14.454 - Macroeconomics 4
Notes on Diamond-Dygvig Model and Jacklin

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1 Setup

- Continuum of consumers, mass 1 of individuals each endowed with one unit of currency.

- $t = 0, 1, 2$

- At $t = 0$, individuals can either invest in short-run project with return equal to 1, or invest in a long-run project that yields a return $R > 1$ at $t = 2$

- If liquidate the long-run project at $t = 1$, return is $L < 1$ only

- At $t = 1$, fraction π of individuals gets liquidity shock and only value consumption at $t = 1$. The remaining fraction $1 - \pi$ is patient and only values consumption at $t = 2$

- Ex-ante expected utility is

$$U = \pi u \left(c_1^I \right) + (1 - \pi) u \left(c_2^P \right)$$

  where c_1^I is consumption in period 1 if impatient and c_2^P is consumption in period 2 if patient.

- In general, we can think of utility being $U = \mathbb{E}_\theta \left(u \left(c_1, c_2, \theta \right) \right)$ for some "taste shock" θ. In this case:

$$u \left(c_1, c_2, \theta \right) = \theta u \left(c_1 \right) + (1 - \theta) u \left(c_2 \right)$$

  where $\theta \in \{0, 1\}$ and $\theta = 1 \iff$ type is impatient

- Consumers always have access to storage technology.
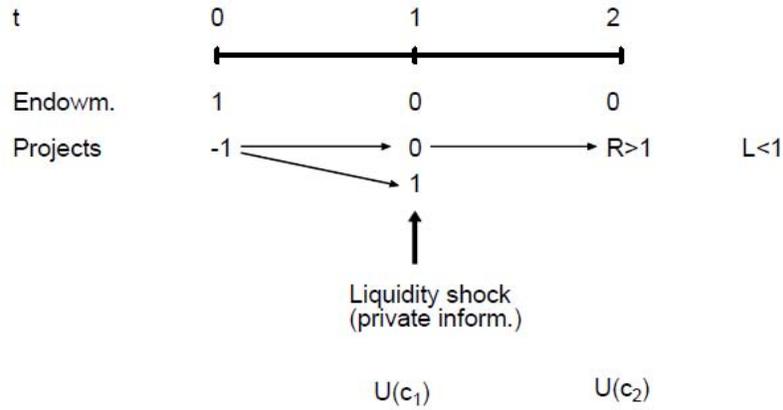
Figure 1: Timeline of model

2 Warm up: Equilibrium with ex-post trades at $t = 1$

Suppose that at $t = 1$ a market opens for bonds that pay 1 unit of consumption good c_2 at $t = 2$ per unit invested at time 1. Budget constraints are then

- $p_2 =$ Price of time $t = 2$ consumption good at $t = 0$

- $p_1 =$ Price of time $t = 1$ consumption good at $t = 0$.

- $q_t =$ Price of claims of time t consumption goods at $t-1$. Normalize $q_1 = 1$

- $y_0 =$ investment in long technology, at $t = 0$

- $x_t =$ investment in storage technology, at time t

- $B_t =$ net purchases (or sales if $B_t < 0$) of claims for consumption goods at time t

- $c_t =$ consumption at time t

Now, because of the linearity of the technology, we have that

$$\text{Profits}_{Long}\left(y\right) = p_2 R y - y = \left(p_2 R - 1\right) y$$

and

$$\text{Profits}_{storage}\left(x_0\right) = p_1 x - x = \left(p_1 - 1\right) x_0$$

So $p_1 = 1$ and $p_2 = \frac{1}{R}$

The budget constraints are then

$$x_0 + y_0 + B_1 \leq 1$$
$$c_1 + q_1 B_2 \leq x_0 + B_1$$
$$c_2 \leq R y_0 + B_2$$

We will show that $q_1 = \frac{1}{R}$ in equilibrium.

- Suppose $q_1 < \frac{1}{R} \iff \frac{1}{q_1} > R \implies$ I want to invest all my endowment in storage, since the return of the following strategy gives higher returns:

  If I want to consume at $t = 2$:

  $$1 \text{ unit storage from } t = 0 \text{ to } t = 1 \implies \frac{1}{q_1} \text{ unit in bonds from } t = 1 \text{ to } t = 2$$
  $$\implies \frac{1}{q_1} \text{ consumption at } t = 2$$

  Then, rate of return is $\frac{1}{p} > R$, so in equilibrium $y = 0$. Moreover, if agents can get indebted in the first period, then they would get an infinite demand for storage in the first period, which is not part of an equilibrium.

