics



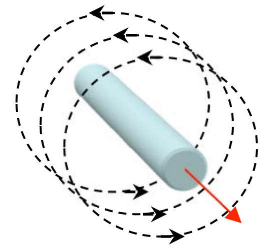
Superposition ...



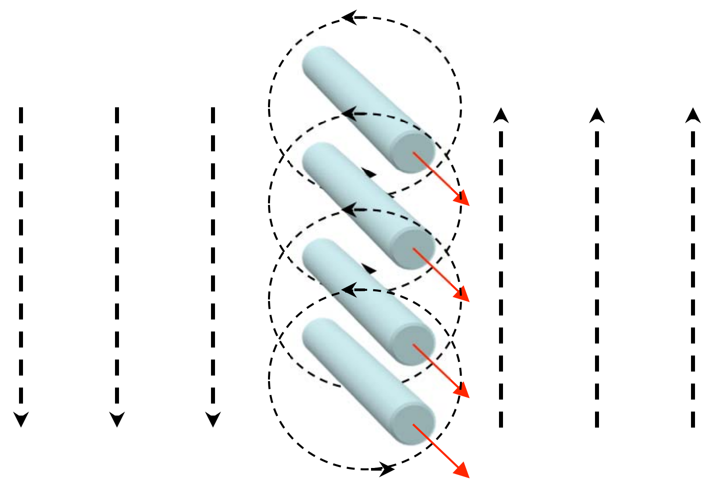
The normal electric field is discontinuous across a surface charge.

$$\overline{n} \cdot \left(\epsilon_0 \overline{E}_1 - \epsilon_0 \overline{E}_2\right) = \sigma_s$$

Magnetostatics



Superposition ...



The tangential magnetic field is discontinous across a surface current.

$$\overline{n} \times \left(\overline{H}_1 - \overline{H}_2\right) = \overline{K}$$

Electric Fields

$$\oint_S \epsilon_o \overline{E} \cdot d\overline{A} = \int_V \rho dV$$

$$= Q_{enclosed}$$

$$\oint_C \overline{E} \cdot d\bar{l} = -\frac{d}{dt}\left(\int_S \overline{B} \cdot d\overline{A}\right)$$

