# Labor and Development

March 4, 2013

# Part I

# Sharing Wage Risk

# Introduction

- Three innovations to the standard risk sharing model are introduced in this paper:
  1. Labor supply is explicitly recognized and modeled as an endogenous variable that responds to exogenous shocks
  2. Explicitly consider non labor income and income from wages
  3. Allow for heterogeneity within and between households

# Example: Setting

- Two agent household, two commodities (consumption, leisure)
- Utility functions for each agent

$$U(C^1) = C^1, \; U(C^2, L^2) = \frac{\left(C^2 L^2\right)^{1-\gamma}}{1-\gamma}$$

- Assume that $\gamma \geq \frac{1}{2}$
- Let $w_2$ be the real wage, $y$ the non labor income and $T$ time.
- The consumption good is the numeraire with price normalized to 1.
- The budget constraint is given by

$$C^2 + w_2 L^2 = w_2 T + y$$

- Agent 1 is risk neutral. **Will he bear all risk in a PO allocation? Not necessarily.** We will show that agent 2 will not face non labor income risk, but will face labor income risk, in an Ex Ante PO allocation.

# Example: Risk Sharing Implications I

- Think of these two agents as a Small Open Economy. Decentralization process in two steps:
  - **Step 1: Ex post efficiency**, for every state of nature, split non labor income, give $\rho$ to household 1. From the 2nd Welfare theorem, any function $\rho$ (that will depend on $(w_2, y)$) generates an **expost efficient** sharing rule. Given this sharing rule, we find consumption and leisure for agent 2.
  - **Step 2: Ex ante efficiency**, the ratio of marginal utilities is equal to the ratio of Pareto weights. We will also see that, ex ante efficiency restricts the sharing rule.
- For Step 1, given the sharing rule, we solve agent 2 optimization problem and we get

$$L^2 = \frac{\rho + w_2 T}{2w_2}, C^2 = \frac{\rho + w_2 T}{2}$$

- The indirect utility function is then

$$V^2(\rho, w_2) = \frac{\left(\frac{\rho + w_2 T}{2} \frac{\rho + w_2 T}{2w_2}\right)^{1-\gamma}}{1-\gamma} = \frac{2^{-(1-\gamma)}}{(1-\gamma)} (\rho + w_2 T)^{2-2\gamma} w_2^{-(1-\gamma)}$$

# Example: Risk Sharing Implications II

- For Step 2, ex ante efficient risk sharing implies that the ratio of marginal utilities of income is constant for every state of the world ($K$ is a constant that depends on the Pareto Weights)

$$\frac{V^2_\rho(\rho, w_2)}{V^1_\rho(\rho, w_2)} = 2^\gamma (\rho + w_2 T)^{1-2\gamma} w_2^{-(1-\gamma)} = K$$

- This, constrains the sharing rule to be

$$\rho = K' w_2^{\frac{1-\gamma}{1-2\gamma}} - w_2 T$$

and then consumption and leisure are given by

$$C^2 = K' w_2^{-\frac{1-\gamma}{2\gamma-1}}, L^2 = K' w_2^{-\frac{\gamma}{2\gamma-1}}$$

$$V^2(w_2) = K'' w_2^{-\frac{1-\gamma}{2\gamma-1}}$$

$$C^1 = w_2 T - 2K' w_2^{-\frac{1-\gamma}{2\gamma-1}} + y$$

# Example: Risk Sharing Implications III

- The risk averse consumer has no non labor income risk $y$
- But, faces wage risk, that is shared with the risk neutral consumer. Labor supply $L^2$ and consumption $C^2$ respond to wages even when there is a risk neutral agent
- Utility also fluctuates. More generally

$$v^2 = v^2(w_2, \rho(w_2, y))$$

$$v^1 = v^1(\rho(w_2, y))$$

# General Framework: Setting

- $S$ states of the world
- Risk sharing group consisting of $H$ households.
- Household $h$ has $I_h$ individuals. Household consumption is $C^h = \sum_{i=1}^{I_h} C^{i,h}$
- Aggregate consumption is $C = \sum_h C^h = \sum_h \sum_{i=1}^{I_h} C^{i,h}$
- Preferences $U^{i,h}(C^{i,h}, L^{i,h})$, strictly increasing and concave
- Each household faces a vector of wages, non labor income and transfers for each state $s$

$$(\mathbf{w}_s^h, \mathbf{y}_s^h) = (w_s^{1,h}, ..., w_s^{I_h,h}, y_s^{1,h}, ..., y_s^{I_h,h}, \tau_s^h)$$

- Define $Y_s^h \equiv y_s^{1,h} + .... + y_s^{I_h,h}$ as the sum of non labor income and $X_s^h \equiv Y_s^h + \tau_s^h$ as the sum of non labor income and transfer

# Household Level Efficiency

- Take as given transfers from the village $\tau_s^h$. Break the problem into ex post efficiency and ex ante efficiency.
- The **ex ante** Pareto Problem of the household

$$\max_{\{C_s^{i,h}, L_s^{i,h}\}_{s\in S, i=1\ldots I_h}} \sum_{i=1}^{I_h} \mu^{i,h} \sum_s \pi_s U^{i,h}(C_s^{i,h}, L_s^{i,h})$$

where $\sum_{i=1}^{I_h} \mu^{i,h} = 1$ and subject to (for all $s \in S$)

$$\sum_{i=1}^{I_h} C_s^{i,h} + \sum_{i=1}^{I_h} w_s^{i,h} L_s^{i,h} = \sum_{i=1}^{I_h} w_s^{i,h} T^i + y_s^{1,h} + \ldots. + y_s^{I_h,h} + \tau_s^h$$

- Pareto weights do not depend on the realization of wages. But, they can depend on the distribution of wages, since they are determined when the household signs the risk sharing contract.

# Notation

- Let $H^{i,h}$ be the Marshallian Demand for leisure (a function $H^{i,h}(w_s^{i,h}, \rho_s^{i,h})$) that solves the individuals $i$ (in household $h$) program (HP).
- Let $v^{i,h}$ be the resulting indirect utility function

$$v^{i,h}(w_s^{i,h}, \rho_s^{i,h}) \equiv \max_{L^i, C^i} U^i(L_s^{i,h}, C_s^{i,h})$$

$$C_s^{i,h} + w_s^{i,h} L_s^{i,h} = w_s^{i,h} T^i + \rho_s^{i,h}$$

- Both functions depend only on $i$'s preferences, while one of the arguments, $\rho_s^{i,h}$ depends on the decision (or bargaining) process that occurs in the household.

