6.003 (Fall 2011)

Quiz #2 *October 26, 2011*

Name:

Kerberos Username:

Please circle your section number:

Section	Time
2	11 am
3	1 pm
4	2 pm

Grades will be determined by the correctness of your answers (explanations are not required).

Partial credit will be given for **ANSWERS** that demonstrate some but not all of the important conceptual issues.

You have **two hours.**

Please put your initials on all subsequent sheets.

Enter your answers in the boxes.

This quiz is closed book, but you may use two 8.5×11 sheets of paper (four sides total).

No calculators, computers, cell phones, music players, or other aids.

1	/15
2	/15
3	/30
4	/20
5	/20
Total	/100

1. Find the differential equation   *[15 points]*

Determine a linear differential equation with constant coefficients to represent the relation between the input $x(t)$ and output $y(t)$ of the linear, time-invariant system whose impulse response is $h(t) = e^{-|t|}$.
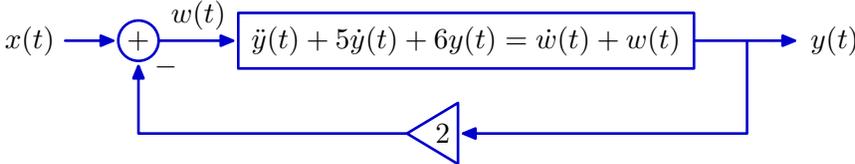
$$x(t) \longrightarrow \boxed{h(t) = e^{-|t|}} \longrightarrow y(t)$$
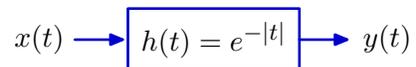
differential equation:

2. **Feedback**   *[15 points]*

A system with input $w(t)$ and output $y(t)$ is represented by the differential equation

$$\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = \dot{w}(t) + w(t) \,.$$

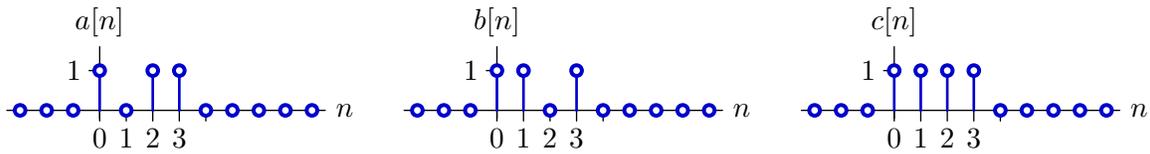This system is placed in a feedback loop as shown below.

Determine a differential equation that relates the input $x(t)$ and output $y(t)$ of the closed loop system. Your answer should **NOT** depend on $w(t)$ or any of its derivatives.

differential equation:

3. Convolutions   *[30 points]*

Consider the convolution of two of the following signals.

Determine if each of the following signals can be constructed by convolving (a or b or c) with (a or b or c). If it can, indicate which signals to convolve. If it cannot, put an X in both boxes.
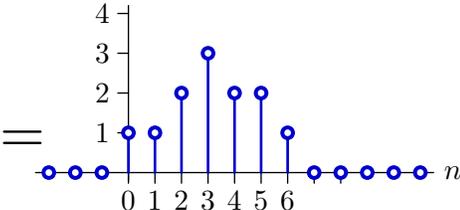
Notice that there are ten possible answers: $(a * a)$, $(a * b)$, $(a * c)$, $(b * a)$, $(b * b)$, $(b * c)$, $(c * a)$, $(c * b)$, $(c * c)$, or (X,X). Notice also that the answer may not be unique.
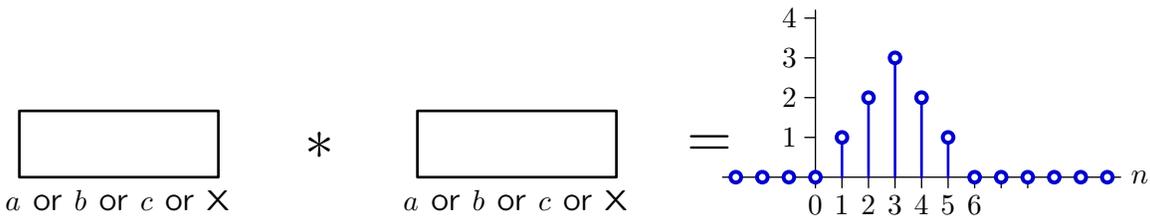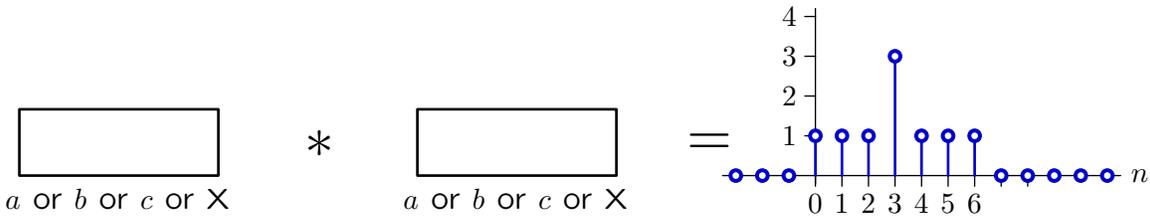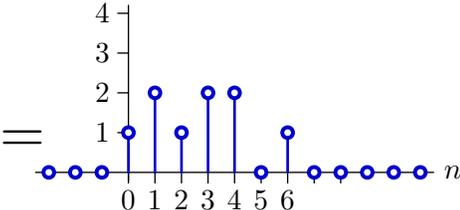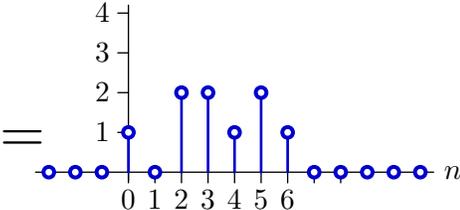
a or b or c or X $*$ a or b or c or X $=$