  On the other hand, if I want to consume at $t = 1$, then since $q_1 < 1$ it is already better than long technology

- Suppose that $q_1 > \frac{1}{R} \implies$ I want to invest all my endowment (and all the debt I can get) in storage. This is because

  If I want to consume at $t = 2 \implies R > \frac{1}{q_1}$ (alternative of storing). Moreover, if I want to consume at $t = 1$, instead of storage I can invest 1 dollar in long technology, then sell $\frac{1}{q_1}$ claims over R units I will get with this. Because $q_1 R > 1$ this is a better alternative.

Therefore, we must have $q_2 = \frac{1}{R}$ in equilibrium. Moreover, in equilibrium we must have $B_1 = 0$

2.1 Supply of bonds for "Impatient" types $(\theta = 1)$

Now, since they have been hit by the liquidity shock, they can either liquidate the long asset (and get Ly) or promise to pay B units tomorrow, in exchange of pB consumption goods today $(t = 1)$. Because the consumer does not derive any utility from period 2 consumption, as long as $p > L$ we will have that

$$B_1^I = -Ry_0$$

$$c_1^I = x_0 - q_1 B_1 = x_0 + q_1 R y_0$$

2.2 Demand of bonds for "Patient" types $(\theta = 0)$

Because patient types only derive utility from consumption of period 2, either they invest their savings again in the storage technology (which renders a return of 1) or rather use it to buy B bonds at $t = 1$ at a price of q_1 from impatient consumers, and get B units at $t = 2$ As long as $q_1 < 1$ (so it is better to buy these bonds than saving in the storage technology) we must have that

$$B_1^P = \frac{x_0}{q_1}$$

$$c_1^P = 0, c_2^P = Ry_0 + \frac{x_0}{q_1}$$

2.3 Equilibrium in claims market

In equilibrium we must have

$$\pi B^I + (1 - \pi) B^P = 0 \iff -\pi R y_0 + (1 - \pi) \frac{x_0}{q_1} = 0 \iff$$

$$q_1 = \left(\frac{1 - \pi}{\pi}\right)\left(\frac{x_0}{Ry_0}\right)$$

Now, since $q_1 = \frac{1}{R}$ and $x_0 + y_0 = 1$ in equilibrium, we must have

$$\frac{1}{R} = \left(\frac{1 - \pi}{\pi}\right)\frac{x_0}{R(1 - x_0)} \iff \left(\frac{1 - \pi}{\pi}\right)\frac{x}{(1 - x)} = 1 \iff x_0 = \pi$$

and therefore, $y_0 = 1 - \pi$

So equilibrium consumption for each type is

$$c_1^I = x_0 + q_1 R y_0 = \pi + \frac{1}{R} R (1 - \pi) = 1$$

$$
\begin{aligned}
c_2^I &= 0 \\
c_1^P &= 0
\end{aligned}
$$

$$c_2^P = R y_0 + \frac{x_0}{q_1} = R (1 - \pi) + R \pi = R$$

2.4 What if different preferences?

Prices are still $q_1 = \frac{1}{R}$ and the rest of the prices are still satisfied. We get a demand function $B_2 (\theta, q_1)$ given by

$$
\begin{aligned}
V (\theta, x_0, y_0) &= \max_{c_1(.), c_2(.), B(.)} u (c_1, c_2, \theta) \\
s.t \quad &: \quad \left\{ \begin{array}{l} c_1 + \frac{1}{R} B_2 \leq x_0 \\ c_2 \leq R y_0 + B_2 \end{array} \right.
\end{aligned}
$$

which will give a demand for bonds $B_2 (\theta, x_0, y_0)$. Then, find x_0 to satisfy equations:

$$
\begin{aligned}
\int B (\theta, x_0, y_0) &= 0 \\
x_0 + y_0 &= 0
\end{aligned}
$$

3 Social Optimum

The social optimum solves

$$\max_{c_1, c_2, x, y} \pi u (c_1) + (1 - \pi) u (c_2)$$

$$
s.t : \left\{ \begin{array}{c} \pi c_1 \leq x \\ (1 - \pi) c_2 \leq R y \\ x + y = 1, x, y \geq 0 \end{array} \right.
$$

