1. Shocks

1.1. VARs, Wold representations and their limits

A brief review of VARs.
Assume a true model, in MA form:

$$\begin{aligned} X &= A_0\ e + A_1\ e(-1) + A_2\ e(-2) + ...; \qquad E(ee') = I \\ &= (A_0 + A_1\ L + A_2\ L^2 + ...)\ e \\ &= A(L)\ e \end{aligned}$$

The "e's" are the shocks, $A(L)$ the propagation mechanism.

Estimate a VAR, rewritten in MA form:

$$\begin{aligned} X &= u + C_1\ u(-1) + C_2\ u(-2) + ...; \qquad E(uu') = \Omega \\ &= (I + C_1\ L + C_2\ L^2 + ...)\ u \\ &= C(L)\ u \end{aligned}$$

- Relation of true shocks to VAR residuals?

$$u = A_0 \ e$$

- Identification of A_0? $A_0 \ A_0' = \Omega$: not enough.

- In 2x2 case, 3 moments in Ω, 4 parameters in A_0. need for one more: Zeros, or short-run, or long-run restrictions.

- Once have A_0, construct

$$A(L) = C(L) \ A_0$$

- Once have $A(L)$, can derive impulse responses, and variance decompositions.

What can go wrong? Many things...

- Underlying true model is non-linear.
- Underlying true model is linear but MA is non-fundamental.
- Underlying true model is linear but corresponding VAR has very long lags.
- Underlying true model is linear but more shocks than included variables. Left to later
- Underlying true model is linear, but get A_0 wrong. Left to later

1. The true model is non-linear

Not all is lost: The Wold decomposition theorem.

Recall definition of covariance stationarity: Let X_t be a random variable or a vector of random variables. Then, X_t is covariance stationary iff:

$$EX_t = \mu \text{ for all } t; \; E(X_t - \mu)(X_{t-k} - \mu) = g_k \text{ for all } t$$

If Y_t is covariance stationary, then it can be represented by a Wold decomposition (an infinite MA representation):

$$Y_t = B(L)\epsilon_t + k(t)$$

where $E\epsilon = 0$, $E\epsilon^2 = \sigma^2$ and $k(t)$ is a deterministic component (Think mean).

Limits: covariance stationarity? Studying the Great Depression, hyperinflations.
Relation of this representation to true underlying process and shocks? Some examples.

Example 1. Markov process

Assume true process is given by 2-state Markov (0, 1) process, with transition probabilities:

$$P(S_t = 1|S_{t-1} = 1) = p, \quad P(S_t = 0|S_{t-1} = 0) = q$$

Check that this process has the following AR(1) representation:

$$S_t = (1 - q) + (p + q - 1)S_{t-1} + \epsilon_t$$

where, conditional on $S_{t-1} = 1$,
$\epsilon_t = (1 - p)$ with probability p, $\quad = -p$ with probability $1 - p$
and conditional on $S_{t-1} = 0$,
$\epsilon_t = -(1 - q)$ with probability q, $\quad = q$ with probability $1 - q$

We can estimate this process and get "shocks". But this gives us a poor understanding of the underlying process.

Example 2. Caplin and Spulber

Consider price setter, with desired price $p^*_{it} = m_t$, actual price p_{it}, non decreasing process for money, and one-sided Ss rule:

- If $p_i > p^*_i + s \quad (s < 0)$, no adjustment
- If $p_i < p^*_i + s$, then adjust to $p^*_i + S$

Then, using the ergodic distribution:

$$E[\Delta p_i] = \Delta m$$

$$E[\Delta p_i | \Delta m, \Delta m_i(-1), ...] = \Delta m$$

So, non linear underlying process (with "stickiness"), but no lags in the regression of Δp_i on Δm. (For much more, see Caballero Engel 2006, and 2004)

Not the (second equation) of the Wold representation however. What about $E[\Delta p_i | \Delta m, \Delta m_i(-1), ..., \Delta p_i(-1), \Delta p_i(-2), ...$?

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2. The true process is non fundamental

Example 1 (from BQ 1993). Estimate process for x_t and find it to be white noise:

$$x_t = u_t$$

Is the true process white noise? Are u_t's the true shocks?

- Not necessarily. True process could be:

$$x_t = \frac{1-\lambda L}{1-\lambda^{-1}L}\, e_t, \quad |\lambda| > 1$$

equivalently

$$x_t = e_t + (1-\lambda^2)\sum_{i\geq 1} \lambda^{-i} e_{t-i}$$

Note: $E(x_t\ x_{t-i}) = 0.$ Exotic?

More examples

1. Hump shaped response of x to a shock (not all hump shapes are non fundamental...) :

True process:

$$x_t = e_t + \theta e_{t-1}, \quad \theta > 1$$

Wold representation:

$$x_t = u_t + \theta^{-1} u_{t-1}$$

where $V(u) = \theta^2 V(e)$ Again, very different impulse responses. Problem arises because true process "non fundamental"

Process "fundamental" if all roots of the MA polynomial are on or outside circle. "Invertible" if all roots are strictly outside circle.

2. Consumption example from Villaverde et al (a bit contrived. A more realistic, but more complex one, in BQ 1993)

Optimizing linear-quadratic consumers, with $\beta = R^{-1} < 1$.
Labor income is white noise:

$$y_t = e_t$$

Consumption is given by a random walk, with shock equal to the annuity value of the change in the PDV of labor income:

$$c_t = c_{t-1} + (1 - R^{-1})e_t$$

Suppose we only observe $s_t \equiv y_t - c_t$, i.e. saving out of labor income (different from saving). Then:

$$s_t - s_{t-1} = R^{-1}e_t - e_{t-1}$$

So non fundamental. Estimate:

$$s_t - s_{t-1} = u_t - R^{-1}u_{t-1}$$

- True process: $dEPDV(s)/de = 0$
- Representation: $dEPDV(s)/du = 1 - R^{-1} > 0$

3. Long lags in the VAR

Chari-Kehoe-McGrattan attack on VAR: Mostly off the mark: If stationarity assumptions about variables, or identification restrictions used to get A_0 are wrong, then implications are false. But one potentially correct and relevant point:

For a class of models, VAR representation may require very long lags.
Example: Simplified RBC:

$$y_t = k_t + e_t; \quad k_t = \rho\, k_{t-1} + \alpha\, e_t$$

where e_t is a productivity shock, assumed white noise, α is small, and ρ is close to one.
Interpretation: Productivity shock with large contemporaneous effect, then smaller, but long lasting effect through capital.

If we observe both output and capital, no problem.

Suppose we only observe output. Then:

$$(\frac{1-\rho L}{1+\alpha-\rho L})\ y_t = e_t$$

or

$$y_t = \frac{\alpha}{1+\alpha}\sum_{i\geq 1}(\frac{\rho}{1+\alpha})^i\ y_{t-i} + e_t$$

If α is small, and ρ close to one, long lags. If not careful, will drop them when estimating VAR.

- Will be more of an issue if identifying structural VAR from long-run restrictions.
- How much of an issue? Depends on class of models. More relevant in standard RBC models than in NK models.
- A natural solution: Include capital (slow moving state variable).