# Risk Sharing Group Level

- At the group level, the Pareto problem is to find the state contingent transfers that maximizes the weighted sum of utilities
- The utility of the household is then given by

$$\omega^h(w_s^h, X_h^s) \equiv \sum_i \mu^{i,h} v^{i,h}(w_s^{i,h}, \rho^{i,h}(w_s^h, X_s^h))$$
$$= \sum_i \mu^{i,h} V^{i,h}(w, X)$$

- The Pareto Problem is given by

$$\max_{\{\tau_s^h\}_{s\in S, h\in H}} \sum_h M^h \sum \pi_s \omega^h(\mathbf{w}_s^h, Z_s^h + \tau_s^h)$$

subject to the resource constraint (for all s)

$$\sum_h \tau_s^h = 0$$

- From the FOC's we can show that (for future reference)

$$\pi_s \frac{\delta \omega^h}{\delta \tau_s^h} = \lambda_s^h \qquad (1)$$

# Risk Sharing: Parametric Example I

- Use the same setting with Cobb Douglas utility functions

$$U^{i,h}(C^{i,h}, L^{i,h}) = \frac{\left(\left(C^{i,h}\right)^{\alpha_{i,h}}\left(L^{i,h}\right)^{1-\alpha_{i,h}}\right)^{1-\gamma_{i,h}}}{1-\gamma_{i,h}}$$

- From the (HP) leisure is given by (allows for corner solution)

$$L_s^{i,h} = \min\left(T, \alpha_{i,h}\frac{Tw_s^{i,h} + \rho_s^{i,h}}{w_s^{i,h}}\right) \tag{2}$$

- The Value functions for work and no work are

$$V_W^{i,h} = [\alpha_{i,h}^{\alpha_{i,h}}(1-\alpha_{i,h})^{1-\alpha_{i,h}}(w_s^{i,h})^{-\alpha_{i,h}}] \times \frac{\left(Tw_s^{i,h} + \rho_s^{i,h}\right)^{1-\gamma_{i,h}}}{1-\gamma_{i,h}} \tag{3}$$

$$V_{NW}^{i,h} = \frac{\left(Tw_s^{i,h} + \rho_s^{i,h}\right)^{1-\gamma_{i,h}}}{1-\gamma_{i,h}} \tag{4}$$

# Risk Sharing: Parametric Example II

- From, (2), the reservation wage is given by

$$\bar{w}_s^{i,h} = \frac{1}{T} \frac{\alpha_{i,h}}{1-\alpha_{i,h}} \rho_s^{i,h} \tag{5}$$

- The agent works iff $w_s^{i,h} \geq \bar{w}_s^{i,h}$
- The agent has either CARA (not working) or HARA (working) utility function with respect to income risk with coefficient equal $\gamma_{i,h}$
- Utility is differentiable at the reservation wage
- Utility is strictly concave in $\rho$

# Risk Sharing: Parametric Example, Summary

- The reservation wage (5) and the demand for leisure yield the following testable predictions.
- An agent is less likely to participate and if he participates he works less when:

  her wage is low
  the household is doing well (the multiplier $\lambda_s^h$ is low)
  her Pareto weight is large: a higher status buys additional leisure

- Conversely, An agent is more likely to participate and if he participates he works more when:

  wage is high
  the household is doing bad (the multiplier $\lambda_s^h$ is high)
  her Pareto weight is small: a lower status cant buy additional leisure

# Risk Sharing: Parametric Example, Labor Supply Equations

- Reservation wage and leisure are then given by

$$\log \bar{w}^{i,h} = \log(\alpha_{i,h}) - \log(T) + \frac{1}{(1-\alpha_{i,h})(1-\gamma_{i,h})-1} \log M^h \mu_{i,h}$$

$$+ \frac{(1-\alpha_{i,h})(1-\gamma_{i,h})}{(1-\alpha_{i,h})(1-\gamma_{i,h})-1} \log(1-\alpha_{i,h}) - \frac{1}{(1-\alpha_{i,h})(1-\gamma_{i,h})-1} \log\left(\frac{\Lambda_s}{\pi_s}\right)$$

$$\log L_s^{i,h} = \left(\log \alpha^{i,h} + \frac{1}{\gamma_{i,h}} \log M^h \mu^{i,h}\right) + \frac{(1-\gamma_{i,h})}{\gamma_{i,h}} \log(\alpha_{i,h}^{\alpha_{i,h}} (1-\alpha_{i,h})^{1-\alpha_{i,h}})$$

$$- \frac{1}{\gamma_{i,h}} \log\left(\frac{\Lambda_s}{\pi_s}\right) + \frac{(1-\alpha_{i,h})(1-\gamma_{i,h})-1}{\gamma_{i,h}} \log w_s^{i,h}$$

- An obvious problem with these two equations is that neither the Pareto weights $M^h, \mu^{i,h}$ nor the marginal utility of income $\Lambda_s$
- Strategy: exploit the specific structure of these equations in terms of variations within and across households.
- We can summarize the system as:

$$\log \bar{w}_s^{i,h} = B_{i,h} + G_{i,h} D_s$$

$$\log(T - l_s^{i,h}) = A_{i,h} + F_{i,h} D_s - E_{i,h} \log w_s^{i,h}$$

# Risk Sharing: Parametric Example, Labor Supply Equations

- Where

$$A_{i,h} = \left(\log \alpha_{i,h} + \frac{1}{\gamma_{i,h}} \log M^h \mu^{i,h}\right) + \frac{(1-\gamma_{i,h})}{\gamma_{i,h}} \log\left((\alpha_{i,h})^{\alpha_{i,h}}(1-\alpha_{i,h})^{(1-\alpha_{i,h})}\right)$$

$$B_{i,h} = \log \alpha_{i,h} - \log T + \frac{1}{(1-\alpha_{i,h})(1-\gamma_{i,h})-1} \log\left(M^h \mu^{i,h}\right)$$
$$+ \frac{(1-\alpha_{i,h})(1-\gamma_{i,h})}{(1-\alpha_{i,h})(1-\gamma_{i,h})-1} \log(1-\alpha_{i,h})$$

