Before I begin today, I thought I would take the first five minutes and show you some fun stuff I have been hacking on for the past three years.

This has to do with 6.002 and circuits and all that stuff, but this is completely optional, this is for fun, this is to go build your intuition, this is to check your answers, whatever you want. This is not a required part of the course. Just for fun.

There is this URL out here that I put down here.

I have been hacking on this system for the past three years, and for the first time this year and very tentatively and gingerly introducing it to students.

The idea here is that it is a, that is kind of defocused.

Any chance of focusing that a little bit better?

The idea of this is that it is a Web-based interactive simulation package that I have pulled together.

And what you can do is you can pull up a bunch of circuits.

Notice that the URL is up here. It is euryale.lcs.mit.edu/websim. And there is the pointer to it.

So you have a bunch of fun things you can play with.

And we have gone through all of these things in lecture.

Let's pick the MOSFET amplifier.

You come to this page. This is something you have seen in class. And let's play with this little circuit. And you see the mouse?

Good. You can set up a bunch of parameters. You can set up the MOSFET parameters VT and K. You can set up the value of R for your resistor, you can establish a bias voltage, and you can have an input voltage vIN.

So you can apply a bunch of input voltages.

You can apply a zero input, unit in pulse, unit step, sine wave, square waves.

Or this was the part that took me the longest to get right.

You can also input a bunch of music.

And so far I just have two clips, so you are going to get bored listening to them. Good.

So you can also input music. And what you can do is you can watch the waveforms, you can listen to the output and do a bunch of fun stuff. One experiment I would love for you guys to try out. Again, remember, this is completely optional. Just for fun.

You can apply some input. Step input, for example, to an RLC circuit and spend 30 seconds thinking about what should the output look like. I divine that the output should look like this and then do this and see if what you thought was correct. And it's fun to kind of play around with it. Let me start with, just as an example, let's say I input classical music. And let us say I would like to listen to the output here that is the voltage at the drain terminal of the MOSFET. For listening it sets up a default timeframe to listen to, so you go ahead and do it.

This shows you the time domain waveform of a clip of the music and then you can listen to it. Lot's of distortion, right? As you can see, there is a bunch of distortion. And that is as you expect because the peak-to-peak voltage is 1 volt, the bias is 2.5, and so this is clipping at the lower end, plus the MOSFET is nonlinear. You can play around with a bunch of things and you can have a lot of fun.

And the reason I created this is that MIT is putting a bunch of its courses on the Web. And one of the hottest things about courses like this is the lab component.

If you are beaming a course to, say, a Third World country or something, how do you get people to set up the massive lab infrastructure? I know you hate your oscilloscopes, I know you hate your wires, I know you hate the clips, but the fact is you have them.

I know a lot of places those are way too expensive to pull together, which is why I have been creating this Web-based kind of interactive laboratory so that people can learn this stuff over the Web. Let's go do another example very quickly. Let's say you learned about, well, let's do RC circuits. Here is the parallel RC circuit. And you can set up capacitor values, resistor values, you can set up input.

Here, let me look at the time domain waveform for the voltage across the capacitor. And this time around let me play a unit step. And let's see what the output is going to look like. You can think in your minds what should the output look like, and then you can go and plot it. There you go.

That's what the output looks like.

So you can play around with it and have fun.

That's all the good news. The bad news is that so far I just have one Pentium III machine behind us.

It is a Linux box, so don't all of you try it at once. However, what I have also done, and that took me another six months of hacking in the small amount of time professors have to hack on stuff, I've hacked an incredibly elaborate cashing system so that once anyone in class tries out some combination of parameters it goes and squirrels away all the outputs.

If anybody else types in the same sets of parameters it will just get all the output and play it back to you.

So if enough of you play with over time, we may end up cashing all the important waveforms and music clips and all of that stuff. I have allocated a few gigabytes of storage, so I am hoping that it may work. Go forth.