Or, simplifying it:

$$\max_{x \in [0,1]} \pi u\left(\frac{x}{\pi}\right) + (1 - \pi) u\left(R\frac{1 - x}{1 - \pi}\right)$$

which yields FOC:

$$u'\left(\frac{x}{\pi}\right) = Ru'\left(R\frac{1 - x}{1 - \pi}\right) \tag{1}$$

Optimal consumption levels come from constraints:

$$x^* = \pi c_1^* \iff c_1^* = \frac{x^*}{\pi} \tag{2}$$

$$y^* = \frac{(1 - \pi) c_2^*}{R} \iff c_2^* = \frac{Ry^*}{1 - \pi} = R\frac{1 - \pi c_1^*}{1 - \pi} \tag{3}$$

And using this into (1) we get

$$u'(c_1^*) = Ru'(c_2^*) = Ru'\left(R\frac{1 - \pi c_1^*}{1 - \pi}\right)$$

(which, as a function of c_1^* is decreasing on LHS, increasing on RHS).

Note that since $R > 1$ we must have

$$u'(c_1^*) > u'(c_2^*) \iff c_1^* < c_2^*$$

Then, even if information about shock is private, the allocation is incentive compatible (i.e. late consumers will not want to behave as if they were early consumers). Note that if $c_2^* < c_1^* \implies$ patient types would pretend to be impatient types and store c_1^* units until $t = 2$, and get higher utility than telling the truth.

Also, note that unless $u = \ln(x)$ the equilibrium $c_1^* = 1, c_2^* = R$ needs not be optimal, since

$$u'(1) \neq Ru'(R)$$

in general. Moreover

$$Ru'(R) = u'(1) + \int_1^R \frac{d[su'(s)]}{ds} ds =$$

$$u'(1) + \int_1^R [su''(s) + u'(s)] ds = u'(1) + \int_1^R u'(s) \left[\underbrace{\frac{su''(s)}{u'(s)}}_{\equiv -\sigma(s)} + 1\right] ds =$$

6

$$u'(1) + \int_1^R u'(s)\left[1 - \sigma(s)\right] ds$$

if $\sigma(s) > 1$ for all s (as authors assume), then

$$Ru'(R) = u'(1) + \int_1^R u'(s)\left[1 - \sigma(s)\right] ds < u'(1)$$

Then $c_1^* > 1$ in the optimum (because LHS is decreasing and RHS increasing) and therefore $c_2^* < R$

3.1 With General preferences:

The mechanism design problem is:

$$\max_{c_1(.),c_2(.),x,y} \int u\left(c_1(\theta), c_2(\theta), \theta\right) dF(\theta)$$

$$s.t: \begin{cases} \int c_1(\theta)\, dF(\theta) \leq x \\ \int c_2(\theta)\, dF(\theta) \leq Ry \\ x + y \leq 1 \\ u\left(c_1(\theta), c_2(\theta), \theta\right) \geq u\left(c_1\left(\widetilde{\theta}\right), c_2\left(\widetilde{\theta}\right), \theta\right) \text{ for all } \theta, \widetilde{\theta} \end{cases}$$

4 Implementing Social Optima

4.1 Classical Banks

The idea is that we make agents play a game: at $t = 0$ they decide whether to pay one unit of their endowment to the bank. In return, the bank offers a deposit contract (c_1, c_2) per unit deposited. The idea is to make all agents eat c_1^* and c_2^*, with associated investments

$$x^* = \pi c_1^*, y^* = \frac{(1 - \pi) c_2^*}{R} \tag{4}$$

In this game, **ex-post trading will be prohibited** (this will be a really important restriction, as we will see). Because θ is private information, agents then self select between the two possible contracts at $t = 1$: either get $(c_1^*, 0)$ (i.e. the optimal contract for impatient types) or $(0, c_2^*)$. If a big enough fraction of the agents decides to go the bank, however, there will not be enough assets

to cover both contracts. In that case, long term assets will be liquidated so as to pay "early birds", which will also result in a reduction in payoffs at $t = 2$ as well. If resources are not enough to pay c_1^* to each agent, then the bank goes bankrupt and pays pro-rata.