Magnetic Fields

$$\oint_S \overline{B} \cdot d\overline{A} = 0$$

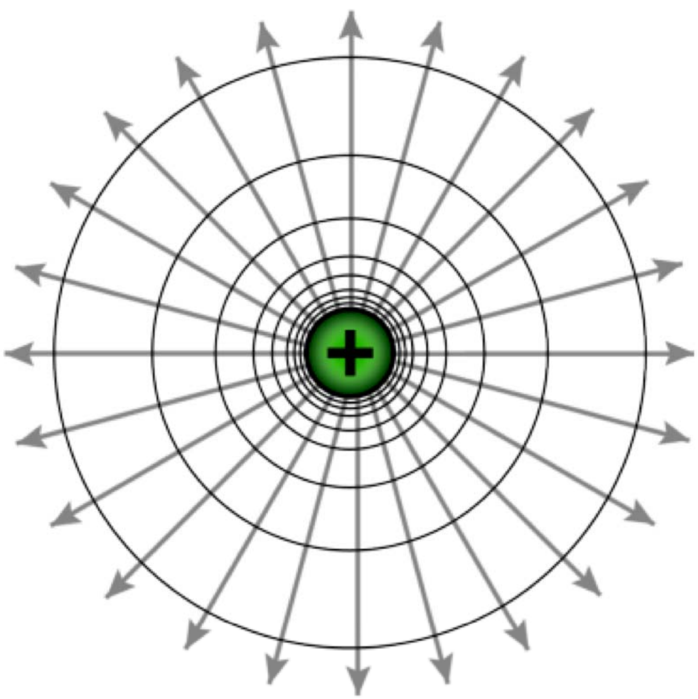
$$\oint_C \overline{H} \cdot d\bar{l}$$

$$= \int_S \overline{J} \cdot d\overline{A} + \frac{d}{dt}\int_S \epsilon E dA$$

Electrostatics

$$\int_S \epsilon_o \overline{E} \cdot d\overline{A} = \int_V \rho dV$$

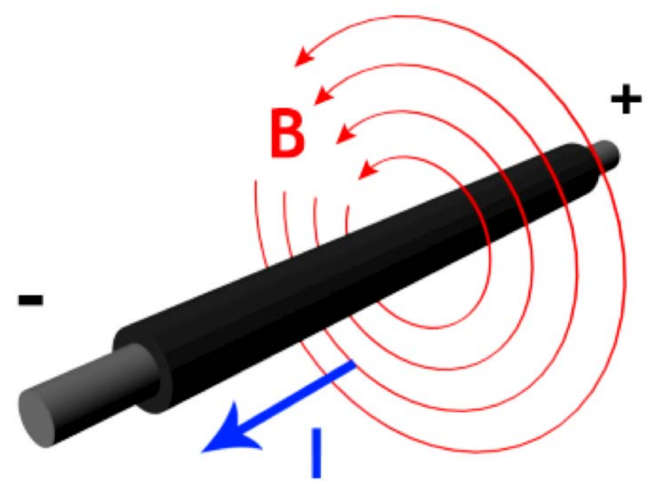
$$= Q_{enclosed}$$



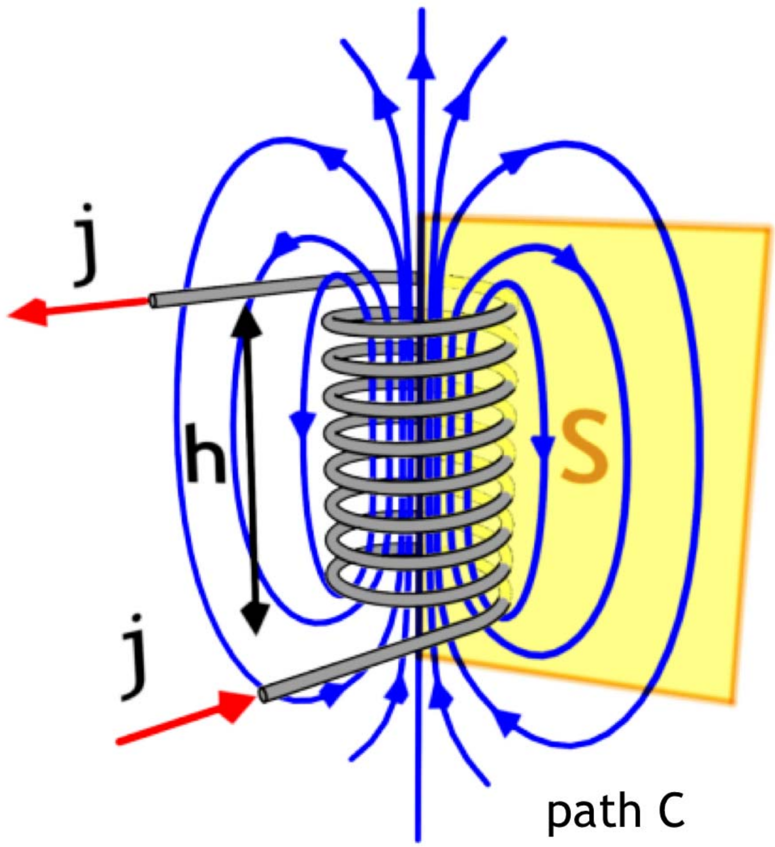
Magnetostatics

$$\int_C \overline{H} \cdot d\bar{l} = \int_S \overline{J} \cdot d\overline{A}$$

$$= I_{enclosed}$$



Fields from a Solenoid



$$\int_C \overline{H} \cdot d\bar{l} = \int_S \overline{J} \cdot d\overline{A}$$

$$= I_{enclosed}$$

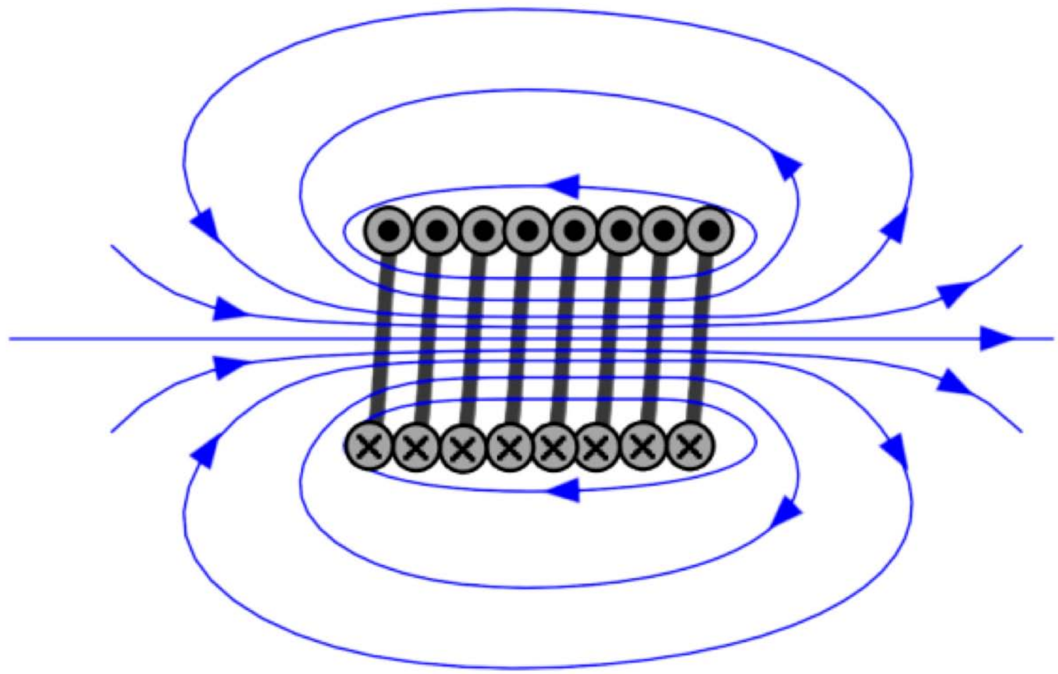
$$H_{inside} \approx \frac{NI}{h}$$

The same result can be obtained using the boundary condition from Ampere's Law with $K = Ni/h$

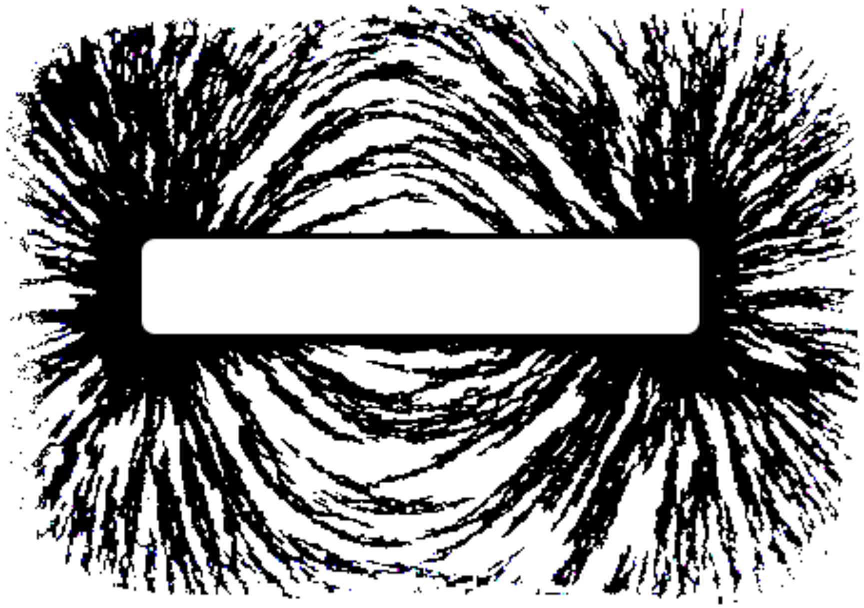
Magnetic Flux Lines

A magnetic pole sets up a magnetic field in the space around it that exerts a force on magnetic materials. The field can be visualized in terms of magnetic flux lines (similar to the lines of force of an electric field). These imaginary lines indicate the direction of the field in a given region. By convention they originate at the north pole of a magnet and form loops that end at the south pole either of the same magnet or of some other nearby magnet. <u>The flux lines are spaced so that the number per unit area is proportional to the field strength in a given area</u>. Thus, the lines converge near the poles, where the field is strong, and spread out as their distance from the poles increases.