$$G_{i,h} = \frac{1}{(1-\alpha_{i,h})(1-\gamma_{i,h})-1}$$

$$F_{i,h} = \frac{1}{\gamma_{i,h}}$$

$$D_s = -\log\left(\frac{\Lambda_s}{\pi_s}\right)$$

$$E_{i,h} = \frac{(1-\alpha_{i,h})(1-\gamma_{i,h})-1}{\gamma_{i,h}}$$

# Part II

# Labor and aggregation

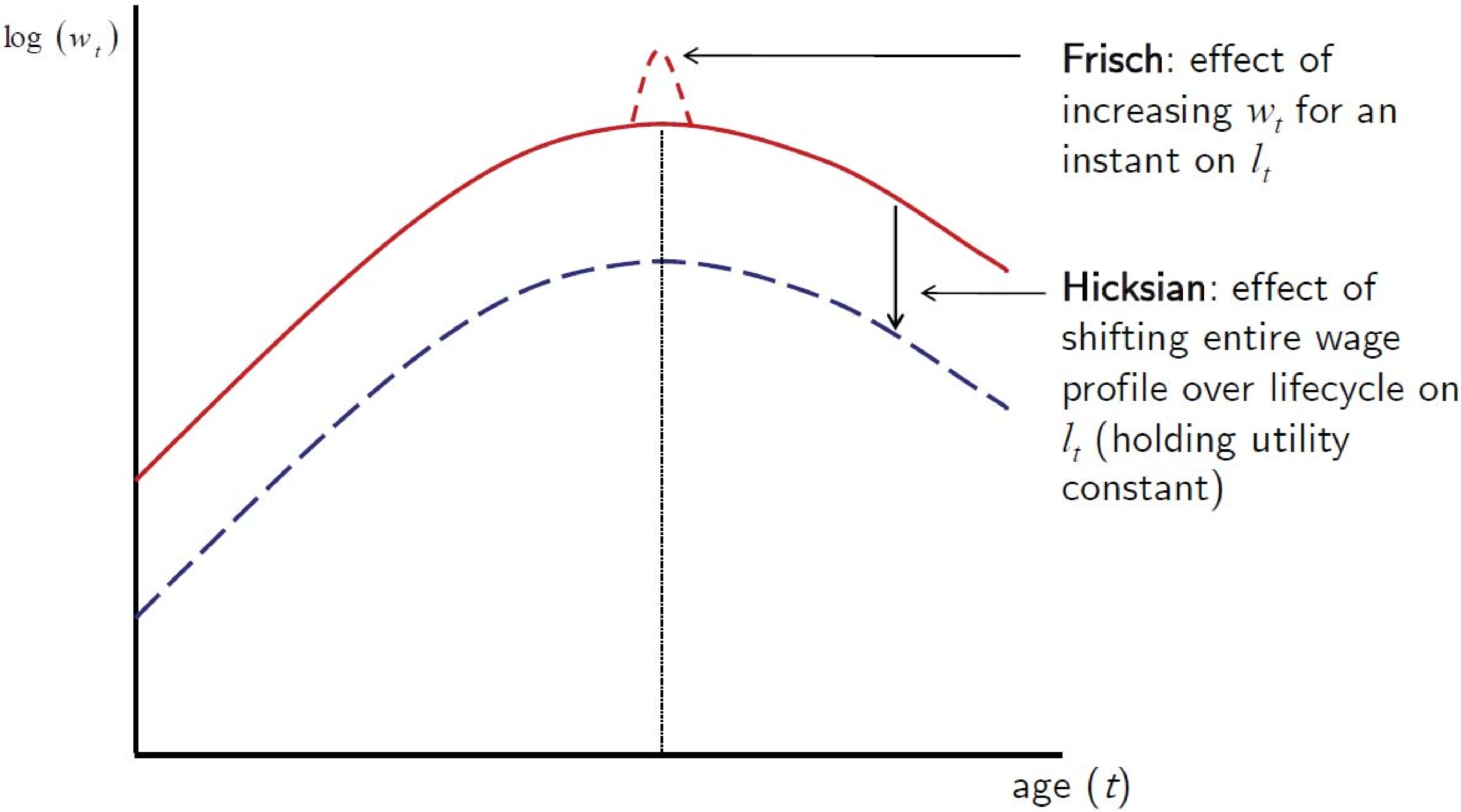
# Introduction

- Tension between the small micro estimates of the intensive and extensive margin labor elasticities and the large values needed to match volatility in aggregate hours or response of hours to changes in taxes.
- What has happened in the literature studying labor supply since the tension was recognized?
  - **Macro** (from Chetty et al): developed models of indivisible labor in which extensive-margin responses make aggregate-hours elasticities larger than intensive-margin elasticities (Richard Rogerson 1988, Gary D. Hansen 1985, Chang Yongsung and S. Kim 2006, Lars Ljungqvist and Thomas J. Sargent 2006).
  - **Micro** (from Chetty et al): Large body of evidence on intensive (hours conditional on employment) and extensive (participation) labor supply elasticities.

# Review of the Evidence: Chetty et al (2012)

- There are four elasticities of interest:
  - steady-state (Hicksian) extensive
  - steady-state (Hicksian) intensive
  - intertemporal (Frisch) extensive.
  - intertemporal (Frisch) intensive.
- Terminology in Macro Literature
  - Macro Elasticity: Frisch Elasticity of Aggregate hours
  - Micro Elasticity: Frisch elasticity on intensive margin

# Review of the Evidence: Chetty et al (2012)



# Review of the Evidence: Chetty et al (2012)

**Micro vs. Macro Labor Supply Elasticities**

| | | Intensive Margin | Extensive Margin | Aggregate Hours |
|---|---|---|---|---|
| Steady State (Hicksian) | Micro | 0.33 | 0.26 | 0.59 |
| | Macro | 0.33 | 0.17 | 0.50 |
| Intertemporal Substitution (Frisch) | Micro | 0.54 | 0.28 | 0.82 |
| | Macro | [0.54] | [2.30] | 2.84 |

Each cell shows a point estimate of the relevant elasticity based on meta analyses of existing micro and macro evidence. Micro estimates are identified from quasi-experimental studies; macro estimates are identified from cross-country variation in tax rates (steady state elasticities) and business cycle fluctuations (intertemporal substitution elasticities). The aggregate hours elasticity is the sum of the extensive and intensive elasticities. Macro studies do not always decompose intertemporal aggregate hours elasticities into extensive and intensive elasticities. Therefore, the estimates in bracket show the value implied by the macro aggregate hours elasticity if the intensive Frish elasticity is chosen to match the micro estimate of 0.54. Source are described in the appendix.

# Review of the Evidence: Chetty et al (2012)

## Frisch Elasticities, Macro

- The fact that employment fluctuations account for 5/6 of the fluctuations in aggregate hours suggests that extensive elasticities above 3 would be needed to match the data in standard RBC models.
- If macro models with an extensive margin were calibrated to match an intensive intertemporal elasticity of 0.54, they would require extensive intertemporal elasticities of 2.84-0.54 $=$ 2.30 on average to match aggregate hours fluctuations.
- This value is an order of magnitude larger than all of the micro estimates considered so far
- Hence, extensive labor supply responses are not large enough to explain the large fluctuations in employment rates at business cycle frequencies.

