Assumptions:

Expected utility of additive preferences with no age variation in period utilities and no other risks

No utility from bequests.

Lifetime utility no annuities, constant interest rate

$$V^{h0}[W] = \text{Max} \sum_{s=1}^{L} \pi_s^h \delta^{s-1} u[c_s]$$

$$\text{s. t. } \sum_{s=1}^{L} R^{s-1} c_s = W \tag{1}$$

FOC:

$$\pi_s^h \delta^{s-1} u'[c_s] = \lambda R^{s-1} \tag{2}$$

Lifetime period-annuities arbitrary prices

$$V^{h1}[W] = \text{Max} \sum_{s=1}^{L} \pi_s^h \delta^{s-1} u[c_s]$$

$$\text{s. t. } \sum_{s=1}^{L} p_s c_s = W \tag{3}$$

FOC:

$$\pi_s^h \delta^{s-1} u'[c_s] = \lambda p_s \tag{4}$$

If no administrative costs (actuarially fair).

$$V^{h2}[W] = \text{Max} \sum_{s=1}^{L} \pi_s^h \delta^{s-1} u[c_s]$$

$$\text{s. t.} \sum_{s=1}^{L} \pi_s^h R^{s-1} c_s = W \tag{5}$$

FOC:

$$\delta^{s-1} u'[c_s] = \lambda R^{s-1} \tag{6}$$

Value of annuitization:

$$V^{h0}[W] = V^{hi}[\theta W] \tag{7}$$

Timing of annuitization
Fully annuitize at start

$$V^{h1}[W] = \text{Max} \sum_{s=1}^{L} \pi_s^h \delta^{s-1} u[c_s] \tag{8}$$

$$\text{s. t. } \sum_{s=1}^{L} p_s c_s = W$$

FOC:

$$\pi_s^h \delta^{s-1} u'[c_s] = \lambda p_s \tag{9}$$

Wait one period and then annuitize, recognizing that there may be learning about survival probabilities and pricing may vary with risk classification. Assume the discount rate does not change with the news.

$$V^{h3}[W] = \text{Max } \pi_1^h u[c_1] + \alpha \sum_{s=2}^{L} \pi_s^h \delta^{s-1} u[c'_s] + (1-\alpha) \sum_{s=2}^{L} \pi_s^h \delta^{s-1} u[c''_s]$$

$$\text{s. t. } c_1 + \sum_{s=2}^{L} p'_s c'_s = W \tag{10}$$

$$c_1 + \sum_{s=2}^{L} p''_s c''_s = W$$

FOC:

$$\pi_1^h u'[c_1] = \lambda' + \lambda''$$
$$\pi_s^h \delta^{s-1} u'[c'_s] = \lambda' p'_s \qquad \text{for } s = 2, 3, \ldots L \tag{11}$$
$$\pi_s^h \delta^{s-1} u'[c''_s] = \lambda'' p''_s \qquad \text{for } s = 2, 3, \ldots L$$