Introduction to Political Economy 14.770
Problem Set 1

Due date: September 22, 2017.

Question 1

Recall Arrow's impossibility theorem which states that if a social ordering is transitive, weakly Paretian and satisfies independence from irrelevant alternatives, then it is dictatorial.

1. Consider a society with two individuals 1 and 2 and three choices, a, b, and c. For the purposes of this exercise, only consider strict individual and social orderings (i.e., no indifference allowed). Suppose that the preferences of the first agent are given by abc (short for $a \succ b \succ c$, i.e., a strictly preferred to b, strictly preferred to c). Consider the six possible preference orderings of the second individual, i.e., $s_2 \in \{abc, acb, bac, ...\}$, etc.. Define a social ordering as a mapping from the preferences of the second agent (given the preferences of the first) into a social ranking of the three outcomes, i.e., some function f such that the social ranking is $s = f(s_2)$. Illustrate the Arrow impossibility theorem using this example [Hint: start as follows: $abc = f(abc)$, i.e., when the second agent's ordering is abc, the social ranking must be abc; next, $f(acb) = abc$ or acb (why?); then if $f(acb) = abc$, we must also have $f(cab) = abc$ (why?); and proceeding this way to show that the social ordering is either dictatorial or it violates one of the axioms].

2. Now suppose we have the following aggregation rule: individual 1 will (sincerely) rank the three outcomes, his first choice will get 6 votes, the second 3 votes, the third 1 vote. Individual 2 will do the same, his first choice will get 8 votes, the second 4 votes, and the third 0 vote. The three choices are ranked according to the total number of votes. Which of the axioms of the Arrow's impossibility theorem does this aggregation rule violate?

3. With the above voting rule, show that for a certain configuration of preferences, either agent has an incentive to distort his true ranking (i.e., not vote sincerely).

4. Now consider a society consisting of three individuals, with preferences given by:

1	$a \succ b \succ c$
2	$c \succ a \succ b$
3	$b \succ c \succ a$

   Consider a series of pairwise votes between the alternatives. Show that when agents vote sincerely, the resulting social ordering will be "intransitive". Relate this to the Arrow's impossibility theorem.

5. Show that if the preferences of the second agent are changed to $b \succ a \succ c$, the social ordering is no longer "intransitive". Relate this to "single-peaked preferences".

6. Explain intuitively why single-peaked preferences are sufficient to ensure that there will not be intransitive social orderings. How does this relate to the Arrow's impossibility theorem?

Question 2

1. Consider the example of a three-person three-policy society with preferences

1	$a \succ b \succ c$
2	$b \succ c \succ a$
3	$c \succ b \succ a$

   Voting is dynamic: first, there is a vote between a and b. Then, the winner goes against c, and the winner of this contest is the social choice. Find the subgame perfect Nash equilibrium with weakly undominated strategies within each stage strategy profiles in this two-stage game (recall that each player's strategy has to specify how they will vote in the first round, and how they will vote in the second round as a function of the outcome the first round).

2. Suppose a generalization whereby there are finite number of policies, $Q = \{q_1, q_2, ..., q_N\}$ and M agents (which you can take to be an odd number for simplicity). Voting takes $N - 1$ stages. In the first stage, there is a vote between q_1 and q_2. In the second stage, there is a vote

between the winner of the first stage and q_3, until we have a final vote against q_N. The winner of the final vote is the policy choice of the society. Prove that if preferences of all agents are single peaked (with a unique bliss point for each), then the unique subgame perfect Nash equilibrium with weakly undominated strategies within each stage implements the bliss point of the median voter.

Question 3

Consider party competition in a society consisting of a continuum of mass 1 of agents, where the set of agents is $\mathcal{H}$. The policy space is the $[0,1]$ interval and assume that preferences are single-peaked. In particular, if an agent $i \in \mathcal{H}$ has bliss point b_i, her utility from policy $q \in [0,1]$ is:

$$u(b_i, q) = -|b_i - q|$$

Finally, assume that the bliss points are uniformly distributed over this space.

1. To start with, suppose that there are two parties, A and B. They both would like to maximize the probability of coming to power. The game involves both parties simultaneously announcing $q_A \in [0,1]$ and $q_B \in [0,1]$, and then voters voting for one of the two parties. The platform of the party with most votes gets implemented. Determine the equilibrium of this game. How would the result be different if the parties maximized their vote share rather than the probability of coming to power?

2. Now assume that there are three parties, simultaneously announcing their policies $q_A \in [0,1]$, $q_B \in [0,1]$, and $q_C \in [0,1]$, and the platform of the party with most votes is implemented. Assume that parties maximize the probability of coming to power. Characterize all pure strategy equilibria.

3. Now assume that the three parties maximize their vote shares. Prove that there exists no pure strategy equilibrium. Characterize the mixed strategy equilibrium (Hint: assume the same symmetric probability distribution for two parties, and make sure that given these distributions, the third party is indifferent over all policies in the support of the distribution).[1]

[1] This part is difficult, and don't get frustrated if it takes some time to get the answer.

Question 4:

Consider the following one-period economy populated by a mass 1 of agents. A fraction λ of these agents are capitalists, each owning capital k. The remainder have only human capital, with human capital distribution $F(h)$. Output is produced in competitive markets, with aggregate production function

$$Y = K^{1-\alpha} H^{\alpha},$$

where uppercase letters denote total supplies. Assume that factor markets are competitive and denote the market clearing rental price of capital by r and that of human capital by w.

1. Suppose that agents vote over a linear income tax, τ. Because of tax distortions, total tax revenue is

   $$Tax = (\tau - v(\tau)) \left(\lambda r k + (1-\lambda) w \int h dF(h) \right)$$

   where $v(\tau)$ is strictly increasing and convex, with $v(0) = v'(0) = 0$ and $v'(1) = \infty$ (why are these conditions useful?). Tax revenues are redistributed lump sum. Find the ideal tax rate for each agent. Find conditions under which preferences are single peaked, and determine the equilibrium tax rate. How does the equilibrium tax rate change when k increases? How does it change when λ increases? Explain.

2. Suppose now that agents vote over capital and labor income taxes, τ_k and τ_h, with corresponding costs $v(\tau_k)$ and $v(\tau_h)$, so that tax revenues are

   $$Tax = (\tau_k - v(\tau_k)) \lambda r k + (\tau_h - v(\tau_h)) (1-\lambda) w \int h dF(h)$$

   Determine ideal tax rates for each agent. Suppose that $\lambda < 1/2$. Does a voting equilibrium exist? Explain. How does it change when λ increases? Explain why this would be different from the case with only one tax instrument?

3. In this model with two taxes, now suppose that agents first vote over the capital income tax, and then taking the capital income tax as given, they vote on the labor income tax. Does a voting equilibrium exist? Explain. If an equilibrium exists, how does the equilibrium tax rate change when k increases? How does it change when λ increases?