14.472 Spring 2004

Social Security in OLG models with certainty and fixed wage, w, and interest rate, r, inelastic labor

This analysis can be extended to the more complicated nonlinear aggregate production function by treating w and r as functions of k, rather than as parameters. The dynamics in the linear case are simpler since there is adjustment to a new steady state equilibrium in one period.

Note there are errors in my BPEA paper - equations 13, 28, and 30.

1 All workers fully rational

n ... population growth rate

Consumer choice, inelastic labor supply, no taxes, no social security

$$\text{Max} \quad u[c_1, c_2]$$

$$\text{s. t.} \quad c_1 + c_2/\left(1+r\right) = w \tag{1}$$

This gives optimal first period consumption: $c^*\left[w, r\right]$

The level of capital per young worker (of the next generation) comes from market clearance:

$$(1+n)\,k = w - c^*\,[w, r] \tag{2}$$

Now add payroll tax financed social security and a lump sum tax (used to finance the public debt). The budget constraint becomes

$$c_1 + c_2/(1+r) = w(1-t) - T + b/\,(1+r) \tag{3}$$

First period consumption becomes $c^*[w(1-t) - T + b/\,(1+r)\,, r]$

Given earmarking, we have government budget balance constraints in the non-social security and social security budgets. Assume government debt outstanding, no other government expenditures.

f=social security funding per young person paying taxes

g=debt per young person in the next cohort

Note that these have different denominators as ratios and so the dollar figures differ by a factor of $(1+n)$.

Lump-sum taxes need to cover the difference between the interest cost on the debt and the part covered by issuing more debt to preserve debt per worker:

$$T = (r - n)\, g \tag{4}$$

Financing for social security benefits come from payroll tax revenues and part of the interest earned by the trust fund:

$$b = tw\,(1+n) + (r-n)\, f \tag{5}$$

Assume that funding is financed by a fixed fraction of revenues:

$$f = \beta tw \tag{6}$$

Then, the social security budget constraint becomes:

$$b = tw\,(1+n) + (r-n)\,\beta tw = tw(1 + (1-\beta)n + \beta r) \tag{7}$$

With these benefits, lifetime income, I, is

$$
\begin{aligned}
I &= w\,(1-t) + b/\,(1+r) \\
&= w\,[(1-t) + t\,(1+n)\,/\,(1+r)] + (r-n)\,f/\,(1+r) \\
&= w - \left(\frac{r-n}{1+r}\right)[tw - f] = w - \left(\frac{r-n}{1+r}\right)(1-\beta)tw \qquad (8)
\end{aligned}
$$

If we have full funding, $\beta = 1$, and $b = tw(1+r)$.

Market clearance

$$
(1+n)\,(k+g) = w\,(1-t) - T - c^*[w(1-t) - T + b/\,(1+r)\,, r] + f \qquad (9)
$$

2 α rational savers $1-\alpha$ 0-savers

Market clearance is now

$$(1+n)(k+g) = \alpha\{w(1-t) - c^*[I,r]\} + f \tag{10}$$

Can change f by a one-time tax surcharge and then preserving the new f (new β). Can do this with lower t or higher b.

Increasing t starting at time zero helps the current elderly if $\beta < 1$ (marginal response only relevant part).

Increasing t hurts steady state rationals when $r > n$ since $dI/dt = (r-n)(1-\beta)w/(1+r)$

Increasing t helps steady state irrationals if the system is not "too large" since

$$\begin{aligned} dc_1/dt &= -w \\ dc_2/dt &= (1 + r\beta + n(1-\beta))w \end{aligned} \tag{11}$$

which can be evaluated using the MRS.

Capital may go up or down

$$(1+n)\,\frac{dk}{dt} = \alpha\{-w + c_I^*\left(\frac{r-n}{1+r}\right)w\,(1-\beta)\} + \beta w \tag{12}$$

3 Changing debt in the presence of interest income taxes

Individual budget constraint

$$c_1 + c_2/\left(1 + r\left(1 - \tau\right)\right) = w - T \tag{13}$$

First period consumption $\quad c^*\left[w - T, r\left(1 - \tau\right)\right]$.

Assume normality: $0 < c_I^* < 1$

Government budget balance

$$\begin{aligned} T &= \left(r - n\right) g - \tau r\left(k + g\right) \\ &= \left(r\left(1 - \tau\right) - n\right) g - r\tau k \end{aligned}$$

Market clearance

$$\left(1 + n\right) k = w - T - c^*\left[w - T, r\left(1 - \tau\right)\right] - g\left(1 + n\right) \tag{14}$$

Increase debt, **w, r given**. Note

$$dT/dg = (r(1-\tau) - n) - r\tau\frac{dk}{dg} = (r-n) - r\tau\left(1 + \frac{dk}{dg}\right) \tag{15}$$

Substituting in market clearance

$$\begin{aligned}(1+n)\,k &= w - (r(1-\tau) - n)\,g + \tau r k \\ &\quad - c^*\left[w - (r(1-\tau) - n)\,g + r\tau k, r(1-\tau)\right] - g(1+n)\end{aligned} \tag{16}$$

Differentiating,

$$dk/dg = -\frac{1 + n + (1 - c_I^*)(r(1-\tau) - n)}{1 + n - (1 - c_I^*)\,r\tau} \tag{17}$$

For $n \leq r(1-\tau)$ $\quad dk/dg < -1$ $\quad$ - a bigger effect.

For $r(1-\tau) \leq n \leq r$ $\quad dk/dg < -1$ $\quad$ when $(1+n > (1-c_I^*)\,r\tau)$

Alternative technology: **k given** - fixed coefficients

Instead of r and w fixed, k endogenous, w and r satisfy

$$y = w + rk \tag{18}$$

This can be used to eliminate w from the analysis.

Differentiating , we have

$$dT/dg = (r(1-\tau) - n) + \{(1-\tau)g - \tau k\}\frac{dr}{dg} \tag{19}$$

Note that g can be larger or smaller than $\tau(k+g)$. We will give a sufficient condition for $\frac{dr}{dg} < 0$

Market clearance

$$(1+n)(k+g) = y - rk - T - c^*[y - rk - T, r(1-\tau)] \tag{20}$$

Note that

$$y - rk - T = y - r\,(1-t)\,k - (r\,(1-\tau) - n)\,g \tag{21}$$

Differentiating market clearance:

$$dr/dg = -\frac{1 + n + (r\,(1-\tau) - n)\,(1 - c_I^*)}{(k+g)\,(1-\tau)\,(1-c_I^*) + (1-\tau)\,c_i^*} \tag{22}$$

If $c_i^* = 0$

12.

$$dr/dg = -\frac{1 + nc_I^* + r\,(1-\tau)\,(1-c_I^*)}{(1-\tau)\,(k+g)\,(1-c_I^*)} < 0 \tag{23}$$

To get more savings the wage must go up, lowering r