# Review of the Evidence: Chetty et al (2012)

## Summing up

- Tension between the small micro estimates of the intensive and extensive margin labor elasticities and the large values needed to match volatility in aggregate hours or response of hours to changes in taxes.
- We will study how different market regimes map a small labor intensive margin elasticity into a higher aggregate elasticity.
- We will find that are a low micro elasticity can be mapped in a higher aggregate elasticity in economies with Complete (Rogerson 1988, Hansen 1985) and Incomplete markets (Kim et al 2006)

# Rogerson (1988): Main Contribution

- Shows that nonconvexities, as a result of aggregation, have a major consequence on the aggregate response to aggregate shocks.
- In particular, an economy with a continuum of identical agents will behave in the same way as one with a representative agent with preferences that are different from the ones of all the individuals in the economy.

# Rogerson (1988): Setting without lotteries

- Three commodities: labor, capital and output.
- Single period, Non Stochastic Environment.
- Representative firm, $f(K,N)$, increasing, concave, continuously diff.
- Continuum of identical (no heterogeneity) individuals $i \in [0,1]$ with:
  - Endowment: 1 unit of time, 1 unit of capital
  - Time is indivisible and divided between leisure and work
  - Identical utility function $u(c) - v(n)$
  - $v(1) = m, v(0) = 0$
  - $c \geq 0, n \in \{0,1\}$, where $n = 1$, means that the individual is working.
  - Define the (non convex!) consumption set

$$X = \left\{(c,n,k) \in \mathbb{R}^3 : c \geq 0, n \in \{0,1\}, 0 \leq k \leq 1\right\}$$

# Rogerson (1988): Equilibrium without lotteries

- An allocation for $E$ is $(c(i), n(i), k(i)) \in X$, and $K, N \geq 0$
- A competitive equilibrium for $E$ is an allocation and prices $w, r$ that:
  - Individual Optimization: For each $i \in [0,1]$, $(c(i), n(i), k(i))$ solves

$$\max_{c,n,k} u(c) - mv(n)$$

    subject to

$$\begin{aligned} c &\leq nw + rk \\ n &\in \{0,1\} \\ 0 &\leq k \leq 1 \end{aligned}$$

  - Firm Optimization: $N, K \geqslant 0$ are a solution to

$$\max_{N,K} f(K,N) - rK - wN$$

  - Markets clear (labor, capital, consumption)
- In economy $E$ a competitive equilibrium exists. But, due to the indivisibility in labor supply, individuals might receive different allocations in equilibrium (see example in the lecture notes). If we allow for randomization, we can make agents better off.

# Rogerson (1988): Setting with lotteries

- Let $\overline{X}$ denote the set of allocations in economy $\overline{E}$:

$$\begin{aligned} X_1 &= \left\{(c,n,k) \in \mathbb{R}^3 : c \geq 0, n = 1, 0 \leq k \leq 1\right\} \\ X_2 &= \left\{(c,n,k) \in \mathbb{R}^3 : c \geq 0, n = 0, 0 \leq k \leq 1\right\} \\ \overline{X} &= X_1 \times X_2 \times [0,1] \end{aligned}$$

- Let $\phi$ be the probability that an individual is assigned to work. An element in $\overline{X}$ is given by:

$$((c_1, 1, k_1), (c_2, 0, k_2), \phi)$$

- The expected utility of this allocation

$$\phi\left[u(c_1) - m\right] + (1-\phi)\left[u(c_2)\right]$$

# Rogerson (1988): Setting with lotteries

- Prices of consumption good, capital and labor are $(1, r, w)$
- There is a randomization device that with probability $\phi$ gives the agent the allocation $(c_1, 1, k_1)$ and with probability $(1-\phi)$ the allocation $(c_2, 0, k_2)$.
- Receiving the allocation $(c_1, 1, k_1)$ means that the agent consumes $c_1$, works, and supplies $k_1$ units of capital (he will receive an income of $w + rk_1$).
- Competitive market for insurance to overcome his income uncertainty with $x_1$ be the premium paid when working and $x_2$ the insurance received when not working
- The budget constraint is then

$$c_1 \leq w + rk_1 - x_1 \tag{6}$$

$$c_2 \leq rk_2 + x_2 \tag{7}$$

- The zero profit condition (Insurance is actuarially fair) for the insurance company

$$\pi = \phi x_1 - (1-\phi) x_2 = 0 \tag{8}$$

# Rogerson (1988): Equilibrium with lotteries

- Individual Optimization. For each $i \in [0,1]$, $(c(i), n(i), k(i))$ solves (**P1**) given by:

$$\max_{c_0, c_1, k_0, k_1, \phi} \phi\,[u(c_1) - m] + (1-\phi)\,[u(c_2)]$$

subject to

$$\begin{aligned} \phi c_1 + (1-\phi) c_2 &\leq w\phi + r\,[\phi k_1 + (1-\phi) k_2] \qquad (9) \\ c_h &\geq 0, h \in \{1,2\} \\ \phi &\in [0,1] \\ 0 &\leq k \leq 1 \end{aligned}$$

- Firm Optimization
- Markets clear

# Rogerson (1988): Equilibrium Characterization

- Simplify (**P1**).
- **Lemma: *If ($c_0, c_1, k_0, k_1, \phi$) is a solution to ($P1$) and* $\phi \in (0,1)$, *then,* $c_1 = c_2$. Also, note that $k_1 = k_2 = 1$.**
- The proof follows from working with first order conditions[1] and comes from separability. The first order conditions for individual optimization for (**P1**) with respect to consumption are

$$
\begin{aligned}
\phi u'(c_1) &= \phi\theta \\
(1-\phi)u'(c_2) &= (1-\phi)\theta
\end{aligned}
$$

  where $\theta$ is the multiplier of the budget constraint (9). This implies that $c_1 = c_2 = c$.
- Note that if $\phi \notin (0,1)$, the requirement that $c_1 = c_2$,has no implications.

[1] We rule out a corner solution for consumption and capital. Sufficient for this would be Inada type of conditions.

# Rogerson (1988): Equilibrium Characterization

- So, from the previous lemma, (**P1**) can be written as (**P2**) that is given by

$$\max_{c,\phi} u(c) - \phi m$$

subject to

$$\begin{aligned} c &= w\phi + r \\ c &\geq 0 \\ 0 &\leq \phi \leq 1 \end{aligned}$$

where we used that $k = 1$.

