MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.007 – Electromagnetic Energy: From Motors to Lasers
Spring 2011

Lab 4: Spectrometer
Tuesday and Wednesday, April 12, 13
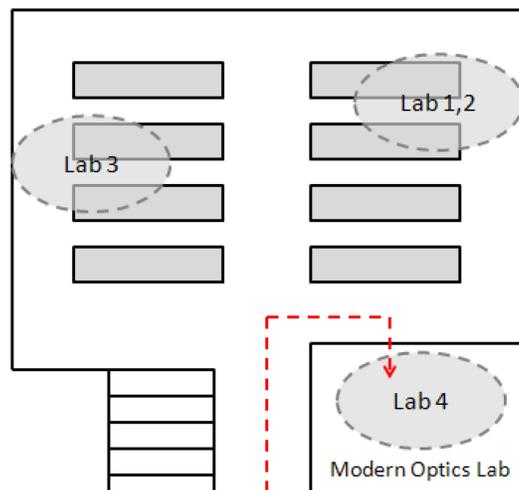*Due: Friday, April 22, 2011*

Introduction

- **The pre-lab component of the lab (the questions and the take home kit) should be completed before coming to the lab.**

- **Have one of the TAs check you off for the prelab**

- **Have one of the TAs check you off for the completion of the lab component before leaving the lab**

In this lab you will be exploring many aspects of optics. In class you learned about light polarization, Snell's law, critical angle, gratings, Fresnel equations and Brewster's angle. We will now be experimentally verifying all of these concepts!

This lab has three components:

- Fresnel coefficient

- Finding the index contrast of the core and the cladding of the fiber used in your pre-lab.

- Spectrometer measurements

This lab will take place in the Modern Optics Lab on the 6th floor of building 38. Because of the presence of chemicals, there is no food or drink in the lab at any time.

Figure 1: Don't get lost!

1 Theory and Background

There are a couple of concepts you need to be familiar with to fully understand this lab. The reading in this section should get you familiar so you can enjoy the lab!

1.1 Fresnel Coefficient

The derivation of the Fresnel equations is given in Chapter 4 of Shen and Kong. Here we will rewrite them in more useful forms. We will start with reflectivity equations for TE and TM given in 4.22 and 4.34 of the book:

$$r_{TE} = \frac{\mu_2 k_z - \mu_1 k_{tz}}{\mu_2 k_z + \mu_1 k_{tz}} \tag{1}$$

$$r_{TM} = \frac{\epsilon_2 k_z - \epsilon_1 k_{tz}}{\epsilon_2 k_z + \epsilon_1 k_{tz}} \tag{2}$$

Now we replace k_z and k_{tz} with the following:

$$k_z = k\cos(\theta_i) \tag{3}$$

$$k_{tz} = k_t\cos(\theta_t) = \frac{n_2}{n_1}k\cos(\theta_t) \tag{4}$$

Substituting Equations 3 and 4 into Equations 1 and 2 gives

$$r_{TE} = \frac{\mu_2 k\cos(\theta_i) - \mu_1 \frac{n_2}{n_1}k\cos(\theta_t)}{\mu_2 k\cos(\theta_i) + \mu_1 \frac{n_2}{n_1}k\cos(\theta_t)} \tag{5}$$

$$= \frac{n_1\cos(\theta_i) - n_2\cos(\theta_t)}{n_1\cos(\theta_i) + n_2\cos(\theta_t)} \tag{6}$$

$$r_{TM} = \frac{\epsilon_2 k\cos(\theta_i) - \epsilon_1 \frac{n_2}{n_1}k\cos(\theta_t)}{\epsilon_2 k\cos(\theta_i) + \epsilon_1 \frac{n_2}{n_1}k\cos(\theta_t)} \tag{7}$$

$$= \frac{n_2\cos(\theta_i) - n_1\cos(\theta_t)}{n_2\cos(\theta_i) + n_1\cos(\theta_t)} \tag{8}$$

For Equation 5, we assumed $\mu_1 = \mu_2$ and for Equation 7 we used the relationship $n = \sqrt{\frac{\epsilon}{\epsilon_0}}$.