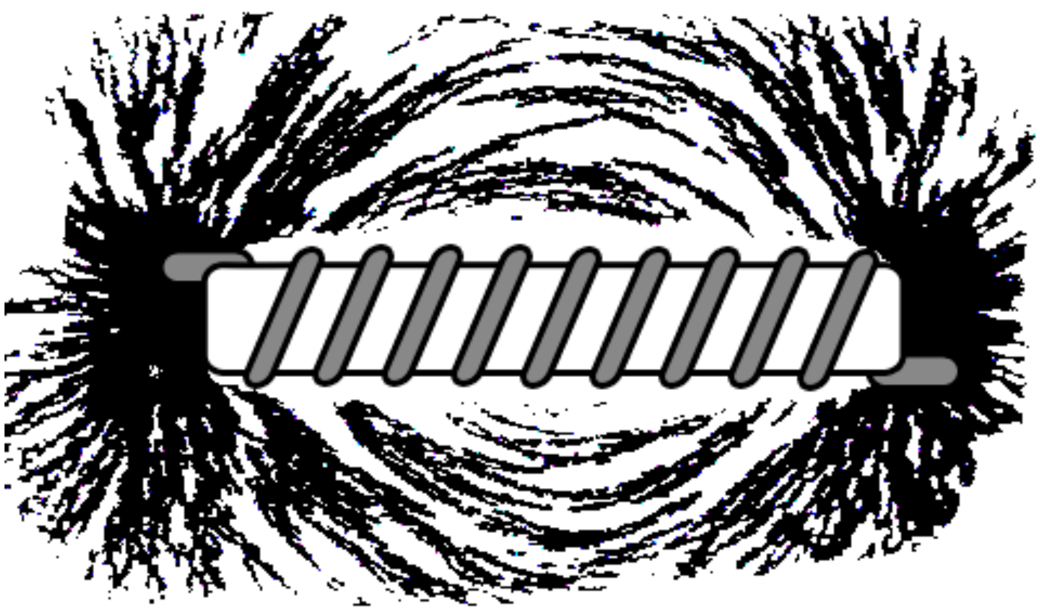
A picture of these lines of induction can be made by sprinkling iron filings on a piece of paper placed over a magnet. The individual pieces of iron become magnetized by entering a magnetic field, i.e., they act like tiny magnets, lining themselves up along the lines of induction.



MAGNETIC FIELD OF A BAR MAGNET



Flux lines close together indicate high magnetic flux density, B, near the poles



We calculated the field of a long solenoid using Ampere's law

$$\int_C \overline{H} \cdot d\bar{l} = \int_S \overline{J} \cdot d\overline{A}$$
$$= I_{enclosed}$$

For long solenoid:

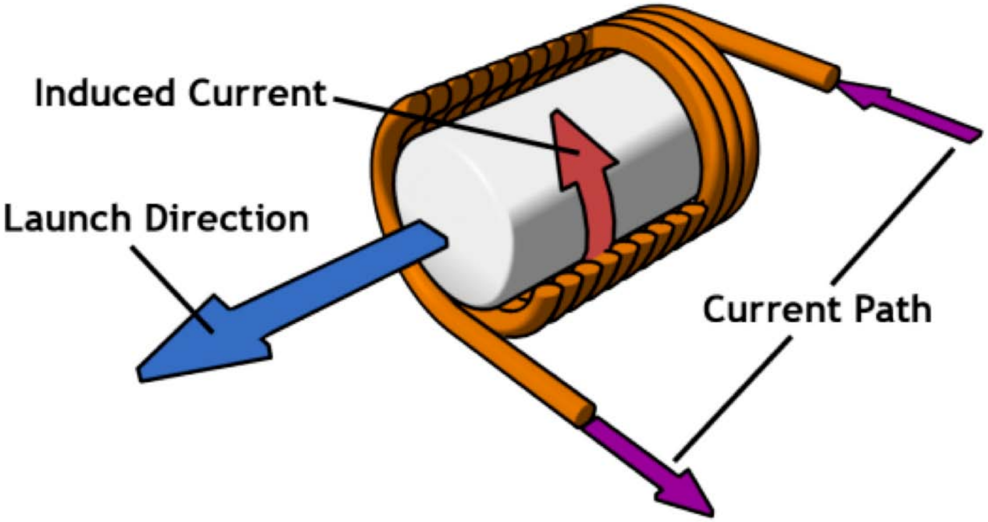
$$H_{inside} = n\,i$$
$$H_{outside} \approx 0$$

Uses of Solenoids

TRANSFORMER

SPEAKER

TUBULAR INDUCTION LAUNCHER



HELMHOLTZ COIL

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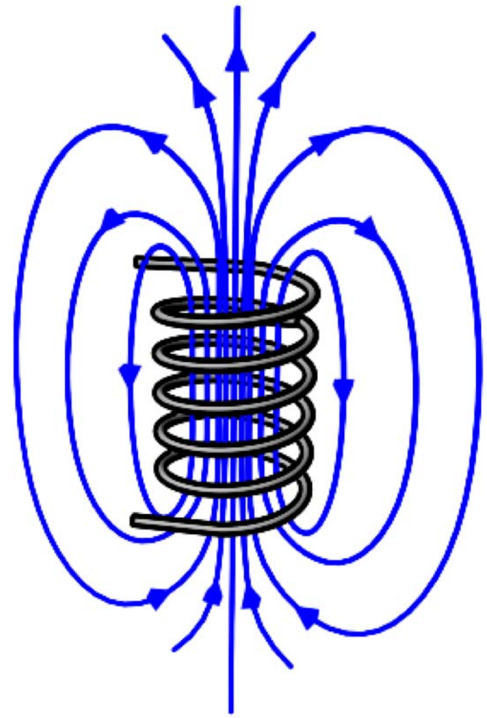
A Preview of Inductance