Play with it. And this is completely my fault, so if there are any bugs or anything simply email them to me. This is the first time this is coming alive so bear with it. Now let me switch back to the scheduled presentation for today.

All right, hope and pray that this works.

Yes. Good.

I am going to do today's lecture using view graphs.

And the reason I am going to do that and not do my usual blackboard presentation which I way, way, way prefer to a view graph presentation. The only reason I am going to do this for today, and maybe one more lecture, is that there is just a huge amount of math grunge in this lecture. What I want to do is kind of blast through that, but you will have it all in the notes that you have, so that you don't waste time in class as you watch me stumbling over twiddles and tildes and all that stuff. The key thing here is that the insight is actually very simple. The beginning and the end are connected very tightly and very simple.

There is a bunch of math grunge in the middle that we are going to work through and, again, follows a complete old established pattern. So, in that sense, there is really nothing dramatically new in there.

Let me spend the next five minutes reviewing for you how we got here, what have we covered so far and set up the presentation. The first ten view graphs I am going to blast through and just tell you where we are in terms of LC and RLC circuits. I began by showing you this little demo, two inverters, one driving.

I can model the inductance here with a little inductor, the capacitor of the gate here. And recall that when I wanted to speed this up by introducing a 50 ohm smaller resistance, I got some really strange behavior.

Just to remind you, for Tuesday's lecture it would help if you quickly reviewed the appendix on complex algebra in the course notes. Remember all the real and imaginary j and omega stuff? It would be good to very quickly skim through that. It is a couple of pages.

Remember this demo? And the relevant circuit that is of interest to us is this one here.

It is the resistor, there is the inductor and there is a capacitor. This is Page 3.

I am just going to blast through the first ten view graphs. It is all old stuff.

Then we observed the following output.

We applied this input at VA and we got this output, a very slowly rising waveform because of the RC transient.

And because of that you saw a delay.

Notice that this delay was because of the slowly rising transient. This waveform took some time to hit the threshold of the neighboring transistor.

So we say ah-ha, let's try to speed this sucker up by reducing the resistance in the collector of the first inverter. And so I had this input.

Now, to my surprise, instead of seeing a nice little much higher and much faster transitioning circuit, well, I did see a much faster transitioning circuit but I got all this strange behavior on the output that I was interested in.

And because of that, if these excursions were low enough, I could actually trigger the output and get a whole bunch of false ones here because of these negative excursions which should not really be there. That was kind of strange.

In the last lecture we said let's take this one step at a time. Let's not jump into an RLC circuit. Let's go step by step.

Let's start with an LC, understand the behavior.

We started off with an LC circuit of this sort, and using the node equation we showed that this was the equation that governed the behavior of the circuit.

And then we said that for a step input and for zero initial conditions, that is the zero state response, let's find out what the output, the voltage across the capacitor looks like. And so we obtained the total solution to be this. And there was a sinusoidal term in there. And the omega nought which was one by square root of LC. And this was the circuit.

And so for this step input notice that the output looked like this. So far an input step I had an output that went like this. Notice that it is indeed possible for the output voltage to actually go above the input value VI. This is kind of non-intuitive but this can happen. So this waveform jumps up and down. But the steady state value, on average if you will, is VI.

On the other hand, it does have sinusoidal excursions and this kind of goes on because there is nothing to dissipate the energy inside that circuit.

By the way, the fact that the capacitor voltage shoots above the input voltage is actually a very important property.

We won't dwell on it in 6.002, but just squirrel that away in your brain somewhere. I promise you that some time in your life you will have to create a little design somewhere that will need a higher voltage than your DC input.

And you can use this primitive fact to actually produce a DC voltage higher than you are given, and then use that somehow. In fact, there is a whole research area of what are called DC to DC converters, voltage converters. Let's say you have 1.5 volt battery, a AA battery, but let's say a circuit needs 1.8 volts. The Pentium IIIs, for example, needed 1.8 volts.

