6.002 CIRCUITS AND ELECTRONICS

Small Signal Circuits

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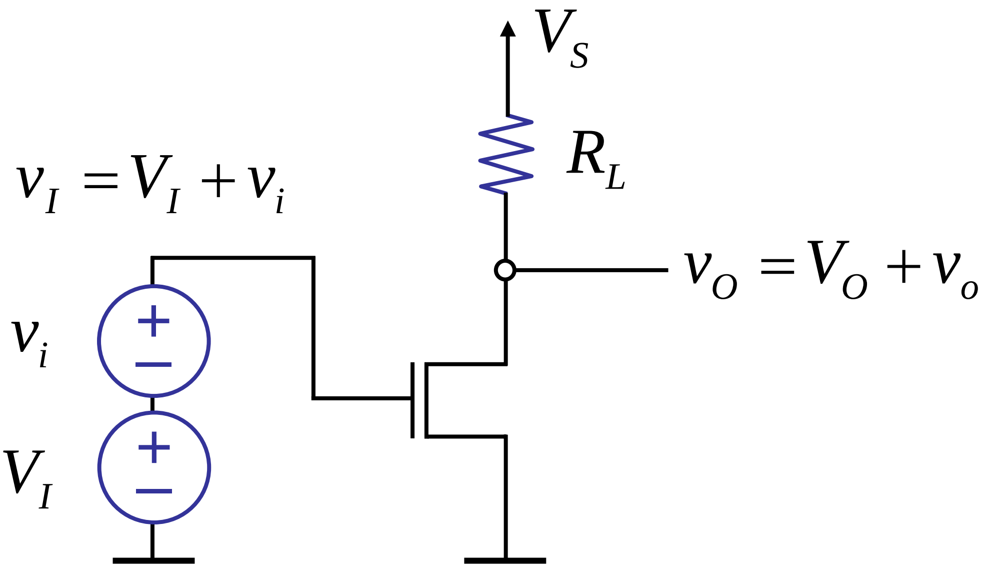
Review:

- Small signal notation

$$\underset{\substack{\uparrow \\ \text{total}}}{v_A} = \underset{\substack{\uparrow \\ \text{operating} \\ \text{point}}}{V_A} + \underset{\substack{\uparrow \\ \text{small} \\ \text{signal}}}{v_a}$$

- $$v_{OUT} = f(v_I)$$
  $$v_{out} = \left.\frac{d}{dv_I} f(v_I)\right|_{v_I = V_I} \cdot v_i$$

-



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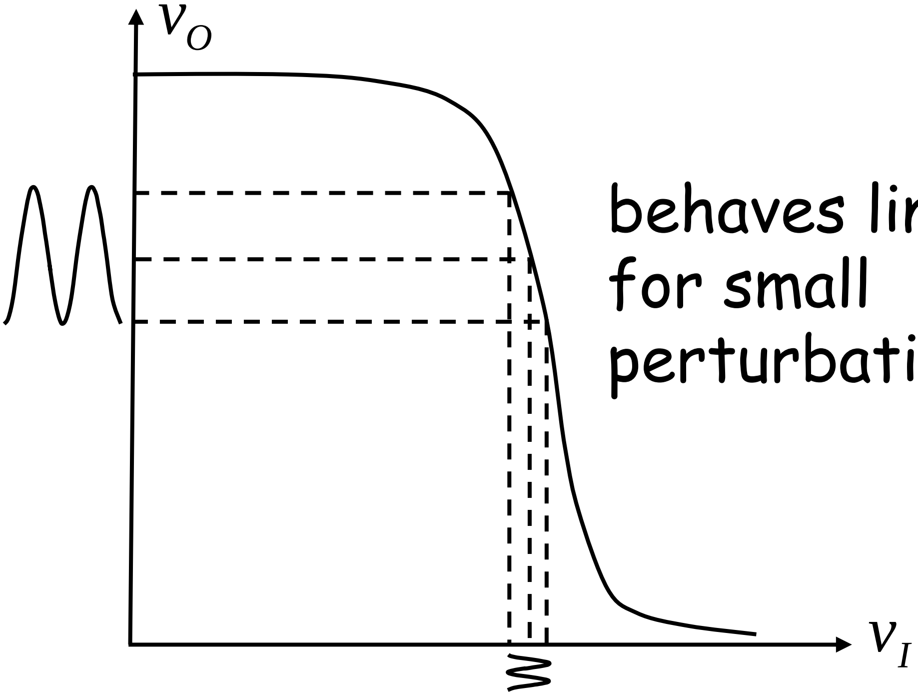
Review:

I Graphical view (using transfer function)



behaves linear for small perturbations

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Review:

II Mathematical view

$$v_O = V_S - \frac{K(v_I - V_T)^2}{2} R_L$$

$$v_o = \left. \frac{d}{dv_I} \frac{\left[V_S - \frac{K}{2} (v_I - V_T)^2 R_L \right]}{} \right|_{v_I = V_I} \cdot v_i$$

$$v_o = -\underbrace{K(V_I - V_T)}_{g_m} R_L \cdot v_i$$

$\rightarrow$ related to V_I

constant for fixed DC bias

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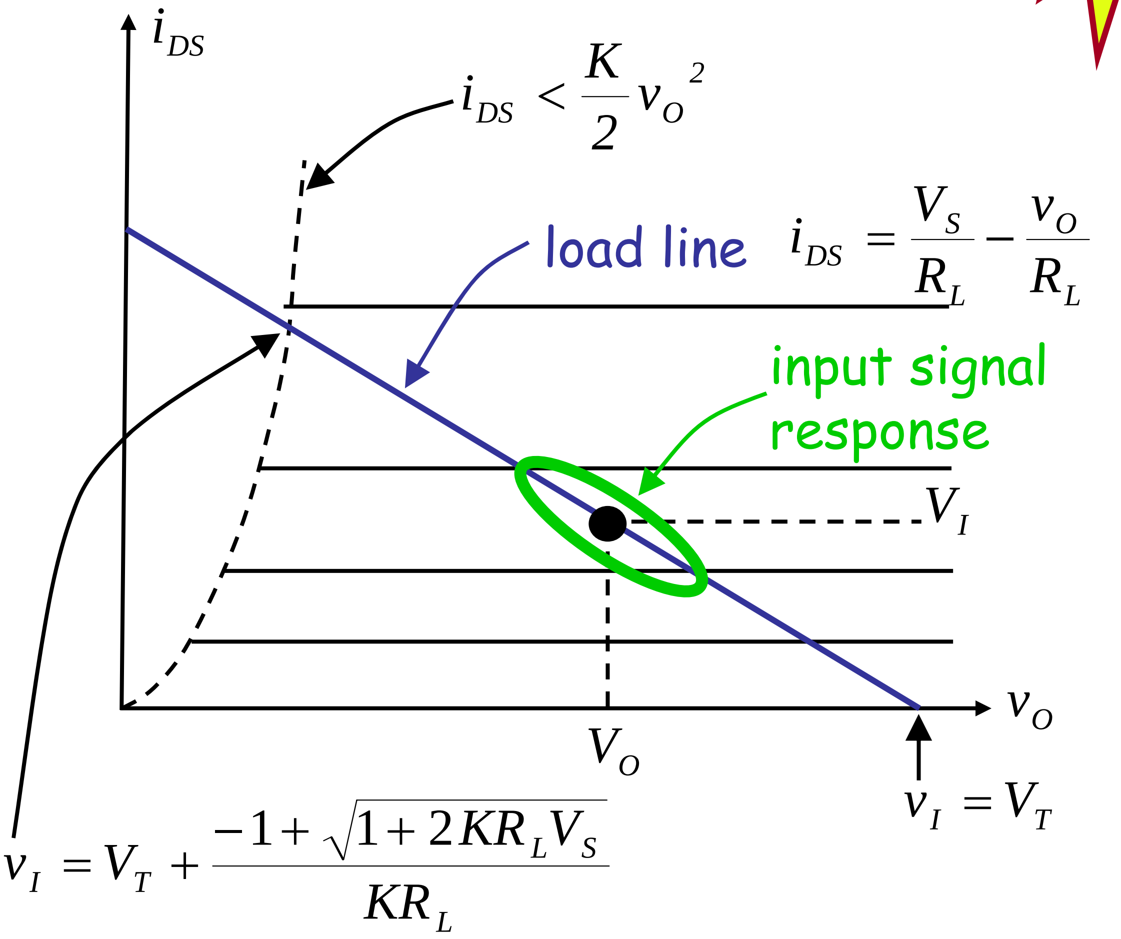
How to choose the bias point, using yet another graphical view based on the load line