- Computing equilibrium now involves finding a list $(c, \phi, K, N, r, w)$:

$(c, \phi)$solves (**P2**)
$(K, N)$ solves firms problem
$\phi = N, K = 1, c = f(K, N)$

# Rogerson (1988): Equilibrium Characterization

- And from this, we can conclude that, equilibrium is identical to the one in an economy with

  Production function $f(K,N)$
  Representative Agent with a utility function given by $u(c)-mn$ and with a consumption set

$$X = \ (c,n,k) \in \mathbb{R}^3 : c \geq 0, 0 \leq n \leq 1, 0 \leq k \leq 1\}$$

- This economy is entirely neoclassical (in particular, no nonconvexity).
- Let (**P3**) be given by

$$\max_{c,\phi} u(c) - m\phi$$

subject to

$$\begin{aligned} c &\leq f(1,\phi) \\ c &\geq 0 \\ 0 &\leq \phi \leq 1 \end{aligned}$$

# Rogerson (1988): Implications for fluctuations

- An economy with a continuum of agents with utility

$$u(c) - v(n)$$

  and indivisibility in labor supply (that is $n \in \{0,1\}$) is isomorphic to an economy with a representative agent and a utility function given by

$$u(c) - mn$$

- So, the economy behaves as one that has a representative agent, with linear utility function; and, linearity in work dis-utility (as shown below), implies, infinite elasticity of substitution, between wages and leisure.
- This motivates the inclusion of an indivisibility in labor supply as a way of generating higher response in aggregate labor in response to an innovation in productivity.
- But, as we will see, there are other realistic ways to generate "indivisibility".

# Hansen (1985): Motivation

- Hansen 1985 applies Rogerson's setting to a Neoclassical Growth model to generate volatility in hours.
- Motivation of the paper
  - There is need to focus on the extensive margin to generate volatility in hours. For example, Heckman and MaCurdy (1980), show the importance of the *extensive margin* (changes in participation) in explaining female labor supply.
  - At the aggregate level, half of the variation in total hours is due to participation. Let $H_t$ be total hours worked, $h_t$ be average hours worked, and $N_t$ be the number of individuals working. Using data from BLS, we can decompose the variance in total hours as

$$var(\log H_t) = \overbrace{var(\log h_t)}^{20\%} + \overbrace{var(\log N_t)}^{55\%} + 2cov(\log h_t, \log N_t)$$

  - To introduce variability in the *extensive margin*, Hansen introduces nonconvexity, as in Rogerson, in preferences: agents work full time or don't work at all. As we saw before, the economy will behave as if it had a representative agent with infinite elasticity of substitution.

# Hansen (1985): Classical Economy

- Neoclassical growth model with adjustment in the intensive margin.
- Let $k_t$ be capital, $h_t$ be total hours, and $z_t$ be productivity. The production function

$$f(z_t, k_t, n_t) = z_t k_t^{\alpha} h_t^{1-\alpha} \tag{10}$$

- Let $c_t$ and $i_t$ be consumption and investment. Aggregate constraint

$$c_t + i_t \leq f(z_t, k_t, h_t) \tag{11}$$

- Law of motion of capital

$$k_{t+1} = (1-\delta)k_t + i_t \tag{12}$$

- Law of motion of productivity

$$\log z_t = \rho \log z_{t-1} + \varepsilon_t \tag{13}$$

- Measure 1 of identical agents utility function

$$u(c_t, 1-h_t) = \log c_t + A\log(1-h_t)$$

- Firms own the production function and rent capital from agents and hire them as workers.

# Hansen (1985): Classical Economy

- Because the are no externalities and other distortions, any Pareto Optimum can be decentralized as a competitive equilibrium. So, to find the competitive equilibrium, we can solve the planners problem that will be given by

$$\max_{\{c_t, h_t\}_{t=0}^{\infty}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t)$$

  subject to (10) to (13), $k_0, z_0$ and $F(\varepsilon)$.
- From first order conditions we can recover prices that support the equilibrium.

# Hansen (1985): Indivisible Labor

- Following Rogerson, introduce a contract between the firm and a household that commits the household to work $h_0$ hours at period $t$ with probability $\alpha_t$ and zero otherwise.
- The household will get paid independently if it works or not.
- Since all the households are identical ex ante they choose the same contract $\alpha_t$.
- But, they differ expost, so, expected utility is given by

$$\begin{aligned} U(c_t, \alpha_t) &= \alpha_t (\log c_t + A \log(1 - h_0)) + (1 - \alpha_t)(\log c_t + A \log(1)) \quad (14) \\ &\equiv \log c_t + A\alpha_t \log(1 - h_0) \quad (15) \end{aligned}$$

- Per capita hours are given by

$$h_t = \alpha_t h_0 \quad (16)$$

- Again, we can solve the planner problem, now, with an additional constraint (16).
- The key property of this economy is that the elasticity of substitution between leisure in different periods for the representative agent is infinite.

# Hansen (1985): Indivisible Labor

- Plugging (16) in (15) we get that

$$\begin{aligned} U(c_t, \alpha_t) &= \log c_t + A\alpha_t \log(1-h_0) \\ &= \log c_t + A\frac{h_t}{h_0}\log(1-h_0) \end{aligned}$$

- So, the utility function of the representative agent is given by

$$u(c_t, 1-h_t) = \log c_t - Bh_t$$

where

$$B = \frac{-A\log(1-h_0)}{h_0}$$

- Then, the planners problem is

$$\max_{\{c_t,\alpha_t\}_{t=0}^{\infty}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t (\log c_t - Bh_t)$$

subject to (6) to (13), $k_0, z_0$ ,$F(\varepsilon)$ and (16).

# Hansen (1985): Solution Method

- The solution strategy can be summarized as follows:

1. Find first order conditions.
2. Compute the steady state.
3. Compute a first order approximation around the steady state.
4. Solve for the law of motion of endogenous variables.
5. Compute the moments of simulated data.

# Hansen (1985): Calibration

- To simulate data we need parameter values for $(\alpha, \beta, \rho, \delta, A, \sigma_\varepsilon)$ and $h_0$.
  - $\alpha$ : share of capital in total production.
  - $\delta$ : such that implies an annual rate of depreciation of 10 percent.
  - $\beta$ : set to 0.99 since this implies a real interest rate of 4 percent for quarterly data.
  - $A$ : set equal to 2 and implies that hours worked in steady state in the model with divisible labor are $1/3$ of total.
  - $h_0$ : such that the two models (with and without nonconvexity) have the same hours in steady state.
  - $\rho$ : is set to 0.95 such that $z$ is log normal with mean 5 percent.
  - Different values are set for $\sigma_\varepsilon$.