And the strong arm is another chip that required 1.8 volts a few years ago, but the AA cell was 1.5 volts.

How do get 1.8 from 1.5? Well, you have to step it up somehow. And this basic principle where the voltage can jump up above the input is actually used, of course with additional circuitry, to kind of get higher voltages. It is a really key point that you can squirrel away. This was pretty much where we got to in the last lecture. This starts off the material for today. What we are going to do is take that same circuit, but instead we are going to put in this little resistor here. This is what we set out to analyze. And for details you can read the course notes Section 13.6. The green curve here was the behavior of the LC circuit. And what we are going to show today is that the moment we introduce R this sinusoid here gets damp. It kind of loses energy.

And I am going to show you that the behavior is going to look like this. By introducing R this guy doesn't keep oscillating forever.

Rather it begins to oscillate and then kind of loses energy and kind of gets tired and settles down at VI.

And remember the demo. This is exactly what you saw in the demo. You had a step input and you had this funny behavior. And for the RLC that is exactly what it was. So today's lecture will close the loop on what you saw in the demo and the weird behavior, and I am going to show you the mathematics foundations for that today. Let's go ahead and analyze the RLC circuit. I purposely created the entire presentation to follow as closely as possible both the discussion of the RC networks and the LC networks so that the math is all the same. Exactly the same steps in the mathematics are in the exposition of the analysis.

What's different are the results because the circuit is different. So don't get bogged down or whatever in the mathematics. Just remember it is the same set of steps that you are going to be applying.

We start by writing down the element rules for our elements.

Nothing new here. For the inductor V is Ldi/dt.

The integral form which is simply 1/L integral vLdt=i.

We saw this the last time. And for the capacitor, the current through the capacitor is simply Cdv/dt.

Those are the two element rules for the capacitor and inductor.

The element rule for the resistor, of course, is V=iR. You know that.

And for the voltage source we know that, too, the voltage is a constant. Just follow the same established pattern. By the way, just so you are aware, I have booby trapped the presentation a little bit to prevent you from falling asleep. You see the dash lines here?

Whenever you see a dash line, that stuff needs to be copied down. Don't trip over that.

Don't say I didn't warn you. We start by using the usual node method. And I have two nodes in this case. Unlike the LC circuits, I have two unknown nodes. One is this node here with the node voltage vA and the second node is the node with voltage vT. Let me start with vA and write the node equation for that. It is simply 1/L, the node equation for this is the current going in this direction with is vI-vA integral and that equals the current going this way which is vA-v/R, node equation.

I then write the node equation for the node v, for this node here, and that is simply (vA-v)/R=Cdvdt. And that is what I have here, two node equations. Let me summarize the results for you and then show you a view graph where I grind through the math as to how I got the result. Here is the result I am going to get. If I take these two node equations and I massage some of the mathematics, I am going to get this result. And I will show you that in a second. By grinding through some math and solving these two equations and expressing this in terms of v, I get a second order differential equation, d^2v blah, blah, blah.

Notice that this is different from the LC in this term.

Every step of the way you can check to see if I am lying or I am correct. I will indulge you, indulge myself rather with a little story here.

Richard Fineman was a known smart guy.

And one of the reasons that he was that was in the middle of talks he was known to get up and ask some of the darndest, hardest questions and say ah-ha, you have a bug in this proof here or a bug in this equation that is not right.

And usually he would be correct.

So his trick in doing this and which is one reason how he became a known smart guy. What he would do is, as the speaker went on talking he would kind of follow along and think of a simple initial primitive case.

In this case, I have an RLC circuit.

So think of a simpler case of this.

A simpler case of this is R=0. Whenever you set R to be zero, you should get exactly what we got in the last lecture, correct? That is what Fineman would do.

He would boil this down to a simpler case, make some assumptions and just follow along.