Demo



$i_{DS} < \frac{K}{2} v_O{}^2$

$i_{DS} = \frac{V_S}{R_L} - \frac{v_O}{R_L}$

$v_I = V_T$

$v_I = V_T + \frac{-1 + \sqrt{1 + 2KR_L V_S}}{KR_L}$

Choosing a bias point:

1. Gain $g_m R_L \propto V_I$
2. Input valid operating range for amp.
3. Bias to select gain and input swing.

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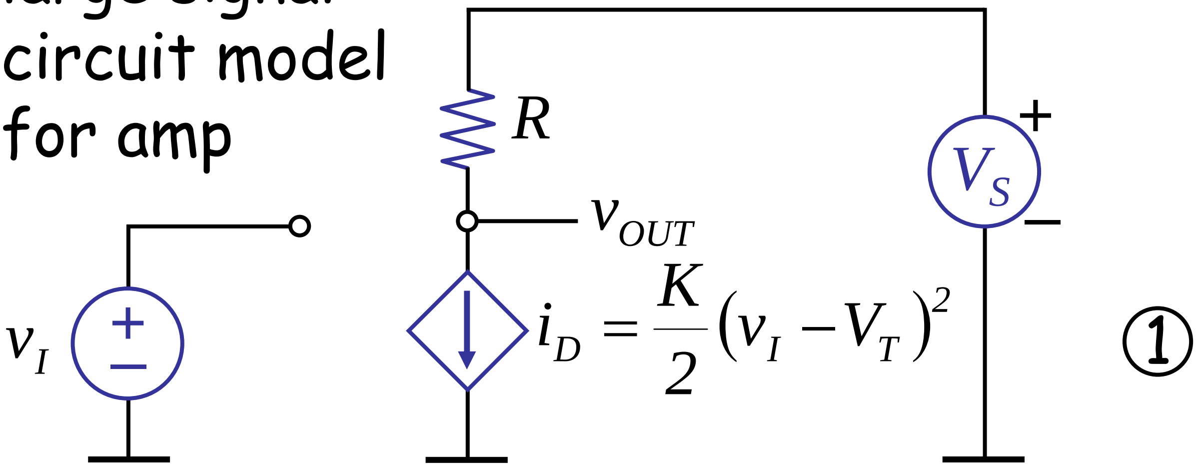
III The Small Signal Circuit View

We can derive small circuit equivalent models for our devices, and thereby conduct small signal analysis directly on circuits

e.g. large signal circuit model for amp



We can replace large signal models with small signal circuit models.

Foundations: Section 8.2.1 and also in the last slide in this lecture.

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Small Signal Circuit Analysis

① Find operating point using DC bias inputs using large signal model.

② Develop small signal (linearized) models for elements.

③ Replace original elements with small signal models.

Analyze resulting linearized circuit…

Key: Can use superposition and other linear circuit tools with linearized circuit!

