14.581 International Trade
Class notes on 3/6/2013[1]

1 Two-by-two-by-two Heckscher-Ohlin model

1.1 Basic environment

- Results derived in previous lecture hold for small open economies

  - relative good prices were taken as exogenously given

- We now turn world economy with two countries, North and South

- We maintain the two-by-two HO assumptions:

  - there are two goods, $g = 1,2$, and two factors, k and l
  - identical technology around the world, $y_g = f_g(k_g, l_g)$
  - identical homothetic preferences around the world, $d_g^c = \alpha_g(p)I^c$

- **Question**
  What is the pattern of trade in this environment?

1.2 Strategy

- Start from **Integrated Equilibrium** $\equiv$ competitive equilibrium that would prevail if *both* goods and factors were freely traded

- Consider **Free Trade Equilibrium** $\equiv$ competitive equilibrium that prevails if goods are freely traded, but factors are not

- Ask: Can free trade equilibrium reproduce integrated equilibrium?

- *If factor prices are equalized through trade*, the answer is yes

- In this situation, one can then use homotheticity to go from differences in factor endowments to pattern of trade

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[1]The notes are based on lecture slides with inclusion of important insights emphasized during the class.

1.3   Integrated equilibrium

- **Integrated equilibrium** corresponds to (p, ω, y) such that:

$$(ZP) \quad : \quad p = A'(\omega)\,\omega \tag{1}$$

$$(GM) \quad : \quad y = \alpha(p)\,(\omega'v) \tag{2}$$

$$(FM) \quad : \quad v = A(\omega)\,y \tag{3}$$

where:

- $p \equiv (p_1, p_2)$, $\omega \equiv (w, r)$, $A(\omega) \equiv [a_{fg}(\omega)]$, $y \equiv (y_1, y_2)$, $v \equiv (l, k)$, $\alpha(p) \equiv [\alpha_1(p), \alpha_2(p)]$
- $A(\omega)$ derives from cost-minimization
- $\alpha(p)$ derives from utility-maximization

1.4   Free trade equilibrium

- **Free trade equilibrium** corresponds to $(p^t, \omega^n, \omega^s, y^n, y^s)$ such that:

$$(ZP) \quad : \quad p^t \leq A'(\omega^c)\,\omega^c \text{ for } c = n, s \tag{4}$$

$$(GM) \quad : \quad y^n + y^s = \alpha(p^t)\,(\omega^{n\prime}v^n + \omega^{s\prime}v^s) \tag{5}$$

$$(FM) \quad : \quad v^c = A(\omega^c)\,y^c \text{ for } c = n, s \tag{6}$$

where (4) holds with equality if good is produced in country c

- **Definition** *Free trade equilibrium replicates integrated equilibrium if* $\exists$ $(y^n, y^s) \geq 0$ *such that* $(p, \omega, \omega, y^n, y^s)$ *satisfy conditions* (4)-(6)

.Two-by-two-by-two Heckscher-Ohlin model
Factor Price Equalization (FPE) Set

- **Definition** (v^n, v^s) *are in the FPE set if* $\exists$ $(y^n, y^s) \geq 0$ *such that condition* (6) *holds for* $\omega^n = \omega^s = \omega$.

- **Lemma** *If* (v^n, v^s) *is in the FPE set, then free trade equilibrium replicates integrated equilibrium*

- **Proof:** By definition of the FPE set, $\exists$ $(y^n, y^s) \geq 0$ such that
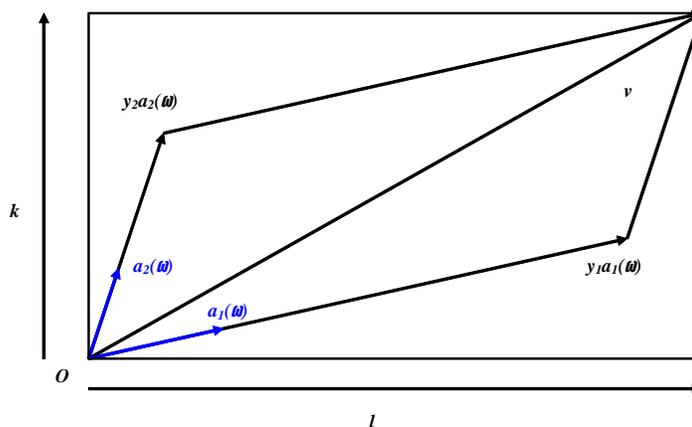
$$v^c = A(\omega)\,y^c$$