And whenever he found a discrepancy between the math here and his simple case he would say oh, there is a bug there. If you want you can catch me that way. Here, what Fineman would do is replace R being zero, and notice then this equation here is exactly what we got the last time with R being zero.

Just remember that Fineman trick.

This is the equation we get, the second-order differential equation with an R term in there.

And let me just grind through the math and show you how I got this from this. So the two node equations again. And what I do is I start by taking these two equations and differentiating this with respect to t and this is what I get.

And, at the same time, I have replaced (vA-v)/R here by this term. I replace this with this and differentiate. Then I simply divide the whole thing by C. Then I take this expression here and write down vA is equal to this stuff here.

Next I am going to substitute this back for vA and eliminate vA. So I take this vA, stick the sucker in here, and thereby eliminate vA and get this. And then I simplify it and here is what I get. That is what I get.

I just grind through the two equations and get that result.

So like a stuck record I will repeat our mantra here, which is here is how we solve the equations that we run across in this course, the same three steps.

Find the particular solution. Find the homogenous solution.

Find the total solution and then find the constants using the initial conditions. Same steps.

You could recite this in your sleep.

And the homogenous solution is obtained using a further four steps. Let's just go through and apply this method to our equation and get the results.

vP is a particular solution and vH is the homogenous solution.

With a particular solution, oh.

Before I go on to do that, let me set up my inputs and my state variables. My input is going to be a step.

Remember, I am trying to take you to the point where the demo left off. The demo had a step input, so I am going to use a step input rising to vI.

And I am going to with the initial conditions being all zeros. So the capacitor voltage is zero, inductor current, another state variable is also zero, and therefore this is also fondly called the ZSR or the zero state response because there is only an input but zero state. Again, remember the dashed lines here. Don't say I didn't warn you.

Let's start with a particular solution.

This is as simple as it gets. I simply write down the particular equation and stick my specific input.

And remember the solution to the particular equation is any old solution, it doesn't have to be a general solution, any old solution that satisfies it.

And I am going to find a simple solution here.

And V particular is a constant VI.

It works. Because remember this has been working all along. And I am going to keep pushing this and see if this works until the end of the course.

Guess what? It will.

So this is a solution. I'm done.

That is my particular solution. Simple.

Second, I go and do my homogenous solution.

And the homogenous equation, remember, is the same old differential equation with the drive set to zero.

Remember that sometimes this equation with the drive set to zero is the entire equation you have to deal with in situations where you have zero input, for example.

Or in other situations in which you have an impulse at the input. And the impulse simply sets up the initial conditions like a charge in the capacitor or something like that. So we are going to blast through this four-step method. The method simply says that four steps, I am going to assume a solution of the form Ae^{st}.

And if you think you've seen that before, yes, you have seen it many times before.

And you will see it again, again and again.

And we need to find A and s. We want to form the characteristic equation, find the roots of the equation and then write down the general solution to the homogenous equation as this. Same old same old.

Let me just walk through the steps here.

Step A, assume a solution to the form Ae^{st}.

And so I substitute Ae^{st} as my tentative solution to the equation. Again, let me remind you that the differential equations that we solve here are really easy because the way you solve them is you begin by assuming you know the solution and stick it in and find out what makes it work. I am going to stick Ae^{st} into this differential equation, and A comes out here.

Differentiate this d squared, I get s squared down here, A s here and this simply gets stuck down here with the 1/LC coefficient. The next step I begin eliminating what I can, so I eliminate the A's, then eliminate the e^{st}'s, and I end up with this equation here. I end up with this equation.

This is my characteristic equation.

It is an equation in s. Do people remember the characteristic equation we got for the LC circuit?

Remember the Fineman trick? That's right, LC. $S^2+1/LC=0$.

This thing wasn't there. All you do is simply follow the R. Just follow the R.

Just imagine this is a dollar sign and kind of follow it.