INDUCTOR

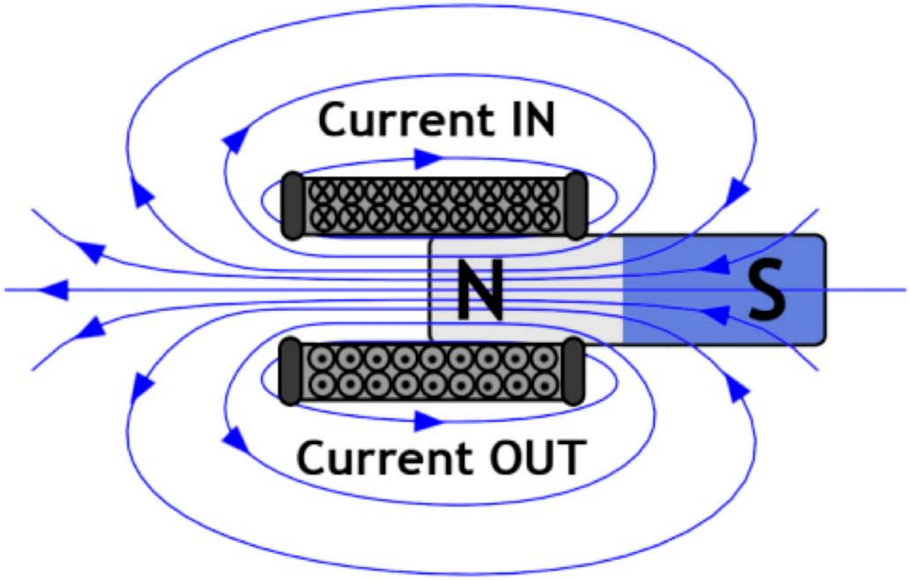


Cross-sectional area A →

$$\lambda = NA\ \mu_0\ Ni/h \rightarrow$$

$$L = \mu_0 N^2 A / h$$

Ding Dong!



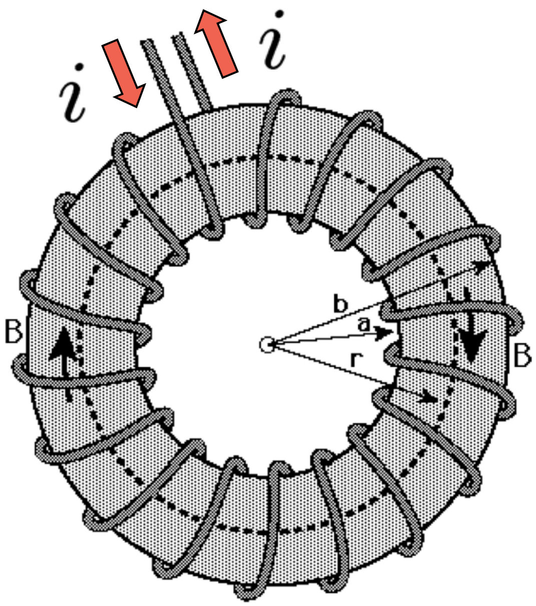
A 1920 explanation of a commercial solenoid used as an electromechanical actuator. Source: Wikipedia

Image by takomabibelot http://www.flickr.com/photos/takomabibelot/3917734943/ on flickr

Fields from a Toroid

$$\int_C \overline{H} \cdot d\bar{l} = \int_S \overline{J} \cdot d\overline{A}$$

$$= I_{enclosed}$$



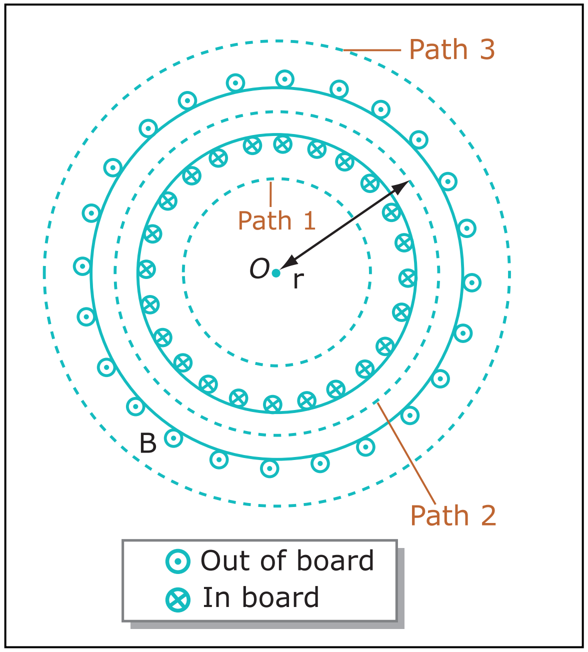


Image by MIT OpenCourseWare.

Path 1

$H_\phi =$ [ ]