Finally, we can write r_{TE} and r_{TM} as only a function of θ_i by substituting $\cos(\theta_t) = \sqrt{1 - (\frac{n_1}{n_2}\sin(\theta_i))^2}$ (Pythagorean trignometric identity and Snell's law):

$$r_{TE} = \frac{n_1\cos(\theta_i) - n_2\sqrt{1 - (\frac{n_1}{n_2}\sin(\theta_i))^2}}{n_1\cos(\theta_i) + n_2\sqrt{1 - (\frac{n_1}{n_2}\sin(\theta_i))^2}} \tag{9}$$

$$r_{TM} = \frac{n_2\cos(\theta_i) - n_1\sqrt{1 - (\frac{n_1}{n_2}\sin(\theta_i))^2}}{n_2\cos(\theta_i) + n_1\sqrt{1 - (\frac{n_1}{n_2}\sin(\theta_i))^2}} \tag{10}$$

You will be using Eqs. 9 and 10 to plot the theoretical curves for the Fresnel coefficient experiment.

1.2 Beer's Law

We learned in class that a Lorentz oscillator with damping results in an imaginary component to the index of refraction. If we write the total index of refraction as $n - j\kappa$, we see that the equation for a propagating wave becomes

$$E(z,t) = E_0 e^{jwt} e^{-jkz} = E_0 e^{jwt} e^{-j\frac{nw}{c}z} e^{-\frac{\kappa w}{c}z} \tag{11}$$

In addition to the propagating wave, we now have an exponential decay of the electric field. Since the only way to measure the electric field is by measuring the intensity of the light, we will define a new variable

$$\alpha = \frac{2\kappa w}{c} \tag{12}$$

where the factor of two results from squaring the electric field to get intensity.

If we look at the magnitude or intensity of the wave in a lossy material, the imaginary exponentials evaluate to unity and we see that the intensity behaves as

$$I(z) = I_0 e^{-\alpha z} \tag{13}$$

Since α comes from the lorentz oscillator model, we also know that α includes the number of dipoles responsible for the oscillations. If we are looking at a dilute number of oscillators, such as we would find in a liquid or a gas, we can decompose alpha into a molar extinction coefficient ϵ or molar absorptivity and a concentration.

$$\alpha = \epsilon c \tag{14}$$

where ϵ has units of $\frac{m^{-1}}{Moles/Liter}$ and c has units of $\frac{Moles}{Liter}$. This decomposition of α will be useful for determining the absorption associated with different concentrations of dyes in liquid.

If we are interested in finding the value of α or ϵ of a material, we can take measurements of intensity for different values of z or c through a medium and use the exponential decay relationship. To do this, we rewrite equation 13 by taking the natural log

$$\ln(I(z)) = \ln(I_0) - \alpha z \tag{15}$$

$$\alpha = \frac{1}{z}\left(\ln(I_0) - \ln(I(z))\right) \tag{16}$$

2 In Lab

There are three sections to the lab. You do not necessarily have to complete them in the order listed here. Work on whichever experiments are available but make sure you have done them all in the end.