The game induced by the contracts is as follows: at $t = 1$ agent i chooses whether to report $r_i \in \{$"Patient", "Impatient"$\} = \{0, 1\}$. Define

$$f = \int_0^1 r_i di$$

as the fraction of the population that reports being impatient. Payoffs are as follows:

$$U_i\left(r_i = 1, f, \text{"Impatient"}\right) = \begin{cases} u\left(c_1^*\right) \text{ if } f \le \overline{f} \equiv \pi + \frac{1-\pi}{\pi} \frac{c_2^*}{c_1^*} \frac{L}{R} \\ u\left(\frac{f^*}{f}\right) \text{ if } f > \overline{f} \end{cases}$$

$$U_i\left(r_i = 0, f, \text{"Impatient"}\right) = u\left(0\right)$$

$\overline{f}$ is the threshold fraction of the population such that if $f \le \overline{f}$ there are still enough resources (by liquidating some or all of the long term assets) to pay each consumer that goes to the bank c_1^*. It is defined by the constraint

$$c_1^* f \le x + Ly = \pi c_1^* + L\left(\frac{1-\pi}{R} c_2^*\right) \iff f \le \pi + \frac{1-\pi}{\pi} \frac{c_2^*}{c_1^*} \frac{L}{R} \equiv \overline{f}$$

If the consumer is patient, then even if she does not derive utility she may derive utility from going to the bank at $t = 1$, since she can store the consumption to $t = 2$ (although at a lower return, so it is not a dominating strategy). Payoffs are

$$U_i\left(r_i = 1, f, \text{"Patient"}\right) = \begin{cases} u\left(c_1^*\right) \text{ if } f \le \overline{f} \\ u\left(\frac{f^*}{f}\right) \text{ if } f > \overline{f} \end{cases}$$

$$U_i\left(r_i = 0, f, \text{"Patient"}\right) = \begin{cases} u\left(c_2^*\right) \text{ if } f \le \underline{f} \equiv \pi \\ u\left(\frac{R(1-c_1^* f)}{1-f}\right) \text{ if } f \in \left[\underline{f}, \overline{f}\right] \\ u\left(0\right) \text{ if } f > \overline{f} \end{cases}$$

In any equilibrium, no matter what f is, impatient agents will go to the bank, so we always have $f \ge \pi$. What is the best response of patient agents?

$$\text{if } f < \underline{f} \implies r_i = 0 \text{ is optimal}$$

$$\text{if } f \in \left[\underline{f}, \overline{f}\right] \Longrightarrow \begin{cases} \text{if } \frac{R(1-c_1^* f)}{1-f} > c_1^* \Longleftrightarrow f < f^* \text{ then } r_i = 0 \text{ optimal} \\ \text{if } f \geq f^* \text{ then } r_i = 1 \text{ optimal} \end{cases}$$

where f^* is defined by inequality

$$\frac{R\left(1 - c_1^* f\right)}{1 - f} > c_1^* \Longleftrightarrow R - Rc_1^* f > c_1^* - fc_1^* \Longleftrightarrow$$

$$R - c_1^* > fc_1^*\left(R - 1\right) \Longleftrightarrow f < \frac{R - c_1^*}{\left(R - 1\right)c_1^*} \equiv f^*$$

Two equilibria:

- If Patient types expect $f < f^* \Longrightarrow$ all patient agents will choose $r_i = 0 \Longrightarrow$

$$f = \int r_i d_i = \pi < f^*$$

  and hence only impatient agents choose $r_i = 1$. Therefore, consumption for each agent is

$$\begin{aligned} c_1^I &= c_1^*, c_2^I = 0 \\ c_1^P &= 0, c_2^P = c_2^* \end{aligned}$$

  implementing the social optimum!

- If Patient types expect $f > f^* \Longrightarrow$ all patient agents will choose $r_i = 1 \Longrightarrow$

$$f = 1 > f^*$$

  and hence all agents choose $r_i = 1 \Longrightarrow$ Bank run!!

Problem: Nash equilibrium implementation of mechanisms has the feature of multiplicity of equilibria (so we cannot know for sure if we will implement the equilibrium or not). Better strategies for implementation are "Dominant strategy implementation", in which one makes sure that not only it is an equilibrium, but rather that it is an optimal strategy to report own type regardless of strategies of other agents. A way of doing this is by "suspension of convertibility"

4.2 Suspension of Convertibility

Suppose now that on top of the previous contract, the bank stipulates the following rule: if more than $\widehat{f}$ fraction of the population comes to ask for deposits, then the bank pays c_1^* to only $\widehat{f}$ of them (selected randomly), and nothing to the other guys.