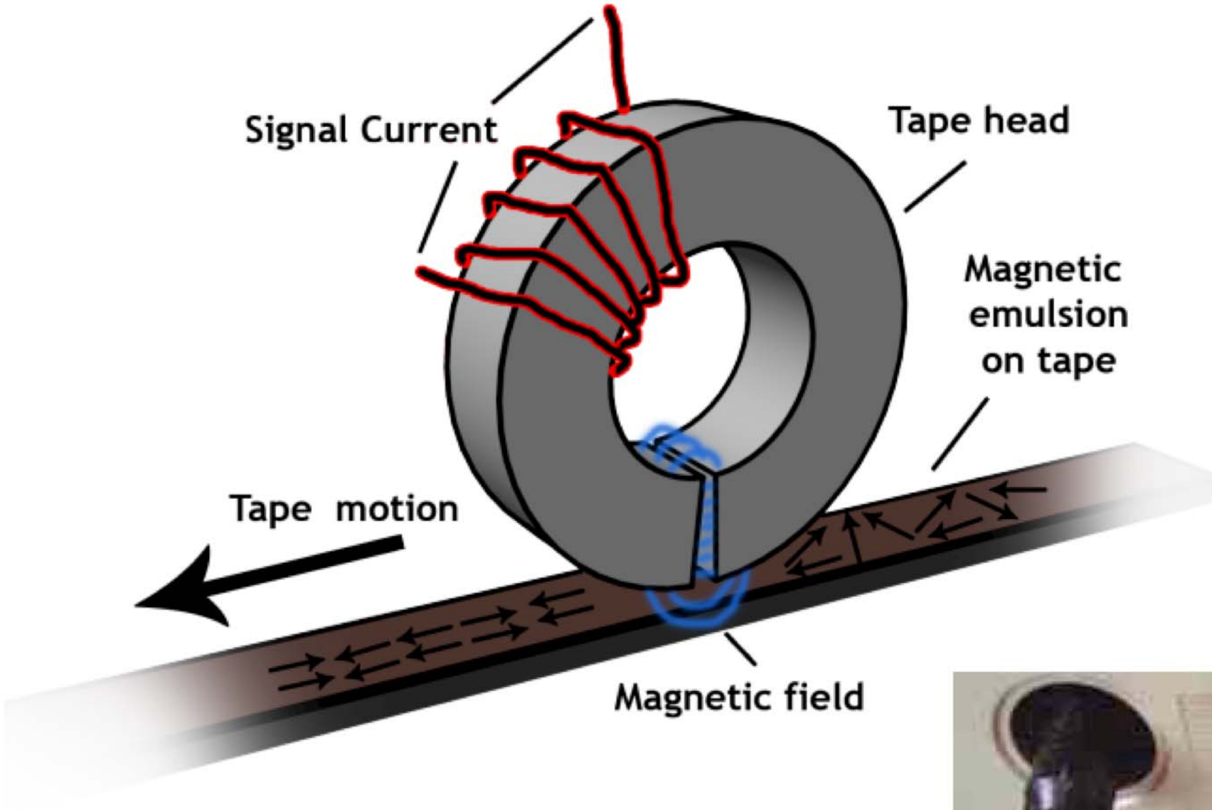
Path 2

$H_\phi =$ [ ]

Path 3

$H_\phi =$ [ ]

Toroids in the Living Room



Permanent Magnets

Neodymium-Boron-Iron, Niobium–Iron–Cobalt and Samarium-Cobalt permanent magnets all produce very large magnetic fields.

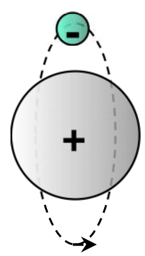
$$d = 0.012\,\text{meters}$$

$$B = 0.5\ \text{Tesla}$$

$$h = 0.019\,\text{meters}$$

Microscopic & Macroscopic Magnets

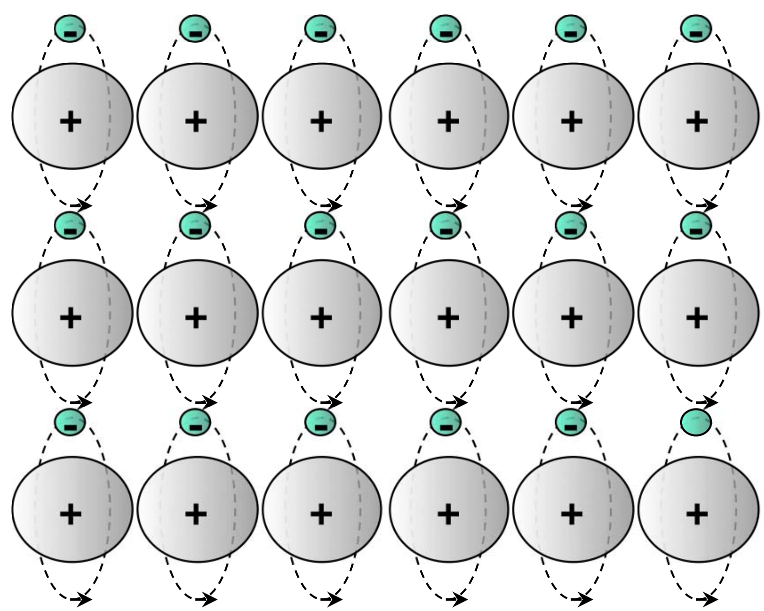
The current within an atom due to electron orbit creates a magnetic moment and field.



The current within an atom due to electron spin creates a magnetic moment and field.

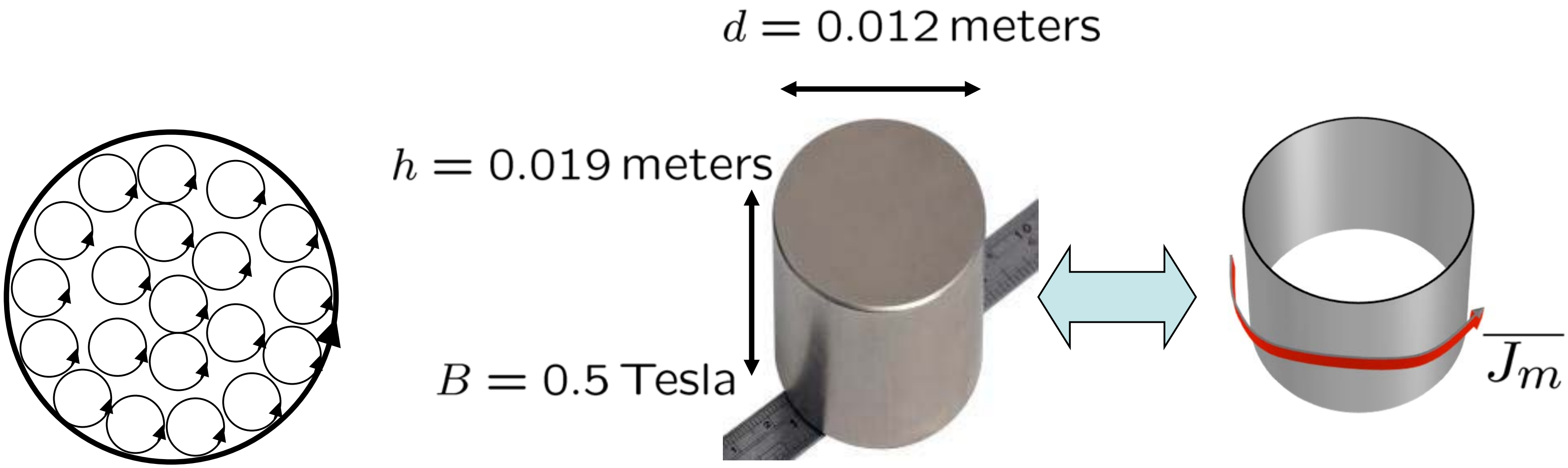
Permanent magnet ...