And you will see what the differences are between the LC and the RLC. So this is the characteristic equation. What I am going to do, iss much as I wrote the characteristic equation for the LC circuit, by substituting omega nought squared for 1/LC.

Let me do the same thing here but introduce something for R and L as well. What I will do is let me give you this canonic form. The very first second-order equation I learned about when I was a kid was this one, S^2+2AS+B^2 or something like that.

Let me write it in that form where I get 2 alpha s plus omega nought squared. Again, remember the alpha comes about because of R. So omega nought squared is 1/LC and alpha is RL/2. Omega nought squared is 1/LC and R/L is equal to two alpha. I am just writing this in a simpler form so that from now on going forward I am just going to deal with alphas and omega noughts.

Once I get to this characteristic equation, after that I can give you one generic way of solving it.

And depending on the kind of circuit you have, a series RLC, which is what we have, or a parallel RLC we will simply get different coefficients for the alpha term. This is going to stay the same but this term will look different, alpha is going to look different. There is a real pattern here.

And what I am doing is simply focusing on what is important, what the differences are between the pattern.

You learned the LC situation and the RLC situation.

Given this I can now write down, I am just simply replacing this as my characteristic equation in dealing with alphas and omegas. I will give you a physical significance of alpha in a little bit.

Do you remember the physical significance of omega nought?

That was the oscillation frequency.

In other words, given an inductor and capacitor, you put some charge on the capacitor and you watch it, it will oscillate. And its oscillation frequency will be one by a square root of LC.

The magnitude of the initial conditions will determine how high are the oscillations or what the phase is in terms of when it starts, but the frequency is going to be the same. Step three, to solve the homogenous equation, is find the roots of the equation, s1 and s2, and here are my roots.

Good old roots for a second-order, little s squared equation here. Finally, given that I have the roots, I can write down the general homogenous solution.

So general solution is simply A1e^s1t, A2e^s2t.

That's it. That's the solution.

This looks big and corny, but we are going to make some simplifications as we go along and show that it ends up boiling down to something cos omega t. The math is kind of involved but we get down to something very simple, a cosine.

Hold this general solution. From that, as a step three of the differential equation solution, I write the total solution down. And my total solution is the sum of the particular and the homogenous, so therefore I get this. VI was my particular and this term here is my homogenous solution.

Now, if I wasn't doing circuits and simply trying to solve this mathematically here is what I would do.

I would find the unknown from the initial conditions, so I know that v(0)=0. And so therefore if I substitute zero for V(0) I get this.

If I substitute zero here, t is 0, t is 0, so I simply get V1+A1+A2. And let me just blast through because I am going to redo this differently.

i=Cdv/dt. And so that's what I get.

I substitute zero and this is what I would get.

I hurried through this. Don't worry.

I'm going to do it again. If you just do it mathematically, you can solve this equation here and these two simultaneous equations in a1 and a2 and get the coefficients and you are done.

But it doesn't give us a whole lot of insight into the behavior of these terms here. What I am going to do for now is kind of ignore that. Ignore I did that and instead try to go down a path that is a little bit more intuitive.

Let's stare at this expression we got for the total solution.

That is the expression we got. All I did is, I had alpha in there, I simply pulled out the alpha outside. So this is my total solution, V1-A1e^(-alpha t) something else and something else.

Three cases to consider depending on the relative values of alpha and omega nought. If alpha is greater than omega nought then I get a real quantity here.

The square root of a positive number, I get a real number, and that number will add up to the minus alpha and I am going to get a solution that will look like, oh, I'm sorry.

Let me just do it a little differently.

There are three situations here.

One is alpha greater than omega nought.

Alpha equal to omega nought. Alpha less than omega nought.

Alpha is greater, alpha is less, alpha is equal to this term inside the square root sign.

For reasons you will understand shortly, we call this "overdamped" case, the "underdamped" case and the "critically damped" case. When alpha is greater than omega nought this term gives me a real number, and I get something as simple as this.