# Hansen (1985): Simulation Results

Table 1

Standard deviations in percent (a) and correlations with output (b) for U.S. and artificial economies.

| | Quarterly U.S. time series[a] (55,3–84,1) | | Economy with divisible labor[b] | | Economy with indivisible labor[b] | |
|---|---|---|---|---|---|---|
| Series | (a) | (b) | (a) | (b) | (a) | (b) |
| Output | 1.76 | 1.00 | 1.35 (0.16) | 1.00 (0.00) | 1.76 (0.21) | 1.00 (0.00) |
| Consumption | 1.29 | 0.85 | 0.42 (0.06) | 0.89 (0.03) | 0.51 (0.08) | 0.87 (0.04) |
| Investment | 8.60 | 0.92 | 4.24 (0.51) | 0.99 (0.00) | 5.71 (0.70) | 0.99 (0.00) |
| Capital stock | 0.63 | 0.04 | 0.36 (0.07) | 0.06 (0.07) | 0.47 (0.10) | 0.05 (0.07) |
| Hours | 1.66 | 0.76 | 0.70 (0.08) | 0.98 (0.01) | 1.35 (0.16) | 0.98 (0.01) |
| Productivity | 1.18 | 0.42 | 0.68 (0.08) | 0.98 (0.01) | 0.50 (0.07) | 0.87 (0.03) |

[a] The U.S. time series used are real GNP, total consumption expenditures, and gross private domestic investment (all in 1972 dollars). The capital stock series includes nonresidential equipment and structures. The hours series includes total hours for persons at work in non-agricultural industries as derived from the *Current Population Survey*. Productivity is output divided by hours. All series are seasonally adjusted, logged and detrended.

[b] The standard deviations and correlations with output are sample means of statistics computed for each of 100 simulations. Each simulation consists of 115 periods, which is the same number of periods as the U.S. sample. The numbers in parentheses are sample standard deviations of these statistics. Before computing any statistics each simulated time series was logged and detrended using the same procedure used for the U.S. time series.

# Chang Kim (2006): Introduction

- In an incomplete markets setting, a high aggregate elasticity, can still be obtained
- They present a model economy where workforce heterogeneity stems from idiosyncratic productivity shocks:
    - The model economy exhibits the cross-sectional earnings and wealth distributions that are comparable to those in the micro data (not presented here, but useful as a cross check for the model)
    - They find that the aggregate labor-supply elasticity of such an economy is around 1, greater than a typical micro estimate

# Chang Kim (2006): Setting

- Continuum of families of measure 1
- Family consists of a pair of male and female and the utility function is

$$U = \max_{\{c_t, h_{mt}, h_{ft}\}_{t=0}^{\infty}} E\left\{\sum_{t=0}^{\infty} \beta^t u(c_t, h_{mt}, h_{ft})\right\}$$

$$u(c_t, h_{mt}, h_{ft}) = 2 \times \ln(0.5c_t) - B_m \frac{h_{mt}^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} - B_f \frac{h_{ft}^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}$$

- Utility of consumption is given by $2 \times \ln(0.5c_t)$ since $c_t$ is measured for each household (assume they share equally and they have equal weight)
- As we saw before, $\gamma$ measures the inter-temporal elasticity of labor
- Workers differ from each other in productivity that follows a Markov process.
- Labor enters as efficiency units. Worker earns $w_t x_t h_t$ if works $h_t$ hours, when the aggregate wage is $w_t$, and his productivity is $x_t$.

# Chang Kim (2006): Setting

- There is no intensive margin: the worker works 0 or $\bar{h}$ hours
- capital market is incomplete: the only asset is physical capital that yields a rate $r$ and depreciates at rate $\delta$. No market for insurance against idiosyncratic shocks as in Aiyagari (1994) and Hugget (1993)
- Budget constraint of the (entire) family

$$c_t = w_t(x_{mt}h_{mt} + x_{ft}h_{ft}) + (1 + r_t)a_t - a_{t+1}$$

$$a_{t+1} \geq \bar{a}$$

- Firms have Cobb Douglass Technology. This will not be important for the results, but closes the model. The production function is given by $Y_t = F(L_t, K_t, \lambda_t) = \lambda_t K_t^{\alpha} L_t^{1-\alpha}$ where $\lambda_t$ is an aggregate shock.

# Chang Kim (2006): Equilibrium

- Consider a recursive equilibrium where $\mu(a, x_m, x_f)$ measures assets and productivities across families
- Let $V_{ee}$ denote the value if the both the male and female are employed (so, both are working $\bar{h}$ hours).
- The value function solves

$$V_{ee}(a, x_m, x_f; \lambda, \mu) = \max_{a' \in A} \; u(c, \bar{h}, \bar{h}) + \beta E\left[\max\{V'_{ee}, V'_{en}, V'_{ne}, V'_{nn}\}/x_m, x_f, \lambda\right]\}$$

$$c = w(x_m h + x_f h) + (1+r)a - a'$$

$$a' \geq \bar{a}$$

$$\mu' = T(\lambda, \mu)$$

  where $V'_{ee}$ denotes $V'_{ee}(a', x'_m, x'_f; \lambda', \mu')$ (the others are analogous)
- The family labor supply decision is

$$V(a, x_m, x_f; \lambda, \mu) = \max\{V_{ee}, V_{en}, V_{ne}, V_{nn}\}$$

# Chang Kim (2006): Definition

- **Equilibrium:** An equilibrium is a set of value functions

  $$V_{ee}(a,x_m,x_f;\lambda,\mu), V_{en}(a,x_m,x_f;\lambda,\mu), V_{ne}(a,x_m,x_f;\lambda,\mu), V_{nn}(a,x_m,x_f;\lambda,\mu), V(a,x_m,x_f;\lambda,\mu)$$

  a set of decision rules for consumption, asset holdings and labor supply

  $$c(a,x_m,x_f;\lambda,\mu), a'(a,x_m,x_f;\lambda,\mu), h_m(a,x_m,x_f;\lambda,\mu), h_f(a,x_m,x_f;\lambda,\mu)$$

  aggregate inputs $K(\lambda,\mu), L(\lambda,\mu)$ and factor prices $w(\lambda,\mu), r(\lambda,\mu)$, and a law of motion $\mu' = T(\lambda,\mu)$ such that:

1. Individual optimization. Given the wages, the individual decision rules solve the Bellman equations.
2. Firms profit maximization
3. Goods Market clear
4. Factor Markets clear
5. Individual and aggregate behavior are consistent

$$\mu'(A^0,X^0,X^0) = \int_{(A^0,X^0,X^0)} \left\{ \int_{\mathscr{A}\,\mathscr{X}\,\mathscr{X}} I_{a'=a'(a,x_m,x_f;\lambda,\mu)} d\pi_x^m(x_m'/x_m) \times d\pi_x^f(x_f'/x_f) d\mu \right\} da'\, dx_m' dx_f'$$