Some materials have organized/permanent magnetic moments while others do not.



Generating Strong Magnetic Fields

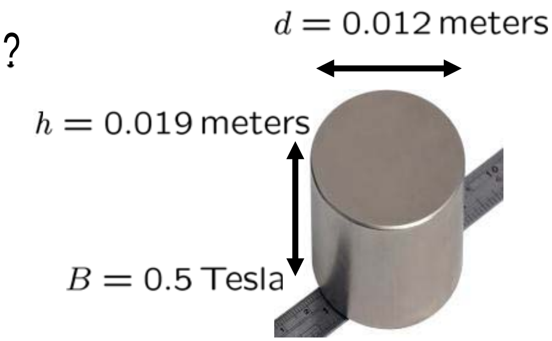
Superposition with current loops…



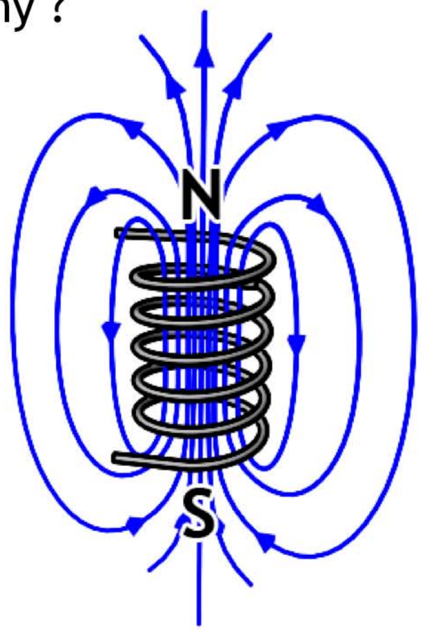
$$\mu_o = 4\pi \times 10^{-7} \left[\frac{\text{T}}{\text{A/m}}\right]$$

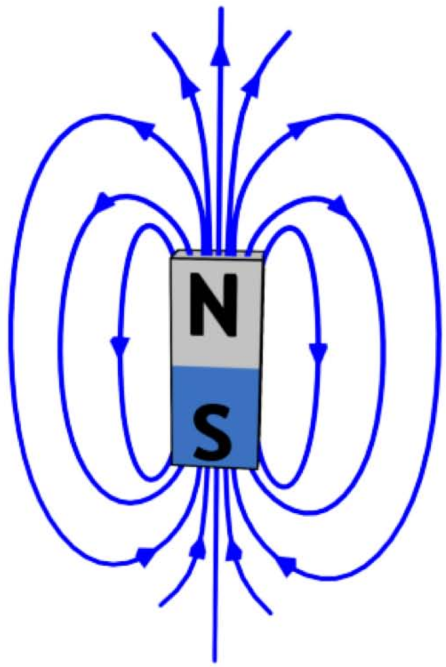
Generating Strong Magnetic Fields

Will it be "easier" to generate a 0.5-T magnetic flux density with a permanent magnet or an electromagnet ?



Why ?