Remember, for the series RLC circuit, alpha was R/2L.

So if R is big, in other words, if in my RLC circuit R is huge then I am going to get this situation. My output voltage on the capacitor is going to look like this, the sum of two exponentials. And if I were to plot it very quickly for you, for a VI step, V would look like this. So v would simply look like this because it is the sum of a couple of exponentials.

All right. Now, alpha is positive here.

Remember alpha1 and alpha2 are both positive.

These two added up, because of this constant VI, give rise to something that increases in the following manner. Let's look at the situation where alpha is less than omega nought, where the term inside the square root sign is negative.

What I can do is pull the negative sign out and express it this way. What I am going to do is since alpha is less than omega nought, I am going to reverse these two and pull out square root of minus one to the outside.

This is what I get. I am just playing around with this so that whatever is under the square root sign ends up giving me a positive real number.

So I pull the j outside and this is what I get.

Now, let me blast through a bunch of math and end up with something very, very simple for this underdamped case. Let me define a few other terms. I am going to call omega nought minus alpha squared the square root of that.

I am going to call it omega d. And here is what I get.

So I have defined three things for you now, alpha, omega nought and omega d. And I get this equation in terms of alpha and omega d. And then, remember from your good-old Euler relationship? e to the j omega d is simply cosine plus a j sine. I am just going to blast through a bunch of math rather quickly.

Once I replace this in terms of a cosine and sine, cosine and a j sine and then collect all the coefficients together, I get an equation of the form VI plus some constant e to the minus alpha t, cosine, the sum of the constant e to the minus alpha t, sine.

Remember the sines and cosines are coming out, but because of my R I am getting this funny alpha here, e to the minus alpha here. So I am getting sums of sine and cosine. And K1 and K2 are some constants which I will need to determine for my initial conditions. I am going to continue on with this and keep on simplifying it because, as I promised you, I want to get to something that is just a cosine.

I want to go down this path. I am not going to cover this case, the critically damped case.

And I will touch upon it later but not dwell on it.

Let me continue down the path of the underdamped case, and this is what we have. Continuing with the math, let's start with the initial conditions, v nought equals zero, and that gives me K1 is simply -VI.

So at v(0)=0 t is zero, so this terms goes away, the cosine becomes a 1, e^(alpha t) goes away, and K1=-VI. Then I know that i(0) and i is simply Cdv/dt. And I get this nasty expression. I substitute t=0 and I get something that looks like this. I know what K1 is, and so therefore K2 is simply -V1alpha divided by omega nought. I have taken this expression where the unknowns K1 and K2 are to be found.

I set the initial conditions down at t=0 and I get K1 and K2 as follows, which gives me the following solution.

This is the solution I get where I do not have any unknowns anymore. Remember that omega d and alpha are directly related to circuit parameters.

Alpha was R/2L and omega d was square root of alpha squared minus omega nought squared. ** omega d = sqrt(alpha^2 - omega_0^2) ** And omega nought squared was 1 by square root of LC. So I know it all now.

I still have sines and cosines here, so I am going to simplify this a little further. Oh, before I go on to do that, let's do the Fineman trick again and notice if I am still true to the LC circuit I did the last time.

Remember when R goes to zero alpha goes to zero.

Because alpha is R divided by 2L.

If alpha was zero what happens? If alpha was zero, this guy goes to one, this whole term goes to zero and omega dt now ends up becoming omega nought, and I get this term here. I get VI-VIcosine(omega t), which is exactly what I expected in my equation.

This is the same as the LC case that I got.

Let's go back to this situation and simply if further.

If you look at Appendix B.7 in your course notes, Appendix B.7 is a quick tutorial on trig.

And in that trig tutorial you will see that, and you have probably seen this before, too, multiple times, the scaled sum of sines are also sines.

This is an incredibly cool fact of sinusoids.