So Condition (6) holds. Since $v = v^n + v^s$, this implies

$$v = A(\omega)\,(y^n + y^s)$$

Combining this expression with condition (3), we obtain $y^n + y^s = y$. Since $\omega^{n\prime}v^n + \omega^{s\prime}v^s = \omega'v$, Condition (5) holds as well. Finally, Condition (1) directly implies (4) holds.
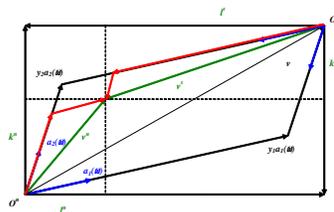
1.5   Integrated equilibrium: graphical analysis

- Factor market clearing in the integrated equilibrium:


1.6   The "Parallelogram"

- **FPE set** $\equiv (v^n, v^s)$ inside the parallelogram

- When v^n and v^s are inside the parallelogram, we say that they belong to the same **diversification cone**

- This is a very different way of approaching FPE than FPE Theorem

  - Here, we have shown that there can be FPE iff factor endowments are not too dissimilar, whether or not there are no FIR

  - Instead of taking prices as given—whether or not they are consistent with integrated equilibrium—we take factor endowments as primitives

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1.7 Heckscher-Ohlin Theorem: graphical analysis

- Suppose that (v^n, v^s) is in the FPE set

- **HO Theorem** *In the free trade equilibrium, each country will export the good that uses its abundant factor intensively*
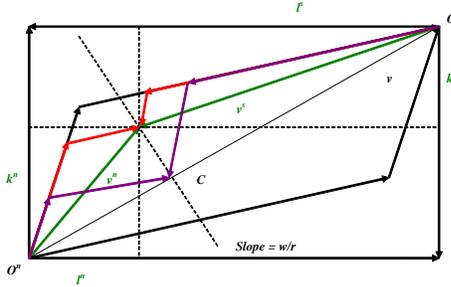
- Outside the FPE set, additional technological and demand considerations matter (e.g. FIR or no FIR)

Heckscher-Ohlin Theorem: alternative proof

- HO Theorem can also be derived using Rybczynski effect:

    1. Rybczynski theorem $\Rightarrow y_2^n/y_1^n > y_2^s/y_1^s$ for any p
    2. Homotheticity $\Rightarrow c_2^n/c_1^n = c_2^s/c_1^s$ for any p
    3. This implies $p_2^n/p_1^n < p_2^s/p_1^s$ under autarky
    4. Law of comparative advantage $\Rightarrow$ HO Theorem

## 1.8 Trade and inequality

- Predictions of HO and SS Theorems are often combined:

    - HO Theorem $\Rightarrow p_2^n/p_1^n < p_2/p_1 < p_2^s/p_1^s$
    - SS Theorem $\Rightarrow$ *Moving from autarky to free trade, real return of abundant factor increases, whereas real return of scarce factor decreases*
    - If North is skill-abundant relative to South, inequality increases in the North and decreases in the South

- So why may we observe a rise in inequality in the South in practice?

    - Southern countries are not moving from autarky to free trade

    – Technology is not identical around the world

    – Preferences are not homothetic and identical around the world

    – There are more than two goods and two countries in the world

1.9   Trade volumes

- Let us define trade volumes as the sum of exports plus imports

- Inside FPE set, iso-volume lines are parallel to diagonal (HKa p.23)

    – the further away from the diagonal, the larger the trade volumes

    – factor abundance rather than country size determines trade volume

volumes

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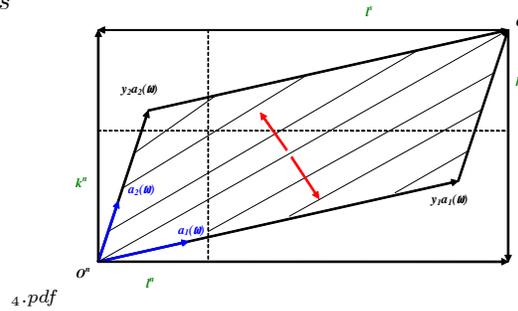
- If country size affects trade volumes in practice, what should we infer?