2.1 Fresnel coefficient

In this section, you will be measuring the Fresnel coefficient of acrylic glass-air interface using a setup shown in Figure 2. You will see that the reflectivity is dependent on the angle of incidence and will also observe Brewster's angle as well as the critical angle. The light source is a red laser (with output of approximately 1 mW). The laser beam is directed through a linear polarizer to polarize the beam in the TE or TM direction with respect to the glass-air interface. The beam then hits a semi-circular prism mounted onto a rotating stage as shown in Figure 2. Note that the beam passes through reflective surfaces three times: air-glass (going into the glass prism), glass-air (reflected back in the glass prism), and glass-air (going out of the glass prism). However, only the second reflection will be dependent on the rotation of the stage because the other two surfaces are always normal with respect to the laser beam (which is why we use a semi-circular prism here!). The power of the reflected laser beam is then measured using a photodetector.
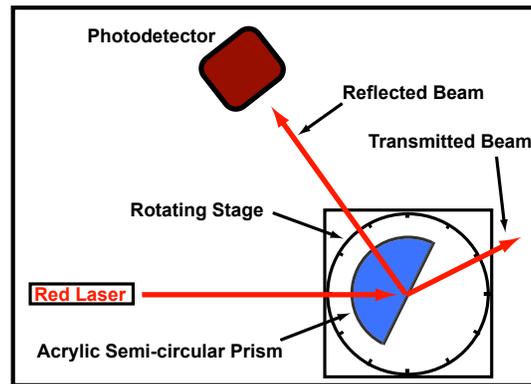

Figure 2: Setup to measure Fresnel coefficient

Here are the instruction for this experiment:

(1) Rotate the linear polarizer to the TM position.

(2) Make sure the beam is directed to the center of the prism. Place the photodetector before the prism to measure the power of the laser beam going into the prism. Incident power: _____

(3) Remove the photodetector out of the beam's path. Rotate the prism until the reflected beam from the prism's flat surface is directed straight back towards the laser pointer. Read the angle indicated on the stage. This is your "0" degrees. Angle: _____. For subsequent angle measurements, remember that the angle you should be recording is the angle with respect to this "0" degrees.

(4) Rotate the stage until the reflected beam from the flat acrylic surface disappears. Measure this angle. This is the Brewster angle. Now, keep rotating the stage until the transmitted beam from the flat acrylic surface disappears. Measure this angle. This is the critical angle. Brewster angle: _____. Critical angle: _____.

Recalling that the critical angle is given by

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \tag{17}$$

calculate the index of refraction of the prism. n_{prism}: _____

(5) Now, rotate the stage back so the angle of incidence at the flat acrylic surface is 6 degrees. Using the photodetector, measure the power of the reflected beam. Increase the angle by 4 degrees at a time and measure the reflected power each time. Record these values in the table given below. When you measure the power, it is important for your measurements to be precise. Make sure the entire

reflected beam is detected by the photodetector and that the beam is hitting the photodetector at normal incidence. Remember that each tick mark on the rotation stage is 2°.

(6) Rotate the linear polarizer to the TE position.

(7) Make sure the beam is directed to the center of the prism. Place the photodetector before the prism to measure the power of the laser beam going into the prism. Incident power: _____

(8) Rotate the stage until the transmitted beam from the flat acrylic surface disappears and measure the critical angle. Critical angle: _____.

(9) Now, rotate the stage back so the angle of incidence at the flat acrylic surface is 6 degrees. Using the photodetector, measure the power of the reflected beam. Increase the angle by 4 degrees at a time and measure the reflected power each time. Record these values in the table given below.

TM: Angle [degree]	TM: Reflected Power [μW]
6	
10	
14	
18	
22	
26	
30	
34	
38	
42	
46	
50	

TE: Angle [degree]	TE: Reflected Power [μW]
6	
10	
14	
18	
22	
26	
30	
34	
38	
42	
46	
50	

2.2 Index contrast of the fiber

Take your completed take-home kit out (LED attached to a fiber). Turn your LED on and observe the light coming out from the end of the fiber. If you lay your fiber down onto a flat sheet of paper, you see that the emitted light from the fiber comes out in a cone. Recall that the sine of the cone half-angle is the numerical aperture, NA, which is given by

$$\text{NA} = \sin\theta_{\max} = \sqrt{n_{\text{core}}^2 - n_{\text{clad}}^2} \qquad (18)$$

Measure this angle using a protractor and calculate the index ratio between the core and the cladding of the fiber. You may want to turn off the lights to see the full extent of the cone.