If you take two sinusoids of the same frequency and you scale them up in any which way and add them up you also end up with a sinusoid. It is hard to believe but it is true. It is an incredible property of sinusoids. Take any two sinusoids, scale them in any way you like, same frequency, add them up, you will get a sinusoid.

What that is saying is that, look, here is a sinusoid, here is a sinusoidal function, and I am scaling them up in some manner. So I should be able to add them up and be able to express that as single sine.

And to be sure you can, look at the Appendix, and there is an expression for a1 sinX plus a2 cosX is equal to a cosine of blah, blah, blah.

This is what you get. No magic here.

Just math. From here I directly get this.

And look at what I have. It is absolutely unbelievable.

v(t) is simply VI, there is a constant here, this an e to the minus alpha term and there is a cosine.

Again, to pull the Fineman trick, if this alpha were to go to zero here then you would end up with the expression you had for the LC situation. Let's stare at this a little while longer. There is a constant plus a minus, a cosine term, so there is a sinusoid at the output, and there is an e to the minus alpha which ends up giving you the decay you have seen before.

In other words, to a step input, the LC circuit would give you a sinusoid.

That is what the LC circuit would do if alpha was zero.

But because of this alpha term here, e to the minus alpha t, that gives rise to a damping effect, so this causes this thing to become smaller and smaller as time goes by until this term goes to zero at t equals infinity.

This guy damps down and so therefore you end up getting the curve that you saw like this. Twenty minutes of juggling math solving a second-order differential equation, but what ends up is the same sinusoid but it is damped in the following manner such that the frequency, rather the amplitude keeps decaying until it starts off at zero and then settles down at vI. This is exactly what you saw in the demo that we showed you earlier.

The critically damped case, I am not going to do it here.

I am going to point you to the following insight.

The underdamped case looked like this.

It was a sinusoid that kind of decayed out.

That is the underdamped case. And then I showed you the overdamped case. The overdamped case looked like this. And, as you might expect, the critically damped case is kind of in the middle and looks like this. So the overdamped case would look like this, underdamped like this, and the critically damped case kind of goes up and kind of settles down almost immediately. This is when alpha equals omega nought. I won't do that case here, but I will simply point you to Section 13.2.3.

Just to tie things together, recall this demo here that we showed you in class yesterday. This is exactly the kind of form of the sinusoid you saw because of that input step.

If you want to see a complete analysis of inverter pairs and look at the delays and so on because of that, you can look at Page 170 and example 898.

In the next five or six minutes, what I would like to do is stare at the RLC circuit. And much like I showed you some intuitive methods to get the RC response, what we are going to do is do the same thing for the RLC.

In the RLC situation, much like the RC situation, experts don't go around writing 15 pages of differential equations and solving them each time they see an RLC circuit.

They stare at it and boom, the response pops out, the sketch pops out. This one is going to be another one like our Bend it Like Beckham series here.

And this one is in honor of Leslie Kolodziejski.

And I call it "Konquer it like Kolodziejski".

Again, as I said, experts don't go around solving long differential equations and spending ten pages of notes trying to get a sinusoid. They look at a circuit and sketch response. I am going to show you how to do that, too. And what you can do is, to practice, go to Websim and try out various combinations of inputs and initial conditions and sketch it, time yourself, give yourself 30 seconds or a minute if you like, and sketch it and check it against the Websim response. If it doesn't match either you are wrong or there is a bug in Websim.

What I am going to do is, the response to the critically damped and underdamped case was very easy to sketch out.

You started with an initial condition, you settled at VI and just kind of drew it like that. The interesting case is the underdamped case, and that is what I am going to dwell on. Before we go on and I show you the intuitive method, as a first step I would like to build some intuition. Let's stare at this response here and try to understand what is going on.

This is the response that we saw.

And this fact that you see an oscillation happening is also called "ringing". You say that your circuit is ringing. All right.

You see some interesting facts. You see that frequency of the ringing is given by omega d. This cosine omega d, so that is the frequency omega d.