$$B = 0.5\ T = \mu_o H$$
$$= 0.5\ T = \mu_o ni/h$$

$$\mu_o = 4\pi \times 10^{-7} \left[\frac{\text{T}}{\text{A/m}}\right]$$

$$ni = $$

0.5 T Electromagnets

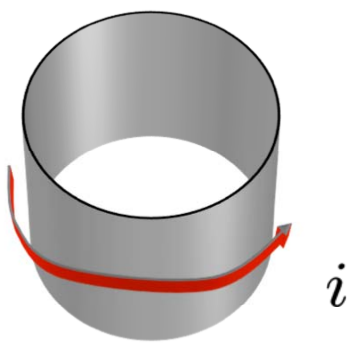
$h = 0.019$ meters
$d = 0.012$ meters

$$B = \mu_o H = \mu_o n i = 0.5\,\text{T}$$

$$\Rightarrow \quad i = \frac{B}{\mu_o}\frac{h}{N}$$

0.5 Tesla with current loop…

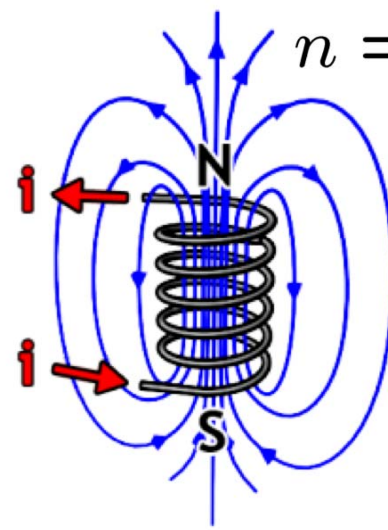
$n = 1$



$i_{0.5T} \approx$ 7600 Amps

0.5 Tesla with 1000 turn solenoid…

$n = 1000$

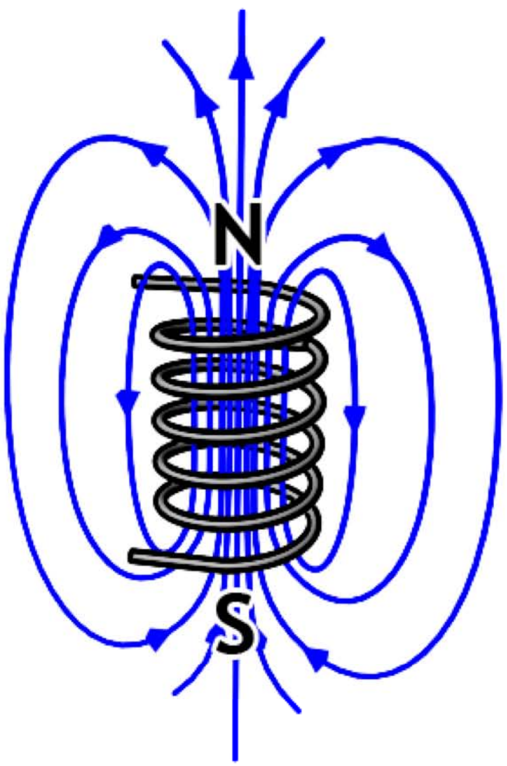


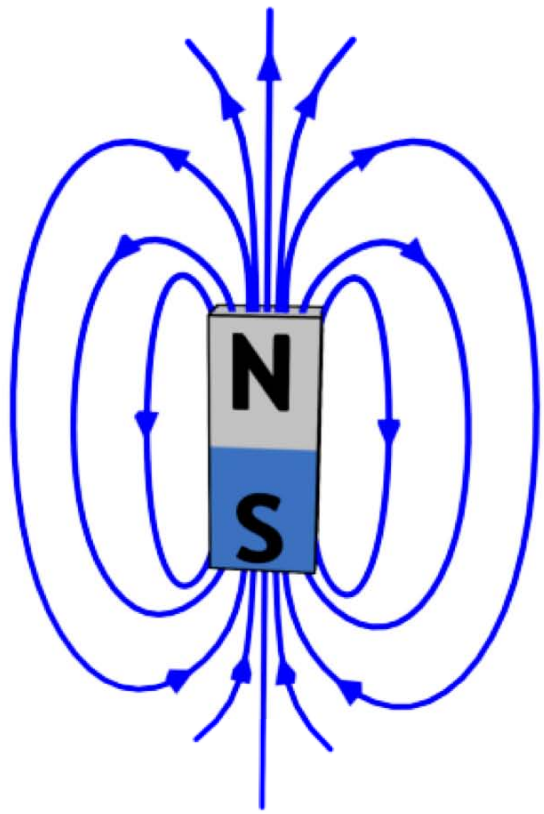
$i_{0.5T} \approx$ 7.6 Amps

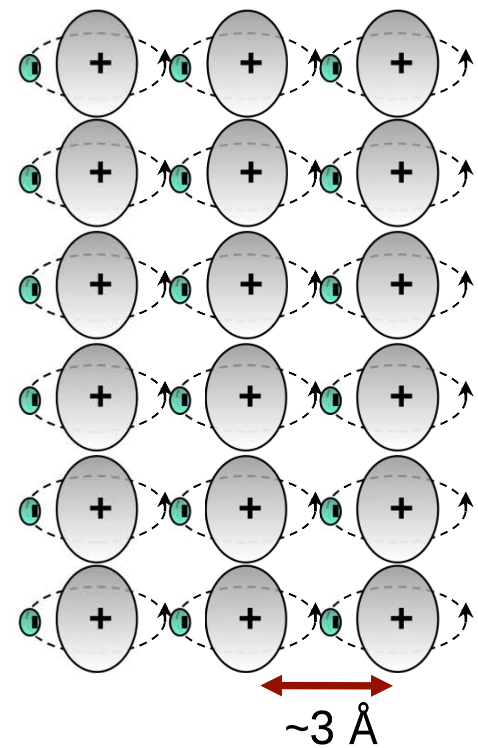
...but the wires are microscopic !

Generating Strong Magnetic Fields

Will it be "easier" to generate a 0.5-T magnetic flux density with a permanent magnet or an electromagnet ?