# Calibration I

- **Individual productivity** follows an AR(1) and iid across the population
$$\ln x' = \rho_x \ln x + \varepsilon_x$$
- For each worker, the wage is $\ln w_t^i = \ln x_t^i + \ln w_t$, an aggregate wage rate, and the specific productivity shock of the worker. Differencing this equation, we get
$$\ln w_t^i = \rho_x \ln w_{t-1}^i + (\ln w_t - \rho_x \ln w_{t-1}) + \varepsilon_{x,t}^i \tag{17}$$
- To correct for selection bias, they apply a Heckman type of estimator to estimate the previous equation. Authors treat $\ln w_t$ as time dummies. The selection equation will be $d_t^i = Z_t^i b + u_t^i$ where $Z_t^i$ includes (age, years of schooling, marital status, age 2 , schooling 2 , age $\times$ schooling). Results can be found in the paper.
- They estimate two versions of the model. Model 1, does not inlcude individual characteristics in (17). Model 2, does, and uses predicted wages (they regress $w_t^i$ on individual characteristics) instead of measured wages.

# Calibration II

- For the **other parameters**, the calibration follows standard parameters in the business cycle literature.
- $\alpha = 0.64$ is the labor share
- $\delta = 0.25\%$ is the quarterly depreciation rate.
- When they work, individuals supply $h = \frac{1}{3}$.
- Most micro estimates of the intertemporal subtitution range (the authors claim) 0 and 0.5 and they use $\gamma = 0.4$.
- The disutility if work $B_m$ and $B_f$ are used to match the average employment rates of males and females.
- The discount factor is chosen so that the quarterly return on capital is 1%.
- The borrowing constrint $\bar{a} = -4.0$ which is one a half of quarterly earnings of the household.
- For business cycle fluctuations, they use an AR(1) process for the aggregate productivity shock.

# Chang Kim (2006): Individual Response

- Consider a sample of 50.000 households in steady state (when $\mu(a, x_m, x_f)$ is invariant) and simulate their histories for 120 quarters. Then, aggregate them for annual frequencies.
- Run a panel regression for individuals that have positive hours of the following form

$$\ln h_{it} = \gamma(\log w_{it} - \log c_{it}) + \varepsilon_{it}$$

  separately for men and women.
- Key findings summarized in table 6

  small elasticities for both men and women
  larger elasticity for women

# Chang Kim (2006): Individual Response

## Aggregate Labor Supply

Implied elasticity from the steady-state reservation-wage distribution

| Model | Male | Female | Aggregate |
|---|---|---|---|
| Model I | 0.84 | 1.36 | 0.94 |
| Model II | 0.96 | 1.71 | 1.12 |

Note: The numbers reflect the elasticity of the labor-market participation rate with respect to reservation wage (evaluated around the steady state) based on the steady-state reservation-wage distribution.

# Chang Kim (2006): Aggregate Response

- Simulate the model assuming that there is an AR(1) aggregate productivity disturbances for 30000 quarters, compute the aggregates.
- First, they run the previous regression

$$\ln h_{it} = \gamma(\log w_{it} - \log c_{it}) + \varepsilon_{it}$$

- Results are in Table 8. Main point: the value is much higher than in the individual response.
- Second, they consider a unique household with preferences given by

$$E\left\{\sum_{t=0}^{\infty} \beta^t \log c_t - \alpha \frac{h_t^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}\right\}$$

  and divisible labor. They simulate this model, compute the moments of the simulated data as in Kydland and Prescott (1982), and pick $\gamma$ to replicate the business cycle moments generated by the model with the non convexity.
- Results are in Table 8. Main point: the value is $\gamma$ needed is around 2, much higher than Micro estimates.

# Chang Kim (2006): Aggregate Response

**Compensated Labor Supply Elasticities from The Model-Generated Data**

| Model | Individual panel | | Aggregate time series |
|---|---|---|---|
| | Male | Female | |
| Model I | 0.41 | 0.78 | 1.08 |
| Model II | 0.45 | 0.89 | 1.15 |

Note: All estimates are based on the OLS of equation (13) using model-generated data. The individual labor supply elasticities are based on the annual panel data of 50,000 worker for 30 years. The aggregate estimates are based on the quarterly time series of 3,000 periods.

# Chang Kim (2006): Aggregate Response

**Comparison With Representative-Agent Economies**

| | Model I | Model II | Representative Agent | | | | U.S. Data 1948:I-2000:IV |
|---|---|---|---|---|---|---|---|
| | | | $\gamma = 0.4$ | $\gamma = 1$ | $\gamma = 2$ | $\gamma = 4$ | |
| $\sigma(Y)$ | 1.53 | 1.58 | 1.22 | 1.38 | 1.54 | 1.71 | 2.22 |
| $\sigma(C)$ | 0.42 | 0.40 | 0.41 | 0.45 | 0.49 | 0.52 | 0.96 |
| $\sigma(I)$ | 5.00 | 5.22 | 3.72 | 4.26 | 4.81 | 5.39 | 4.67 |
| $\sigma(N)$ | 0.72 | 0.79 | 0.25 | 0.50 | 0.75 | 1.01 | 1.78 |
| $\sigma(N)/\sigma(Y)$ | 0.47 | 0.50 | 0.20 | 0.36 | 0.49 | 0.59 | 0.80 |
| $\sigma(N)/\sigma(Y/N)$ | 0.82 | 0.92 | 0.23 | 0.55 | 0.91 | 1.37 | 1.61 |

Note: All variables are detrended by the H-P filter. $\gamma$ denotes the Frisch labor supply elasticities. The statistics for data are based on per capita values (divided by civilian noninstitutional population over 16) from the Citibase: Y = nonfarm business GDP (GPBUQF); C = consumption of nondurables and services (GCNQ+GCSQ); I = nonresidential fixed private investment (GIFQ); N = total employed hours in private nonagricultural sector based on the establishment survey (LPMHU).

# Seema (2006): Introduction

- The paper builds and tests a model in which productivity shocks cause larger changes in the wage when workers are poorer, less able to migrate, and more credit-constrained because of such workers' **inelastic labor supply**.
- The equilibrium wage effect hurts workers but acts as insurance for landowners.
- Agricultural wage data for 257 districts in India for 1956–87 are used to test the predictions, with rainfall as an instrument for agricultural productivity.
- The results show that with fewer banks or higher migration costs, the wage is **much more responsive** to fluctuations in productivity.