Numerical Aperture: _____

The core of the fiber is made of polymetyl-methacrylate resin, which has a refractive index of 1.49. What is the refractive index of the cladding? $n_{\text{clad}} =$ _____

Next you will fill the 100mm-long hollow teflon tube fiber with ethylene glycol and couple the light from the LED fiber into this new fiber with ethylene glycol as its core. First, try inserting your LED-coupled fiber into the empty tube fiber as practice. Next, position the 100mm-long hollow teflon tube fiber as shown in Figure 3. Ask a TA to help you fill the tube; slightly overfill it so that there is a drop of ethylene glycol extending out from both sides. Insert the ethylene glycol fiber into the V-groove holder against the glass slide to flatten the meniscus and secure the fiber with the screw. Make sure that the liquid in the tube wets the glass slide and that there are no bubbles in the tube or between the tube exit and the glass slide. Insert the LED-coupled fiber into the other side.
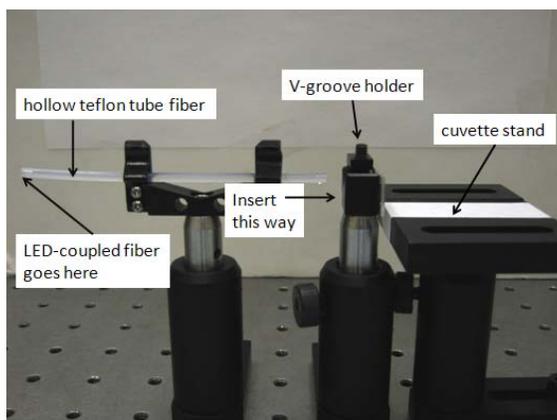

Figure 3: Setup for holding the hollow tube fiber.

Measure the cone angle of the output of the liquid waveguide with the protractor and a piece of paper. Numerical Aperture: _____.

If the index of the teflon tubing is 1.32, what is the index of ethylene glycol?
Ethylene Glycol Refractive Index: _____.

2.3   Spectrometer

Take a spectrometer kit and take out the components. You should have the following items.

- 1.5 inch optical posts
- 2 inch optical post holders
- Fiber V-groove holder
- Glass mounted plastic linear grating
- Grating holder
- Digital camera
- Black cardboard
- Hollow teflon tubing of lengths 100 mm, 50 mm
- 1000 μm multimode fiber
- Cuvette holder

2.3.1　Building the Spectrometer

Pick a station on the optical table to build your setup. You will need approximately an area of 2 ft x 1 ft to build your setup. For reference, the threaded 1/4"-20 screw holes on the optical table are 1 inch apart. Build your setup according to the schematic provided in figure 4. The spectrometer consists of 3 components, an input waveguide source, a grating, and a digital camera. The digital camera will be imaging the output of the waveguide through the grating. Since the 1st order diffraction occurs at an angle, when we image the waveguide output at the 1st order diffraction angle of the grating, we will see a line of waveguide outputs spread due to wavelength dependent diffraction. When setting up the system, make sure that the V-groove is higher than the cuvette holder such that you can place a vial of liquid in the light path. Ask your TA for help if necessary.
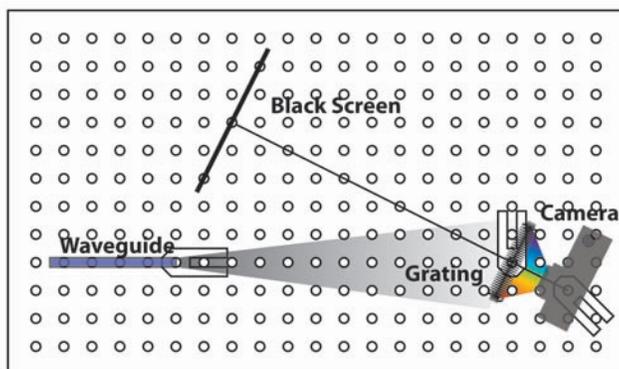

Figure 4: Schematic for building the spectrometer.