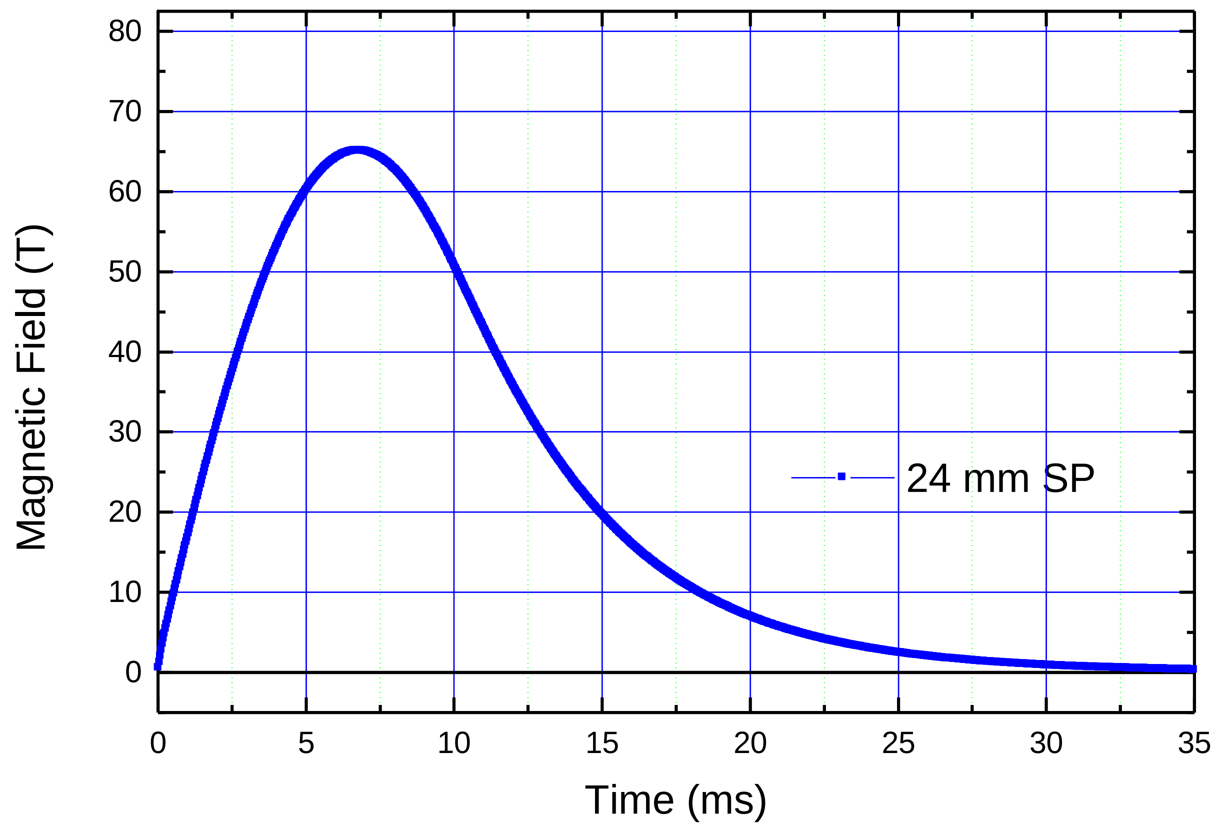




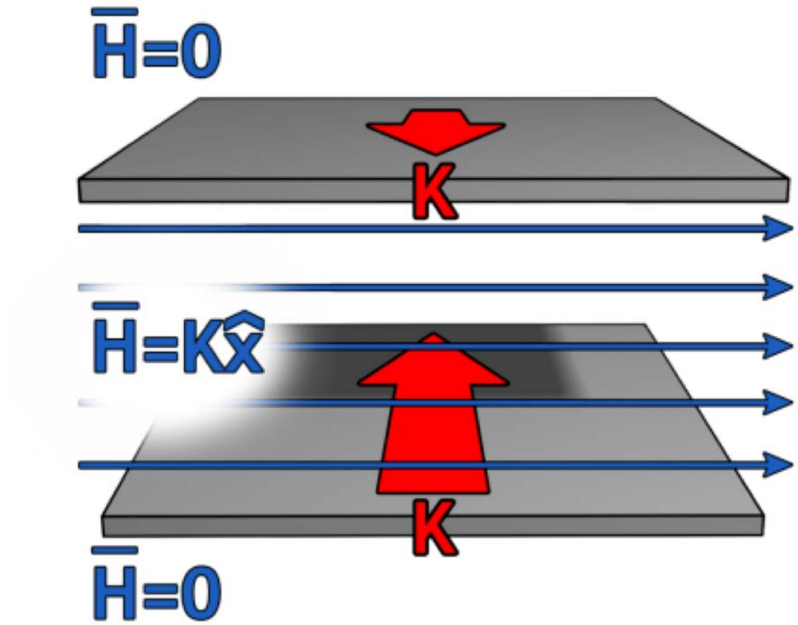
$i_{atom} \approx 10^{-4}$ A and $K_{atom} \approx 3000$ A/cm !

Record Breaking (Pulsed) Electromagnets

- Coil consists of few turns to keep the coil inductance low
- Magnet cooled to 77 K prior to pulsing increases conductivity



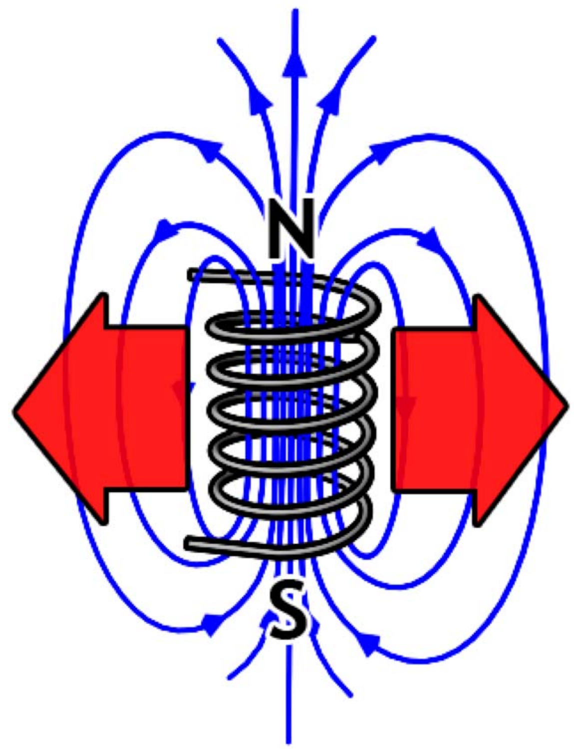
Force on Current Sheet



The force acts to
SEPARATE the plates.

Note that the direction of the force tends to increase the volume that the field is stored in.

What about forces on a solenoid ?



What Sets the Limit ?

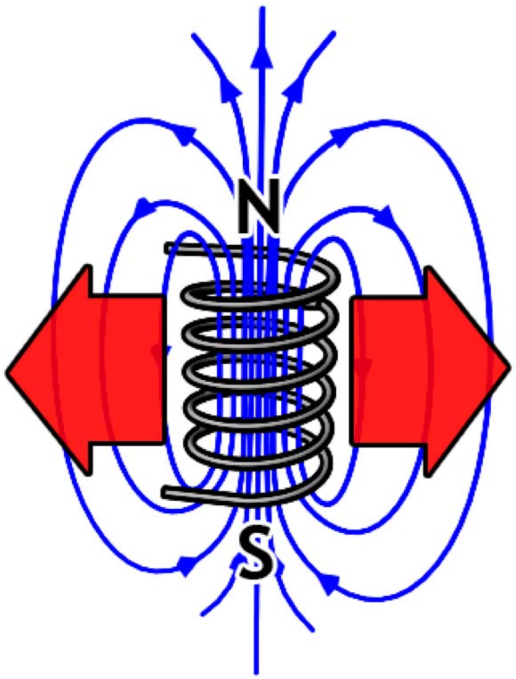
1 atmosphere = 14.7 pounds per square inch

Pressure Under Water

1000 m	**Submarine**	**1000 psi**
4000 m	**Ocean Floor Submersible**	**6000 psi**

80-T Pulsed Magnet **200,000 psi**

... *exceeds the practical strength of most materials* ...



Strong Electromagnets Generate
HUGE Forces- this can be disastrous if not controlled!

Summary

• Maxwell's Equations (in Free Space with Electric Charges present):

	DIFFERENTIAL FORM	INTEGRAL FORM
E-Gauss:	$\nabla \cdot \epsilon_o \vec{E} = \rho$	$\oint\!\!\oint_S \epsilon_o \vec{E} \cdot dS = \iiint_V \rho dV$
Faraday:	$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\mu_o \vec{H}$	$\oint_C \vec{E} \cdot d\vec{C} = \iint_S -\frac{\partial}{\partial t}\mu_o \vec{H} \cdot d\vec{S}$
H-Gauss:	$\nabla \cdot \mu_o \vec{H} = 0$	$\oint\!\!\oint_S \mu_o \vec{H} \cdot d\vec{S} = 0$
Ampere:	$\nabla \times \vec{H} = \vec{J} + \frac{\partial}{\partial t}\epsilon_o \vec{E}$	$\oint_C \vec{H} \cdot d\vec{C} = \iint_S (\vec{J} + \frac{\partial}{\partial t}\mu_o \vec{H}) \cdot d\vec{S}$

6.007 Electromagnetic Energy: From Motors to Lasers
Spring 2011