14.54 International Trade
——Lecture 3: Preferences and Demand——

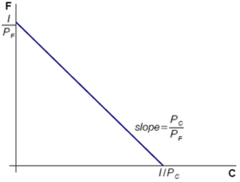
Today's Plan

1. Utility maximization
   1. Budget set
   2. Preferences
   3. Solution
   4. Relative demand
2. Homothetic Preferences
   1. Definition
   2. Properties
   3. Examples

The small graphs on slides 3-5, 7-19, 21, and 24-28 are courtesy of Marc Melitz. Used with permission.

Budget Set

- 2 goods: Cloth (C) and Food (F); Consumption level $D = (D_C, D_F)$
- Given prices p_C and p_F and income I
- Budget set is set of consumption bundles such that $p_C D_C + p_F D_F \leq I$



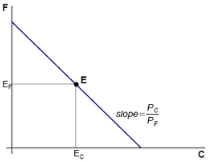
- p_C / p_F is the relative price of C (measured in units of F)

Budget Set With Endowment

- In a trading environment, income is determined by value of endowment $E = (E_C\ ,\ E_F\ )$ (bundle of goods that can be traded)
- So budget line is given by

$$p_C D_C + p_F D_F = p_C E_C + p_F E_F \Leftrightarrow \frac{p_C}{p_F} D_C + D_F = \frac{p_C}{p_F} E_C + E_F$$

$\Rightarrow$ Only relative price p_C / p_F matters! ('nominal' prices are irrelevant)

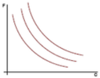


Preferences

- Represented by a utility function $U(D_C, D_F)$
- Recall that utility is an ordinal concept, so units don't matter (only ranking)
  - $U + a$, $a.U$, U^2, $\sqrt{U}$, $logU$, e^U all represent the same preferences
- Marginal utility of each good are assumed to be non-negative:

$$MU_C = \frac{\partial U\left(D_C, D_F\right)}{\partial D_C} \geq 0 \text{ and } MU_F = \frac{\partial U\left(D_C, D_F\right)}{\partial D_F} \geq 0$$

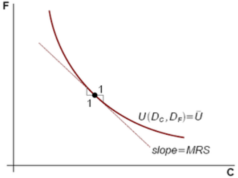
- Preferences are completely summarized by an indifference curve map $U(D_C, D_F) = \overline{U}$ for any $\overline{U}$:



Marginal Rate of Substitution

- At any point on an indifference curve, the marginal rate of substitution is defined as $MRS = MU_C / MU_F$
  - Important note: to avoid confusion, will always refer to MRS in absolute value (a positive number)
  - You may have seen it defined as $MRS = -MU_C / MU_F$
  - The MRS at any consumption point is the slope of the tangent to the indifference curve at that point
- In words: MRS is the amount of F a consumer is willing to trade for one unit of C
  - That is, leaves the consumer on the same indifference curve (utility level remains constant)
  - It is the consumer's valuation of a unit of C –measured in units of F
  - The MRS captures the substitutability between C and F at the current consumption point

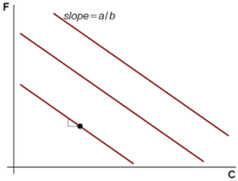
Marginal Rate of Substitution (Cont.)



- Further assumption on preferences: they are (weakly) convex
- Indifference curves are bowed out to the origin
- *MRS* is decreasing as consumption of C increases
- The more C is consumed, the less valuable it becomes relative to F

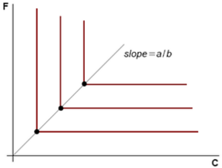
Example: Linear Preferences

- $U(D_C, D_F) = aD_C + bD_F$



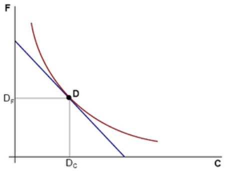
- Consumer is always indifferent between $\Delta D_C = b$ and $\Delta D_F = a$
- MRS is constant at a/b
- What does this imply about the substitutability of C and F?

Example: Leontief Preferences



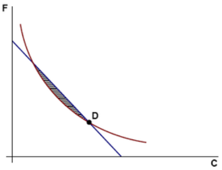
- $U(D_C, D_F) = \min \{aD_C, bD_F\}$
- Consumer always wants to consume b units of C with a units of F
- MRS is undefined
- What does this imply about the substitutability of C and F?

Utility Maximization



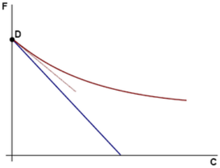
- At an interior optimum, $MRS = p_C / p_F$
- Whenever $MRS > p_C / p_F$, consumer wants to trade F for C
- Whenever $MRS < p_C / p_F$, consumer wants to trade C for F

Tangency of Budget Line and Indifference Curve at the Interior Optimum



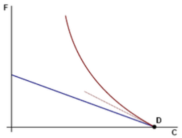
- Why is this a necessary condition?

Corner Solutions to Utility Maximization Problem



- $D_C = 0$ is an optimum if $MRS < p_C / p_F$ at that point. Why?
- Consumer wants to trade C for F, but there is no more C left to trade!

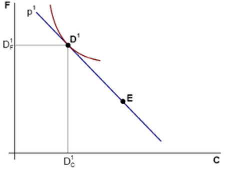
Corner Solutions to Utility Maximization Problem



- $D_F = 0$ is an optimum if $MRS > p_C / p_F$ at that point. Why?
- Consumer wants to trade F for C, but there is no more F left to trade!