2   High-Dimensional Predictions

2.1   FPE (I): More factors than goods

- Suppose now that there are F factors and G goods

- By definition, (v^n, v^s) is in the FPE set if $\exists (y^n, y^s) \geq 0$ s.t. $v^c = A(\omega) y^c$ for $c = n, s$

- If $F = G$ ("even case"), the situation is qualitatively similar

- If $F > G$, the FPE set will be "measure zero": $\{v | v = A(\omega) y^c$ for $y^c \geq 0\}$ is a G-dimensional cone in F-dimensional space

- **Example:** "Macro" model with 1 good and 2 factors
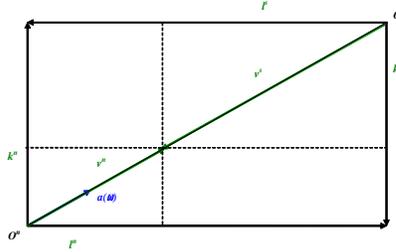
2.2   FPE (II): More goods than factors

- If $F < G$, there will be indeterminacies in production, (y^n, y^s), and so, trade patterns, but FPE set will still have positive measure

- **Example:** 3 goods and 2 factors
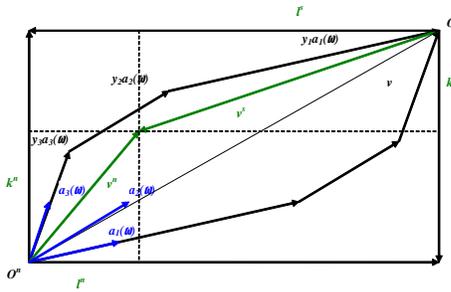
- By the way, are there more goods than factors in the world?

2.3   Stolper-Samuelson-type results (I): "Friends and Enemies"

- SS Theorem was derived by differentiating zero-profit condition

- With an arbitrary number of goods and factors, we still have

$$\widehat{p}_g = \sum_f \theta_{fg} \widehat{w}_f \tag{7}$$

where w_f is the price of factor f and $\theta_{fg} \equiv w_f a_{fg}(\omega)/c_g(\omega)$

- Now suppose that $\widehat{p}_{g_0} > 0$, whereas $\widehat{p}_g = 0$ for all $g \neq g_0$

- Equation (7) immediately implies the existence of f_1 and f_2 s.t.

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\begin{aligned}
\widehat{w}_{f_1} &\geq \widehat{p}_{g_0} > \widehat{p}_g = 0 \text{ for all } g \neq g_0, \\
\widehat{w}_{f_2} &< \widehat{p}_g = 0 < \widehat{p}_{g_0} \text{ for all } g \neq g_0.
\end{aligned}
$$

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- So every good is "friend" to some factor and "enemy" to some other (Jones and Scheinkman 1977)

2.4   Stolper-Samuelson-type results (II): Correlations

- Ethier (1984) also provides the following variation of SS Theorem

- If good prices change from p to p', then the associated change in factor prices, $\omega' - \omega$, must satisfy

$$(\omega' - \omega) A(\omega_0)(p' - p) > 0, \text{ for some } \omega_0 \text{ between } \omega \text{ and } \omega'$$

- **Proof:**

  Define $f(\omega) = \omega A(\omega)(p' - p)$. Mean value theorem implies

$$f(\omega') = \omega A(\omega)(p' - p) + (\omega' - \omega)[A(\omega_0) + \omega_0 dA(\omega_0)](p' - p)$$

  for some ω_0 between ω and ω'. Cost-minimization at ω_0 requires

$$\omega_0 dA(\omega_0) = 0$$

  Combining the two previous expressions, we obtain

$$f(\omega') - f(\omega) = (\omega' - \omega) A(\omega_0)(p' - p)$$

  From zero profit condition, we know that $p = \omega A(\omega)$ and $p' = \omega' A(\omega')$. Thus

$$f(\omega') - f(\omega) = (p' - p)(p' - p) > 0$$

  The last two expressions imply

$$(\omega' - \omega) A(\omega_0)(p' - p) > 0$$

- **Interpretation:**

  Tendency for changes in good prices to be accompanied by raises in prices of factors used intensively in goods whose prices have gone up

- What is ω_0?