2.3.2　Camera focus and alignment

You should already have the 100mm-long tube and fiber coupled LED mounted into the V-groove holder. Now turn on your white LED. Rotate the camera until you see a rainbow on the camera LCD screen. Mount the camera into the post holder and angle the camera until the spectrum appears in the center of the screen and at the bottom half vertically as shown in Figure 5.
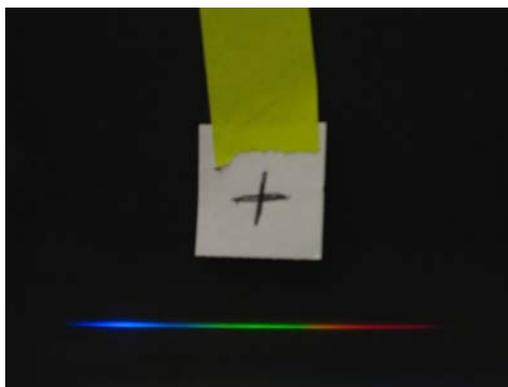

Figure 5: Camera image of an aligned spectrometer.

Since the camera is only imaging a small line, we need to set up an image plane for the camera to focus on. Take the black cardboard and tape a white square to the center of the camera focus. Draw a cross on the square and position it in the center of the camera view as shown in Figure 5. You may choose to zoom in to make the spectrum larger in the image. Align the focus by moving the cardboard back and forth until

imaging the cross results in a focused spectrum. The screen should be roughly at the same distance from the camera as the waveguide output. When everything is set up and aligned, your setup should look similar to Figure 6.
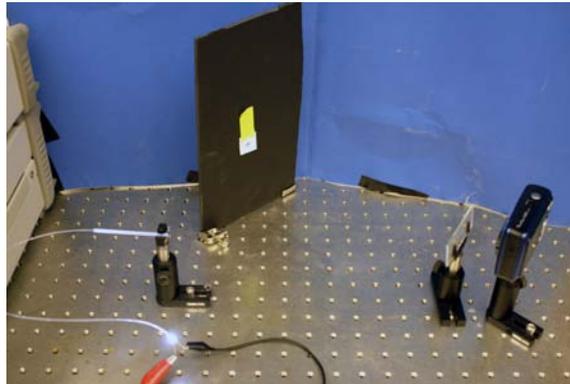

Figure 6: Image of the spectrometer setup on the optical table.

2.3.3 Camera Configuration

Since we want to measure the intensity of the spectrum quantitatively, we need to make sure that the camera is always taking pictures with the same exposure and ISO conditions.

- Mode: To set the options we need, we need to go to program mode on the camera. Press the mode button and select program mode. Hit funtion set (center button) to confirm.

- Auto-Off: We don't want our camera turning off in between measurements. Hit the menu button on the bottom right and go to the settings tab (second tab). Scroll down the list until you get to **Lens Retract**. Set this to **0 sec**. Then go to the item below it, **Power Saving...**, and select it with function set (center button). Change **Auto Power Down** to **OFF** and hit Menu twice to exit.

- ISO: This is essentially the gain of the camera. We want this to be very low to reduce noise so we will set this to 80. Press ISO (top button), select 80, and hit function set to confirm.

- Exposure: This is how we tell the camera how light or dark medium intensity should be. Since our spectrum is most likely going to be brighter than the room lighting, we will set this from 0 to -2. Depending on how well you made your fiber coupled LED, you may not need to go all the way to -2. Press function set and select the first menu item (Exp). Change the number to -2 and hit function set to confirm.

- Spot metering: We want to ensure that the camera determines the light level from the cross and not the spectrum which could be changing intensity. Spot metering tells the camera to only determine the light level from the square in the center of the viewing LCD. Press function set and scroll to the menu button 4th from the top. Spot metering should be the last of the three options. Select it and hit function set. **Note: Even with spot metering enabled, the camera may sometimes choose an exposure that is not consistent. Make sure when you focus (hitting the button half way), the exposure time is consistent. It will be displayed in the lower left of the screen (1/#) along with an aperture size (F#.#). Make sure both of these are the same when you take your images.**

2.3.4   Spectrometer Calibration

Since we are taking pictures of the spectra, we need to calibrate our spectrometer to be able to convert from pixels to wavelength. You should already have your system set up with the spectra of the White LED showing on the camera. Take a picture of this.