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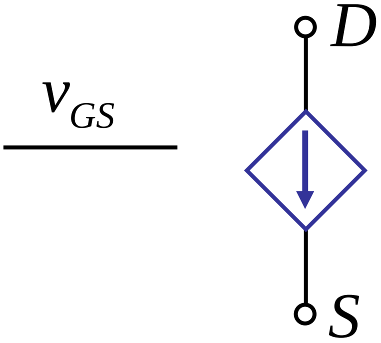
Small Signal Models

Ⓐ MOSFET

large
signal



$$i_{DS} = \frac{K}{2}\left(v_{GS} - V_T\right)^2$$

Small signal?

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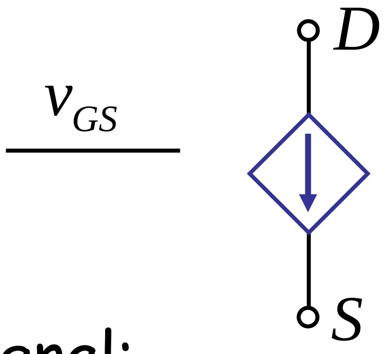
Small Signal Models

Ⓐ MOSFET

large signal



$$i_{DS} = \frac{K}{2}\left(v_{GS} - V_T\right)^2$$

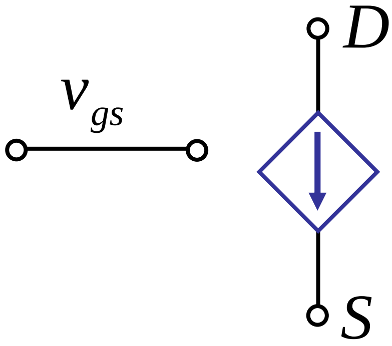
Small signal:

$$i_{DS} = \frac{K}{2}\left(v_{GS} - V_T\right)^2$$

$$i_{ds} = \frac{\partial}{\partial v_{GS}}\left[\frac{K}{2}\left(v_{GS} - V_T\right)^2\right]\Bigg|_{v_{GS}=V_{GS}} \cdot v_{gs}$$

$$i_{ds} = \underbrace{K\left(V_{GS} - V_T\right)}_{g_m} \cdot v_{gs} \quad \Rightarrow \quad i_{ds} \text{ is linear in } v_{gs}\,!$$

small signal



$$i_{ds} = K\left(V_{GS} - V_T\right)v_{gs}$$

$$i_{ds} = g_m v_{gs}$$

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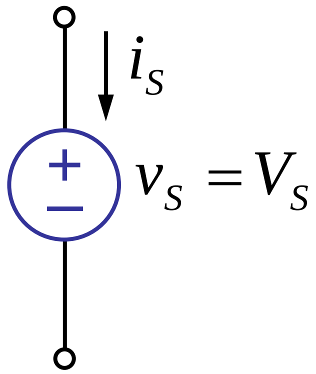
Ⓑ **DC Supply** V_S

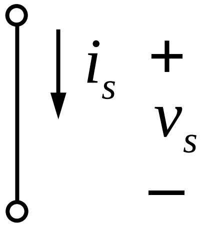
large signal



$$v_S = V_S$$

Small signal

$$v_s = \left.\frac{\partial V_S}{\partial i_S}\right|_{i_S = I_S} \cdot i_s$$



$$v_s = 0$$

DC source behaves as short to small signals.

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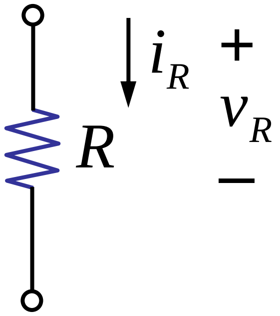
Ⓒ Similarly, R

large
signal

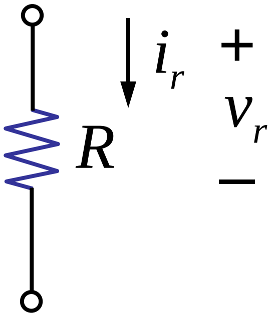


$$v_R = R\,i_R$$

$$v_r = \left.\frac{\partial\left(Ri_R\right)}{\partial i_R}\right|_{i_R=I_R} \cdot i_r$$

