14.462
Problem set 4

Consider the following model

$$p_t = \alpha p_{t+1}^e + \varepsilon_t$$

Where p_t is the endogenous variable (say an asset price), p_{t+1}^e its expected value, and ε_t an iid shock with zero mean and variance σ^2, which is not known a period ahead.

1. What is the rational expectations equilibrium (REE)?

Assume that instead of rational expectations, we have a learning process defined by

$$p_{t+1}^e = p_t^e + \frac{1}{g_t}(p_{t-1} - p_t^e),$$

where g_t is a deterministic sequence of strictly positive numbers/

We will say that the learning process converges to the REE if

$$\lim_{t \to +\infty} E(p_{t+1}^e - E_{t,REE}p_{t+1})^2 = 0,$$

where E is the unconditional expectations operator, and $E_{t,REE}p_{t+1}$ is the value of p_{t+1}^e in the rational expectations equilibrium, given the history of shocks.

2. Show that a necessary and sufficient condition for convergence to the REE is

$$\lim_{t \longrightarrow +\infty} m_t = 0, \tag{1}$$

where

$$m_t = \frac{1}{g_t^2} + \sum_{s=1}^{t} \frac{h_s^2 ... h_t^2}{g_{s-1}^2},$$

and

$$h_t = 1 + (\alpha - 1)/g_t.$$

3. Show that m_t satisfies

$$m_{t+1} = h_{t+1}^2 m_t + 1/g_{t+1}^2 \tag{2}$$

4.Show that for (1) to hold it must be that

$$\alpha < 1$$

$$\lim g_t = +\infty$$

and

$$\lim \prod_{s=0}^{t} h_t = 0$$

5. Show that the latter condition is asymptotically equivalent to

$$\sum_{0}^{+\infty} 1/g_t = +\infty.$$

6. Does the learning process converge to the REE is g_t is a constant? If $g_t = 1/(t+1)^2$?

7. Assume $g_t = 1/(t+1)$. Show that (1) is satisfied (hint: use (2) and look for $\beta > 0$ such that one can show by induction that $m_t < At^{-\beta}$ for t large enough).

8. Intuitively, what are the problems if g_t converges "too fast" or "too slowly" to infinity?