# Seema (2006): Setting

- The economy (village) has a large number $N$ of agents who live for two periods $t \in \{1, 2\}$.
- Each agent $i$ is endowed with landholding $k_i$ . There is no market for land. Total capital in the village is $K$
- All agents have the same endowment of time, $\bar{h}$, which they allocate between labor, $h_i$ , and leisure, $l_i$ .
- There are two types of individuals, landless and landowning. A proportion $\theta \in (0, 1)$ of the village is landless ($k_p = 0$), and the remaining villagers have equally sized plots of land $k_r = \frac{K}{(1-\theta)N}$.
- "$r$" denotes rich and "$p$" poor

# Seema (2006): Setting

- **Period 1**: Landowners produce with production function $f(d_i, k_i) = \tilde{A} d_i^{\beta} k_i^{\beta}$ where $d_i$ is labor input (hired and landowner). TFP shock, $A_H > A_L$ with probability $\frac{1}{2}$. At the moment of production, productivity is known.
- **Period 2**: exogenous (certain) income $y_i$.
- **Assumption:** The parameters are such that the individual wants to save of there is a good shock and borrow otherwise.
- **Preferences:** Stone Geary preferences with subsistence level of consumption

$$u(c_{it}, l_{it}) = \log(c_{it} - \underline{c}) + \frac{1-\alpha}{\alpha} \log l_{it}$$

- **Financial market:** interest rate $r$ and there is a cost of borrowing and saving $\phi$ such that the interest rate on savings is $r - \phi$ and the interest rate on borrowing is . Agents must have nonnegative assets at the end of period 2.

# Seema (2006): Individuals Problem

- Each individual (Landless and Landowner) solves

$$\max_{c_{i1}\geq \underline{c}, c_{i2}\geq \underline{c}, \bar{h}\geq l_i\geq 0, d_i\geq 0} \log(c_{i1}-\underline{c}) + \frac{1-\alpha}{\alpha}\log l_{it} + b\log(c_{i2}-\underline{c})$$

$$c_{i2} \leq [1+(r+\phi)I(c_{i2}<y_i)+(r-\phi)I(c_{i2}>y_i)]$$
$$\times[\tilde{A}d_i^{\beta}k_i^{\beta} - d_i w + w(\bar{h}-l_i) - c_{i1}] + y_i$$

- The term $\tilde{A}d_i^{\beta}k_i^{\beta} - d_i w + w(\bar{h}-l_i)$ measures: income from the plot, minus the total wages paid, plus the income of the wage if he hours he is using.
- For the case of landless the first two terms are zero. If we substract consumption to this, we get the surplus or deficit in first period, that can be saved or borrowed. That, plus the income in period 2, is the maximum amount that can be consumed in the second period.

# Seema (2006): Individuals Problem

- From the first order conditions we get that

$$d_i^* = k_i \left(\frac{\tilde{A}\beta}{w}\right)^{\frac{1}{1-\beta}} \quad \pi_i = \tilde{A}(1-\beta)k_i\left(\frac{\tilde{A}\beta}{w}\right)^{\beta/(1-\beta)}$$

- Note that the distribution of land does not affect labor demand
- An interior solution for labor supply yields

$$h_i^* = \frac{1-\alpha}{1+\alpha b}\left\{\frac{\alpha(1-b)}{1-\alpha}\bar{h} - \right.$$

$$\left.\frac{1}{w}\left[\frac{y-\underline{c}}{1+(r\pm\phi)} - \underline{c} + (1-\beta)\left(\frac{\tilde{A}\beta}{w}\right)^{\beta/(1-\beta)} k_i\right]\right\}$$

# Seema (2006): Equilibrium

- **Def**: A competitive equilibrium is a set of wages $w_H, w_L$(for each state of nature) such that landowners and landless consumers maximize their utility and the labor and savings market clears.
- The labor market clears when

$$\sum_i d_i = \sum_i (\underline{h} - l_i)$$

- The equilibrium wage elasticity is defined as

$$v \equiv \frac{w_H - w_L}{A_H - A_L} \frac{A_H + A_L}{w_H + w_L}$$

# Seema (2006): Testable Implications

- **Proposition 1:** The wage elasticity is increasing in poverty, where poverty is parameterized by the ratio of the subsistence level to average TFP $\left(\frac{\underline{c}}{A}\right)$: for fixed A, $\frac{\delta v}{\delta \underline{c}} > 0$
- **Proposition 2:** The wage elasticity is increasing in banking costs, or $\frac{\delta v}{\delta \phi} > 0$.
- The intuition for this proposition is as follows.
  - Banking costs affect the degree to which individuals save when there is a good shock and borrow when there is a bad shock.
  - When there is a good shock, a worker has a greater incentive to supply labor if he can more easily shift income to period 2. .
  - Without the ability to save, working more will raise his period 1 consumption, which has a decreasing marginal benefit. Raising his period 1 income is more valuable if he can also shift income to period 2, when the marginal utility of consumption is higher.
  - Similarly, when there is a negative shock, if individuals cannot borrow as easily against their period 2 income, they are compelled to work more in period 1, driving down the wage and exacerbating wage volatility. High banking costs therefore imply more inelastic labor supply and, in turn, larger wage responses to TFP shocks.

# Seema (2006): Specification

- Log wages

$$w_{jt} = \beta_1 A_{jt} + \beta_2 S_{jt} + \beta_3 S_{jt} \times A_{jt} + \beta_4 X_{jt} + \beta_5 X_{jt} A_{jt} + \delta_t + \alpha_j + \varepsilon_{jt}$$

  where the unit of observation is a district $j$, $A_{jt}$ is productivity in that district in $t$, $S_{jt}$ is some variable that is presumed to affect the wage elasticity, $X_{jt}$ are control variables
- The coefficient $\beta_1$ measures the average elasticity of the wages with respect to productivity.
- The main testable prediction to be tested is $\beta_3 < 0$: when there are more smoothing mechanisms available, the elasticity of wages with respect to productity is lower (proposition 2).
- To measure productivity, the only measure available is crop yield (crop volume per unit of land)

$$w_{jt} = \beta_1 Yield_{jt} + \beta_2 S_{jt} + \beta_3 S_{jt} \times A_{jt} + \beta_4 X_{jt} + \beta_5 X_{jt} A_{jt} + \eta_t + \lambda_j + u_{jt}$$

# Seema (2006): Data

- The panel comprises 257 rural districts, defined by 1961 boundaries, observed from 1956 to 1987
- The sample covers over 80 percent of India's land area, including the major agricultural regions.
- A district in the sample has, on average, 400,000 agricultural workers.

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