$$v_r = R \cdot i_r$$

small
signal



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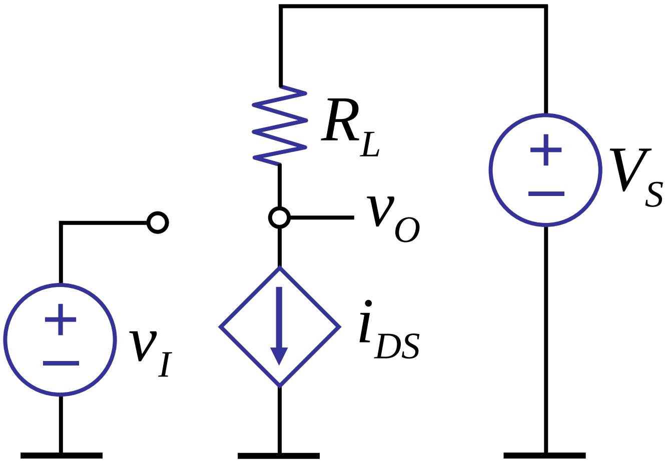
Amplifier example:

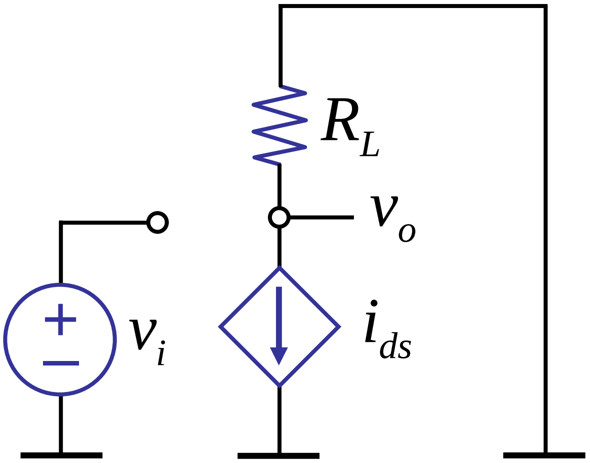
Large signal



$$i_{DS} = \frac{K}{2}\left(v_I - V_T\right)^2$$

$$v_O = V_S - \frac{K}{2}\left(v_I - V_T\right)^2 R_L$$

Small signal



$$i_{ds} = K\left(V_I - V_T\right)\cdot v_i$$

$$i_{ds}R_L + v_o = 0$$

$$v_o = -i_{ds}R_L$$

$$v_o = -K\left(V_I - V_T\right)R_L \cdot v_i$$

$$= -g_m R_L \cdot v_i$$

Notice, first we need to find operating point voltages/currents.

Get these from a large signal analysis.

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III The Small Signal Circuit View

To find the relationship between the small signal parameters of a circuit, we can replace large signal device models with corresponding small signal device models, and then analyze the resulting small signal circuit.

Foundations: (Also see section 8.2.1 of A&L)

KVL, KCL applied to some circuit C yields:

$$\cdots + v_A + \cdots + v_{OUT} + \cdots + v_B + \cdots$$ ①

Replace total variables with
operating point variables plus small signal variables

$$\cdots + \cancel{V_A} + v_a \cdots + \cancel{V_{OUT}} + v_{out} \quad + \cancel{V_B} + v_b + \cdots$$

Operating point variables themselves satisfy the same KVL, KCL equations

$$\cdots + V_A \qquad \cdots + V_{OUT} \qquad + V_B \qquad + \cdots$$

so, we can cancel them out

Leaving

$$\cdots \qquad + v_a \cdots \qquad + v_{out} \qquad + v_b + \cdots$$ ②

But ② is the same equation as ① with small signal variables replacing total variables, so ② must reflect same topology as in C, except that small signal models are used.

Since small signal models are linear, our linear tools will now apply...

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