2.5 Rybczynski-type results

- Rybczynski Theorem was derived by differentiating the factor market clearing condition

- If $G = F > 2$, same logic implies that increase in endowment of one factor decreases output of one good and increases output of another (Jones and Scheinkman 1977)

- If $G < F$, increase in endowment of one factor may increase output of all goods (Ricardo-Viner)

- In this case, we still have the following correlation (Ethier 1984)

$$(v' - v) A(\omega)(y' - y) = (v' - v)(v' - v) > 0$$

- If $G > F$, inderteminacies in production imply that we cannot predict changes in output vectors

2.6 Heckscher-Ohlin-type results

- Since HO Theorem derives from Rybczynski effect + homotheticity, problems of generalization in the case $G < F$ and $F > G$ carry over to the Heckscher-Ohlin Theorem

- If $G = F > 2$, we can invert the factor market clearing condition

$$y^c = A^{-1}(\omega) v^c$$

- By homotheticity, the vector of consumption in country c satisfies

$$d^c = s^c d$$

where $s^c \equiv c$'s share of world income, and $d \equiv$ world consumption

- Good and factor market clearing requires

$$d = y = A^{-1}(\omega) v$$

- Combining the previous expressions, we get net exports

$$t^c \equiv y^c - d^c = A^{-1}(\omega)(v^c - s^c v)$$

2.7 Heckscher-Ohlin-Vanek Theorem

- Without assuming that $G = F$, we can still derive sharp predictions if we focus on the *factor content of trade* rather than *commodity trade*

- We define the *net exports of factor f* by country c as

$$\tau_f^c = \sum_g a_{fg}(\omega) t_g^c$$

- In matrix terms, this can be rearranged as

$$\tau^c = A(\omega) t^c$$

- **HOV Theorem** *In any country c, net exports of factors satisfy*

$$\tau^c = v^c - s^c v$$

- So countries should export the factors in which they are abundant compared to the world: $v_f^c > s^c v_f$

- Assumptions of HOV Theorem are extremely strong: identical technology, FPE, homotheticity

  - One shouldn't be too surprised if it performs miserably in practice...

3 Quantitative Issues

3.1 Basic Idea

- Stolper-Samuelson offers sharp insights about distributional consequences of international trade, but...

  - Theoretical insights are only *qualitative*
  - Theoretical insights crucially rely on 2×2 assumptions

- Alternatively one may want to know the *quantitative* importance of international trade:

  - Given the amount of trade that we actually observe in the data, how large are the effects of international trade on the skill premium?
  - In a country like the United States, how much higher or smaller would the skill premium be in the absence of trade?

3.2   Eaton and Kortum (2002) Revisited

- Eaton and Kortum (2002)—as well as other gravity models—offer a simple starting point to think about these issues

- Consider multi-sector-multi-factor EK (e.g. Chor JIE 2010)

  - many varieties with different productivity levels $z(\omega)$ in each sector s
  - same factor intensity across varieties within sectors
  - different factor intensities across sectors

- Unit costs of production in country i and sector s are proportional to:

$$c_{i,s} = \left[\left(\mu_s^H \right)^\rho \left(w_i^H \right)^{1-\rho} + \left(\mu_s^L \right)^\rho \left(w_i^L \right)^{1-\rho} \right]^{1/(1-\rho)} \tag{8}$$

  where:

  - w_i^H, $w_i^L \equiv$ wages of skilled and unskilled workers.
  - $\rho \equiv$ elasticity of substitution between skilled and unskilled

3.3   Dekle, Eaton, and Kortum (2008) Revisited

- Suppose, like in EK, that productivity draws across varieties within sectors are independently drawn from a Fréchet

- Then one can show that the following gravity equation holds:

$$X_{ij,s} = \frac{T_i \left(\tau_{ij,s} c_{i,s} \right)^{-\theta_s}}{\sum_{l=1}^n T_l \left(\tau_{lj,s} c_{l,s} \right)^{-\theta_s}} E_{j,s}, \tag{9}$$

  where $E_{j,s} \equiv$ total expenditure on goods from sector s in country j

- Two key equations, (8) and (9), are CES:

  - One can use DEK's strategy to do welfare and counterfactual analysis
  - But one can also discuss the consequences of changes in variable trade costs, $\tau_{lj,s}$, or technology, T_i, on skill premium
  - How large are GT compared to distributional consequences?

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