So the time is 2 pi divided by omega d.

The oscillation frequency is omega d, but omega d is simply omega nought squared minus alpha squared.

Once you have a big value of R alpha becomes very small and omega d is very commonly equal to, very close to omega nought.

So omega d and omega nought very commonly are very close together. And when that happens this frequency is directly omega nought.

Alpha governs how quickly your sinusoid decays.

e to the alpha t here is the envelope that governs how quickly my sinusoid decays. And notice that each of these terms, alpha and omega nought, comes directly from my characteristic equation. Which means that once you get your characteristic equation you really don't have to do much else. And up until now you still have to write the differential equation to get the characteristic equation, so you still have to do some differential equation stuff, but in two lectures I am going to show you a way that you can even write down the characteristic equation by inspection.

Look at your circuit and boom, in 15 seconds or less write down the characteristic equation.

It is absolutely unbelievable. What are the other factors that are interesting here? Of course I need to find out initial values. I start off at zero.

This is my capacitor voltage. If I don't have an infinite spike or an impulse my capacitor voltage tries to stay where it is and starts off at zero. And the final value is given by VI, the capacitor is a long-term open so therefore VI appears across the capacitor. In the long-term my final value is going to be VI. There is one other interesting parameter, which I will simply define today but dwell on about a week from today, and that is called the Q.

Some of you may have heard the term oh, that's a high Q circuit. Q is an indication of how ringy the circuit is. And Q is defined as omega nought by 2 alpha. It is called the "quality factor". And it turns out that Q is approximately the number of cycles of ringing.

So if you have a high Q you ring for a long time and if you have a low Q you ring for a very short time.

That is called the quality factor defined by omega nought by 2 alpha. Notice that Q, omega nought, alpha, omega d, all of these come from the terms in the characteristic equation. We will spend more time on Q later. With this insight here is how I can go about very quickly sketching out the form of the response. Here is my circuit.

I want to sketch the form of the response for a step input at vI. Zero to vI step input here, I want to find out what happens at this point.

This is described to you in a lot more detail in Section 13.8 in your course notes. Let's go through the steps.

Let's do the really simple situation first.

Let's also assume for fun that you are given that v(0) starts out being some positive value. Some v(0) which is a positive number. And, to make it harder on ourselves, let's say i(0) starts out being some negative number.

So i(0) is some negative current.

The first thing I know is v(0), the capacitor voltage starts out here, which can change suddenly.

And I also know that in the long-term this is an open circuit. So that this voltage vI will appear directly across the capacitor in the long-term.

So I get starting out at v(0), ending at vI, I am also half the way there. I know the initial and ending point of the curve. And then I know that somewhere in here there must be some funny gyrations here, because remember I am dealing with the underdamped case.

And you can determine that from alpha and omega nought.

If alpha is less than omega nought, you know that you are in the underdamped case and this is what you get.

Let's compute and write the characteristic equation down.

A week from today you will write it by inspection, but for now you will do it by writing down a differential equation. And from the characteristic equation you will get omega d, you will get alpha, omega nought and Q. So omega d gives you the frequency of oscillations. My frequency of oscillation is now known. From Q I know how long it rings, because I know it rings for about Q cycles.

I know that ringing stops approximately here.

And then I know that between that the start and end point my curve kind of looks like this, something like this.

Right there we are 95% of the way there.

The only question is I do not know if it goes like this or it goes like this. I am not quite sure yet if it starts off going high or starts off going low.

Not quite clear. I also do not know what the maximum amplitude is. It turns out this is rather complicated to determine so we won't deal with that.

Just simply so you can draw a rough sketch.

The questions is which way does it start?

I could leave it for you to think about.

Yeah, let me do that. It is given on this page so don't look at it. Think about it, and think about how you can determine whether it goes up or down. It turns out that in this case it is going to down and then ring.

See if you can figure it out for yourselves and then we will talk about it next week.