Utility Maximization and Relative Demand

- Given preferences and endowment E, optimal (util. max) demand D can be calculated for any given relative price p_C / p_F



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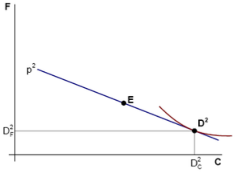

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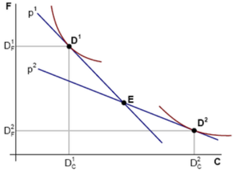
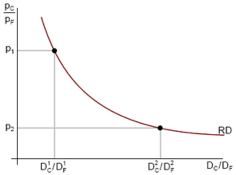


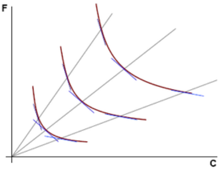
- This pattern of demand can be represented as a relative demand curve i.e. D_C/D_F as a function of p_C/p_F:



- In general, a relative demand curve (RD) will depend on the consumer's endowment point E

Homothetic Preferences

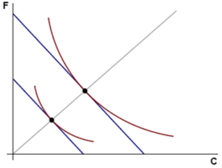
- Definition: MRS is constant along any ray from the origin



- A single indifference curve summarizes all the information about preferences

Important Property of Homothetic Preferences for Demand

- Changes in income are proportionally reflected in the optimal demand for all goods (holding prices fixed)



- This leads to some very important aggregation properties across consumers with different income levels

Special Examples of Homothetic Preferences

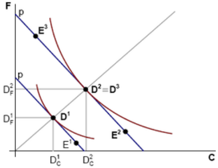
- Cobb-Douglas preferences: $U(D_C, D_F) = (D_C)^a (D_F)^b$ with $a, b > 0$
  - Consumer always spends a constant share of his/her income on both goods:

$$\frac{p_C D_C}{p_C D_C + p_F D_F} = \frac{a}{a+b} \text{ and } \frac{p_F D_F}{p_C D_C + p_F D_F} = \frac{b}{a+b}$$

- Linear preferences
- Leontief preferences

Homothetic Preferences and Relative Demand

- If consumers have the same homothetic preferences, then they will always consume the same relative amount of C and F –regardless of differences in their endowments



- Thus, the RD curve for any homothetic preferences is independent of the consumer's endowment

Aggregation Property of Homothetic Preferences

- Consider N consumers indexed by $i = 1..N$
- For each consumer i : $pD_C^i + D_F^i = pE_C^i + E_F^i$ (budget constraint) where $p = p_C / p_F$ is the relative price
- Now sum the budget constraints:

$$p \sum_{i=1}^{N} D_C^i + \sum_{i=1}^{N} D_F^i = p \sum_{i=1}^{N} E_C^i + \sum_{i=1}^{N} E_F^i \Leftrightarrow pD_C + D_F = pE_C + E_F$$

where $\mathbf{D} = (D_C, D_F)$ is aggregate demand and $\mathbf{E} = (E_C, E_F)$ is the aggregate endowment –over all N consumers

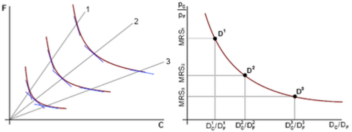
- Also, $D_C^i / D_F^i = RD(p)$ for all consumers i so this must also hold in the aggregate: $D_C / D_F = RD(p)$
  $\Rightarrow$ Aggregate demand is the same as if it were generated by a single consumer who owns the aggregate endowment E and shares the same homothetic preferences as the individual consumers

Aggregation Property of Homothetic Preferences (Cont.)

- Can capture all the properties of aggregate demand for a country by modeling the demand of a single consumer
- Furthermore, this aggregate demand is independent of the distribution of endowments (hence incomes) across consumers
- Important note: If the welfare of this aggregate consumer is increasing (or decreasing) then this will imply that overall welfare is also increasing (or decreasing)
    - But this does not mean that the welfare of all individual consumers is increasing (or decreasing)

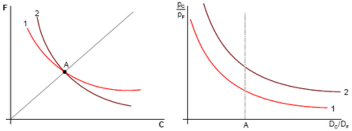
Homothetic Preferences and Relative Demand (Redux)

- Recall that any homothetic preferences can be exactly described by the associated relative demand curve (since it is independent of endowments)



Some Additional Examples

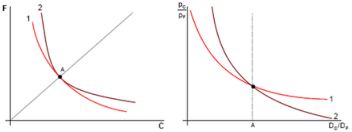
- Consider 2 consumers with different homothetic preferences (1 and 2):



- Who likes C relatively more?
- Consumer 2 does: at same p_C/p_F, he/she will always demand relatively more C ($D_C^1/D_F^1 < D_C^2/D_F^2$)

Some Additional Examples (cont.)

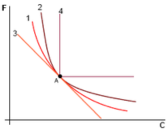
- Consider 2 consumers with different homothetic preferences (1 and 2):



- Who likes C relatively more? What is main difference between preferences?
- Consumer 1 considers C and F to be relatively closer substitutes (than consumer 2 does) –his/her demand is more elastic

Some Additional Examples

- Consider 4 consumers with different homothetic preferences (1-4):



- What are the relative demands?

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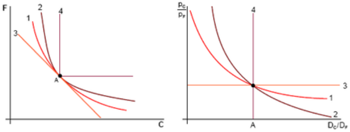

- What are the relative demands?