See that if bank sets $\widehat{f}$, then there's a floor on the fraction of the long asset that there will be liquidated. In particular, if $\widehat{f} = \pi$, then $f \leq \pi$ and the payoffs for the Patient agents are:

$$
\begin{aligned}
U_i \left(r_i = 1, f, "\text{Patient}" \right) &= \begin{cases} u\left(c_1^*\right) \text{ if } f \leq \pi \\ \frac{\pi}{f} u\left(c_1^*\right) < u\left(c_1^*\right) \text{ if } f > \pi \end{cases} \\
U_i \left(r_i = 0, f, "\text{Patient}" \right) &= u\left(c_2^*\right)
\end{aligned}
$$

Since $c_2^* > c_1^* \implies r_i = 0$ is a dominant strategy for any Patient type. Therefore, this game implements the social optimum in dominant strategies.

4.3 Equity Shares (Jacklin)

Suppose now that instead of a bank, there is a single firm that can raise an amount of capital K by issuing shares (at a price of 1) at $t = 0$. Once bought, shareholders decide (unanimously) how much the firm will pay in dividends $D > 0$ at $t = 1$. Therefore, the payoff stream of equity shares is:

- At $t = 0$, $-K$ (invest endowment at a price of 1)

- At $t = 1$, get D

- At $t = 2$, get $R\left(K - D\right)$

We will look at equilibria in which $K = 1$. At $t = 1$, shareholders receive their dividends and a market in the ex-dividend shares opens. Let z be the price of equity shares of the firm at $t = 1$. We will see that if we set

$$D = \pi c_1^*$$

then the equilibrium will implement the social optimum.

Impatient types:

These types consume the dividend and sell their equity shares at a price of z to patient types. Therefore, their consumption at $t = 1$ is

$$c_1 = D + z$$

Patient types:

These types use their dividends to buy the equity shares sold by impatient types. They can buy at most $\frac{D}{z}$ shares, so at $t = 2$ they get

$$c_2 = \left(1 + \frac{D}{z}\right) R (1 - D)$$

In equilibrium in the shares market, we must have that

$$\pi = (1 - \pi) \frac{D}{z} \iff z = \frac{1 - \pi}{\pi} D$$

and hence equilibrium consumption is

$$c_1^I = D + \frac{1 - \pi}{\pi} D = \frac{1}{\pi} D, c_2^I = 0$$

$$c_1^P = 0, c_2^P = \left(1 + \frac{D}{\frac{1-\pi}{\pi} D}\right) R (1 - D) = \frac{1}{1 - \pi} R (1 - D)$$

if we set $D = \pi c_1^*$ then equilibrium price is $z = \frac{1-\pi}{\pi} \pi c_1^* = (1 - \pi) c_1^*$ and allocations are

$$c_1^I = \frac{1}{\pi} D = \frac{1}{\pi} \pi c_1^* = c_1^*$$

$$c_2^P = \frac{1}{1 - \pi} R (1 - \pi c_1^*) = \frac{R (1 - x^*)}{1 - \pi} = \frac{R y^*}{1 - \pi} = c_2^*$$

Moreover, this is the unique equilibrium of this economy.

4.4 Asymmetry between equity shares and bank implementation if preferences are different

In general, implementation through equity shares does not work. Why? Now type θ reports its type to the firm

$$V (\theta, z) = \max_{c_1, c_2, a \in [0,1]} u (c_1, c_2, \theta)$$

$$s.t : \begin{cases} c_1 \leq za + D \\ c_2 \leq (1 - a) R (1 - D) \end{cases}$$

where $a \in [0, 1]$ is the amount of shares that the agent sells. See that in the optimum, we must have

$$a = \frac{c_1 - D}{z}$$

and putting this into the time $t = 2$ budget constraint, we get

$$c_2 = \left(1 - \frac{c_1 - D}{z}\right) R(1 - D) \iff$$

$$c_2 = R(1 - D) - \frac{c_1}{z} R(1 - D) \iff$$

$$c_1 + \frac{z}{R(1 - D)} c_2 = z \iff c_1 + \frac{1}{\widehat{R}} c_2 = z$$

where $\widehat{R} \equiv \frac{R(1-D)}{z}$ is the gross return on investing in shares. Then, we can express the problem as