a or b or c or X $*$ a or b or c or X $=$

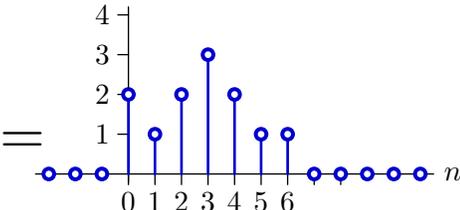a or b or c or X $*$ a or b or c or X $=$
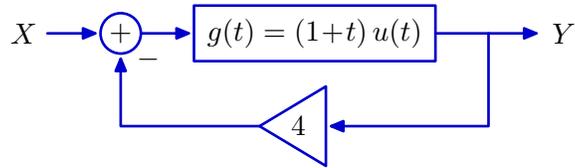

a or b or c or X $*$ a or b or c or X $=$

4. Feedback   *[20 points]*

Let $g(t) = (1+t)\,u(t)$ represent the impulse response of a linear, time-invariant system that is part of the following feedback system. Determine the poles and zeros of the closed-loop system $\frac{Y}{X}$.

Enter the number of poles and zeros and list their approximate values below. If a pole or zero is repeated k times, then enter that value k times. If there are more than 5 poles or zeros, enter just 5 of them. If there are fewer than 5 poles or zeros, write **none** in the remaining boxes.
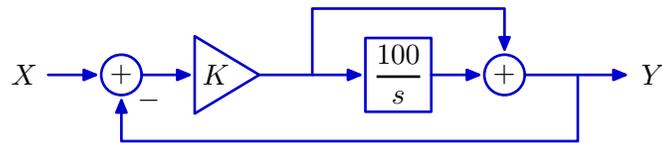
\# of poles:

poles:

\# of zeros:

zeros:

5. Frequency Response of Feedback System   *[20 points]*

Consider the frequency response of the following system, where the input signal is X and the output signal is Y.


Part a. On the axes below, sketch the straight-line approximation (Bode plot) for the magnitude of the frequency response for the case $K = 0.01$.

Show **numerical values** for the magnitudes (dB) and frequencies ω for all of the points of intersection between adjacent straight-line segments.
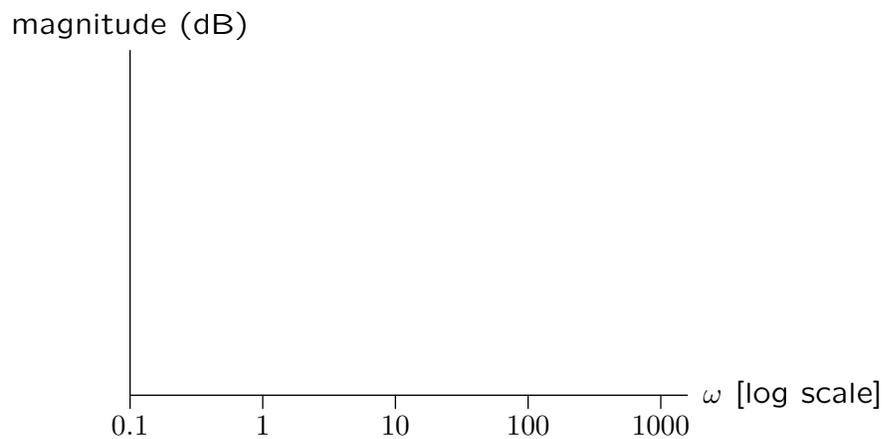
Part b. On the axes below, sketch the straight-line approximation for the angle of the frequency response for the case $K = 0.01$.

Show **numerical values** for the angles (radians) and frequencies ω for all of the points of intersection between adjacent straight-line segments.

angle
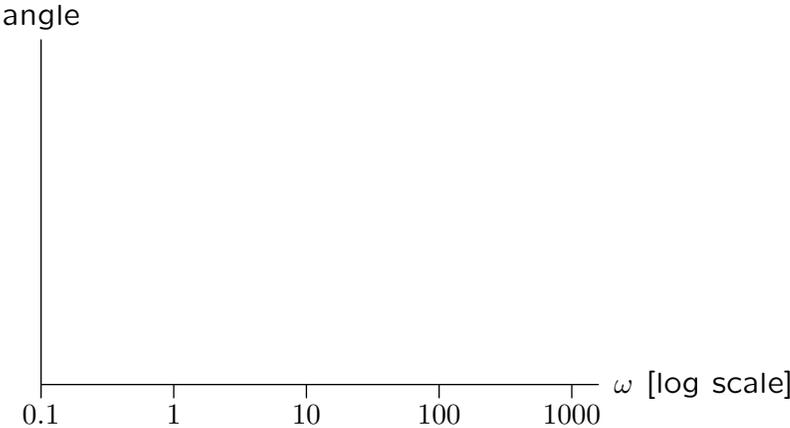
0.1   1   10   100   1000 ω [log scale]

6.003 Signals and Systems
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