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14.462 Spring 2004
Problem set 1

When does it pay to use two technologies at the same time?

Assume that output can be produced using two technologies, labelled by 1 and 2. The production functions are given by

$$Y_1 = K_1^{\alpha} L_1^{1-\alpha}$$

$$Y_2 = K_2^{\beta} L_2^{1-\beta}$$

1. Show that if the two technologies are used, the capital labor ratio in technology 2 must be equal to

$$k_2 = \frac{K_2}{L_2} = \left(\frac{\alpha}{\beta}\right)^{\frac{\alpha}{\beta-\alpha}} \left(\frac{1-\alpha}{1-\beta}\right)^{\frac{1-\alpha}{\beta-\alpha}}$$

$$k_1 = \frac{K_1}{L_1} = \left(\frac{\beta}{\alpha}\right)^{\frac{\beta}{\alpha-\beta}} \left(\frac{1-\beta}{1-\alpha}\right)^{\frac{1-\beta}{\alpha-\beta}}$$

Show that this entirely determines factor prices and the allocation of labor and capital between the two technologies.

2. Show that if $\alpha = 1/3$, $\beta = 2/3$, total capital is $K$ and total labor is $L$, then

$$\begin{aligned} k_2 &= 2 \\ k_1 &= 1/2 \\ L_2 &= \frac{2K - L}{3} \\ L_1 &= \frac{4L - 2K}{3}. \end{aligned}$$

3. Between which bounds must the aggregate $K/L$ lie for the two technologies to be used in equilibrium?

4. Show that if $K/L$ is between these two bounds, then they will actually be used in equilibrium.

5. Can you explain what is going on in the $(K, L)$ plane?