Make sure not to move any of the equipment for the rest of the lab, because if you change your alignment, you will have to redo the calibration. Also remember which picture corresponds to what!

Now pull out your LED-coupled fiber and replace it with a regular fiber. Take pictures of the spectra for the green laser input and the red laser input by shining the lasers into the regular fiber. Make sure that the beam is coupled all the way through the tubing and shows up on your camera. You may want to have one member of your group use their hands to block any stray light from the lasers as this will also show up in the picture. These will be your two calibration points for converting pixels to wavelength.

Make a note of the wavelengths of the two lasers.
Green laser pointer λ: _____
Red laser pointer λ: _____

2.3.5   Dye Absorption Measurements

At this point, you should have your 50 mm tube mounted in the V-groove holder. Set up the cuvette holder so that you can place your cuvettes against the output of your fiber. This is shown in Figure 7.
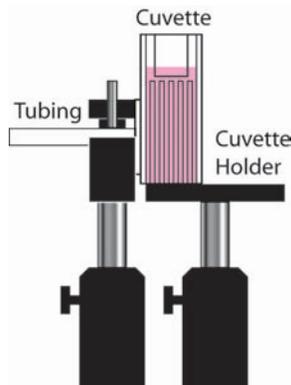

Figure 7: Illustration of the spectrometer configuration for measuring the cuvette.

Go to the TA for a set of 1 cm cuvettes with different dye concentrations. The dye is called "Erythrosin extra bluish" and should look pink. The four vials are numbered with their corresponding concentrations. **Note: The cuvettes are only 1 cm long in one direction. Do not mount your cuvettes the wrong direction.**

- 1: 100 $\frac{\mu Moles}{Liter}$

- 2: 50 $\frac{\mu Moles}{Liter}$

- 3: 10 $\frac{\mu Moles}{Liter}$

- 4: Water

Take pictures of the spectrum for each of the different dye concentrations. Remember that cuvette 4 is just water and will be used to normalize your data.

3 Post-Lab

Here are the items you need to turn in on Friday.

- **(3 Points)** Completed pre-lab questions.

- **(1 Point)** This lab document with all of the data sections filled out.

- **(2 Points)** Plot the Fresnel coefficient measurement from lab section 2.1. Normalize the measured power by dividing by the highest power you measured (power measured for angle greater than the critical angle). Plot the magnitude of the reflectivity as a function of the incidence angle for both TE and TM on the same plot. Remember that $R = |r|^2$. You measured the reflectance R, so you need to take the squareroot to get the reflectivity $|r|$. Also on the same plot, plot the theoretical curves using the Equations 9 and 10. How well does your data match the theory?

- **(1 Point)** Spectrometer Calibration. Plot the spectrum of the white LED, the green laser, and the red laser on the same graph versus wavelength as we did in prelab for the data taken in lab section 2.3.4. Where do you think the loss is coming from? (Hint: You probably saw this experimentally.)

- **(1 Point)** Dye Absorption Measurements. Plot the percent transmission (normalized to that of water) versus wavelength through the 3 different dye concentrations from lab section 2.3.6. What color is absorbed the most? Does the trend you see make sense?

- **(1 Point)** Plot the absorption coefficient versus wavelength for the 3 different dye concentrations. Do you expect the absorption coefficient to change for different concentrations? Does it?

- **(1 Point)** Plot the molar extinction coefficient versus wavelength for the 3 different dye concentrations. Do you expect the molar extinction coefficient to change for different concentrations? Does it?

- **(1 Point Bonus)** Lab participation. You should have been checked off in lab.

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