$$V(\theta, z) = \max_{c_1, c_2, a \in [0,1]} u(c_1, c_2, \theta)$$

$$s.t \quad : \quad c_1 + \frac{1}{\widehat{R}} c_2 \leq z$$

Then, we must have $c_1^*(\theta) + \frac{1}{\widehat{R}} c_2^*(\theta) \leq z$, which is an extra constraint on the set of mechanisms we can implement. Therefore, in general the social optimum will not be implementable.

5   The Need for Trading Restrictions (Jacklin)

On the background, we had two assumptions

- The storage and long technologies are managed by firms, which because of constant returns to scale run with zero profits. Banks are the only agents in the economy that can deal with firms (so consumers cannot invest directly into firms)

- Agents cannot trade among themselves

We will see that if we drop either of these assumptions, then banks no longer implement the social optimum.

5.1 Consumers can trade with firms and among themselves

Imagine that there is a stock market for firms running the long technology open at time $t = 1$ and $t = 2$. Suppose that everyone else is following the strategy of going to the bank, and we consider a deviant consumer that is considering investing in the long technology.

If at $t = 0$ the agent invests his endowment in one of these firms. Two things can happen

- If patient, then just waits until $t = 2$ and get $R > c_2^*$

- If impatient, can sell a patient agent the right to get the dividends tomorrow. Recall that the equilibrium price of a share that pays $R(1 - \pi c_1^*)$ is $(1 - \pi) c_1^*$. Therefore, the price of this share, that pays $R > 0$ units can be then sold at
$$\frac{(1 - \pi) c_1^*}{1 - \pi c_1^*} > c_1^*$$

So agent is better off by deviating if there is a stock market open at all dates

5.2 Reintroducing Bond Markets

Suppose now that agents have non-observable consumption. Moreover, suppose that now agents can not only accept a contract with a financial intermediary, but have also access to the loan market at $t = 1$, in which agents can buy and sell bonds at a price q_1. As we showed before, in any equilibrium of the bond market, we must have $q_1 = \frac{1}{R}$

. This is the private market in which agents can trade. Then, given a contract $\{c_1(\theta), c_2(\theta)\}$ an agent will choose the report (θ') at $t = 1$, and then consumption and borrowing decisions. More specifically, given θ and contract $(c_1(\theta), c_2(\theta))$, agents solve

$$\widehat{V}(\{c_1(\theta), c_2(\theta)\}, \theta) = \max_{x_1, x_2, B, r} u(x_1, x_2, \theta) \qquad (CP)$$

$$s.t : \begin{cases} x_1 + \frac{1}{R}B = c_1(r) \\ x_2(\theta) = c_2(r) + B \end{cases}$$

Note that we can simplify the two conditions with the equation

$$x_1(\theta) + \frac{x_2(\theta)}{R} = c_1(r) + \frac{c_2(r)}{R} \qquad (5)$$

13

Which implies that a consumer, given the contract $\{c_1(\theta), c_2(\theta)\}$ will choose the report r to make the intertemporal budget constraint (5) to be the less slack possible. This implies that, if an intermediary were to design a incentive compatible contract, the present value of the consumption bundle should be constant across types. More specifically, let us write the problem of the intermediary in this setting

The agent will choose the report r (or isomorphically, the par c_1, c_2) to maximize the present value of consumption. Based on the previous argument, we now that any IC allocation has to satisfy

$$c_1(\theta) + \frac{c_2(\theta)}{R} = c_1(\theta') + \frac{c_2(\theta')}{R} \text{ for all } \theta, \theta'$$

In the special case of Diamond-Dygvig, we must have that

$$c_1^I + \frac{c_2^I}{R} = c_1^P + \frac{c_2^P}{R} \iff c_1^* = \frac{c_2^*}{R}$$

So, adding this constraint in the social optimum problem, gives the following system of equations:

$$c_2^* = R\frac{1 - \pi c_1^*}{1 - \pi}$$
$$c_1^* = \frac{c_2^*}{R}$$

which gives solution

$$c_1^* = 1, c_2^* = R$$

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