1 Investment

- Neo-Classical (Jorgenssen)
- irreversability (Bentolila-Bertola / Bertola-Caballero)
- Fixed costs (Caballero-Engle)
  - firm level
  - aggregation
  - general equilibrium

2 User Cost

Value with complete markets

$$\sum_{t,s^t} q_t^0 \left(s^t\right) d_t \left(s^t\right)$$

$$\begin{aligned} d_t \left(s^t\right) &= \pi \left(k_t \left(s^{t-1}\right), s_t\right) - c \left(k_t \left(s^{t-1}\right) \ , \ i_t \left(s^t\right) \ , \ s_t\right) - p_t \left(s^t\right) i_t \left(s^t\right) \\ k_{t+1} \left(s^t\right) &= (1-\delta) k_t \left(s^{t-1}\right) + i_t \left(s^t\right) \end{aligned}$$

subsituting

$$\begin{aligned} d_t \left(s^t\right) &= \underbrace{\pi \left(k_t \left(s^{t-1}\right), s_t\right) - c \left(k_t \left(s^{t-1}\right) \ , \ k_{t+1} \left(s^t\right) - (1-\delta) k_t \left(s^{t-1}\right) \ , \ s_t\right)}_{\text{``bla}_t(s^t)\text{''}} \\ &\quad -p \left(s_t\right) \left(k_{t+1} \left(s^t\right) - (1-\delta) k_t \left(s^{t-1}\right)\right) \end{aligned}$$

- value

$$\sum_{t,s^t} q_t^0 \left(s^t\right) \left(\text{bla}_t \left(s^t\right) - p_t \left(s^t\right) \left(k_{t+1} \left(s^t\right) - (1-\delta) k_t \left(s^{t-1}\right)\right)\right)$$

- if $p_{t+1}(s^{t+1}) = p_t(s^t)$ then

$$\begin{aligned} & q_t^0\left(s^t\right) k_{t+1}\left(s^t\right) - \sum_{s_{t+1}} q_{t+1}^0\left(s^{t+1}\right)(1-\delta) k_{t+1}\left(s^t\right) \\ = \; & q_t^0\left(s^t\right) k_{t+1}\left(s^t\right)\left(1-\frac{1-\delta}{R_{t,t+1}\left(s^t\right)}\right) \\ = \; & \frac{q_t^0\left(s^t\right)}{R\left(s^t\right)} k_{t+1}\left(s^t\right)\left(R\left(s^t\right)-1+\delta\right) \\ = \; & \sum_{s_{t+1}} q_t^0\left(s^t\right) k_{t+1}\left(s^t\right)\left(R\left(s^t\right)-1+\delta\right) \end{aligned}$$

- where risk-free rate

$$R\left(s^t\right) = \frac{q_t^0\left(s^t\right)}{\sum_{s_{t+1}} q_{t+1}^0\left(s^{t+1}\right)}$$

- or at $t+1$ simply

$$k_{t+1}\left(s^t\right)\left(R\left(s^t\right)-1+\delta\right)$$

- user cost (rental rate)

$$\nu\left(s^t\right) \equiv p_t\left(s^t\right)\left(R\left(s^t\right)-(1-\delta)\right)$$

- assume $p_t(s^t) \neq p_{t+1}(s^t, s_{t+1}) = p_{t+1}(s^t, \hat{s}_{t+1})$
  i.e. prices are not constant but predictable

$$\begin{aligned} & p_t\left(s^t\right) q_t^0\left(s^t\right) k_{t+1}\left(s^t\right) - \sum_{s_{t+1}} p_{t+1}\left(s^{t+1}\right) q_{t+1}^0\left(s^{t+1}\right)(1-\delta) k_{t+1}\left(s^t\right) \\ = \; & q_t^0\left(s^t\right) k_{t+1}\left(s^t\right)\left(p_t\left(s^t\right) - p_{t+1}\left(s^{t+1}\right)\frac{1-\delta}{R\left(s^t\right)}\right) \\ = \; & \sum_{s_{t+1}} q_{t+1}^0\left(s^{t+1}\right) k_{t+1}\left(s^t\right)\left(R\left(s^t\right) p_t\left(s^t\right) - (1+\delta) p_{t+1}\left(s^{t+1}\right)\right) \end{aligned}$$

- user cost (rental rate)

$$\nu\left(s^t\right) \equiv p_t\left(s^t\right)\left(R\left(s^t\right) - (1-\delta)\frac{p_{t+1}\left(s^{t+1}\right)}{p_t\left(s^t\right)}\right)$$

  $\rightarrow$ physical and **economic** depreciation

- thus...

  $$d_t\left(s^t\right)=\pi\left(k_t\left(s^{t-1}\right),s_t\right)-c\left(k_t\left(s^{t-1}\right)\ ,\ i_t\left(s^t\right)\ ,\ s_t\right)-p\left(s^t\right)\left(k_{t+1}\left(s^t\right)-(1-\delta)\,k_t\left(s^{t-1}\right)\right)$$

  ...equivalent to

  $$\tilde{d}_t\left(s^t\right)=\pi\left(k_t\left(s^{t-1}\right),s_t\right)-c\left(k_t\left(s^{t-1}\right)\ ,\ i_t\left(s^t\right)\ ,\ s_t\right)-\nu\left(s_t\right)k_t\left(s^{t-1}\right)$$

3 Irreversability or Costly Reversability

Discrete example (see Dixit-Pindyck)

- three periods $t=-1,0,1$

  $t=-1$ invest k_0

  $t=0$ invest k_1

- at $t=0$ learn $A_1\in\{A_H,A_L\}$
- irreversability

  $$k_1\geq k_0\,(1-\delta)$$

- risk neutral pricing

  $$q_t^0\left(s^t\right)\equiv R^{-t}\Pr\left(s^t\right)$$

- constant user cost v

  Firm problem

$$\max_{k_0}\left(A_0F\left(k_0\right)-vk_0+R^{-1}\left(p\max_{k_1\geq k_0(1-\delta)}\left(A^HF\left(k_1\right)-vk_1\right)+(1-p)\max_{k_1\geq k_0(1-\delta)}\left(A_LF\left(k_1\right)-vk_1\right)\right)\right)$$

equivalently

$$\max_{k_0}A_0F\left(k_0\right)-vk_0+R^{-1}\mathbb{E}\ V\left(k_0,A_1\right)$$

$$V\left(k,A\right)=\max_{k'\geq k(1-\delta)}AF\left(k'\right)-vk'$$

- unconstrained optimum

  $$\max_{k'}AF\left(k'\right)-vk'$$

  $$\Rightarrow AF'\left(k^{\#}\left(A\right)\right)-v=0$$

- if $k_1^*(A_L) = k_0(1-\delta)$

$$AF'(k_1) - v \leq 0$$

$$\Rightarrow A_0 F'(k_0) - v + \frac{1-p}{R} \underbrace{(A_L F'(k_1) - v)}_{-} (1-\delta) = 0$$

$$\Rightarrow k_0^* < k_0^{\#}(A_0)$$

- another way of seeing this:

$$A_0 F'(k_0) - v + \mathbb{E}\ [V_k(k_0, A_1)] = 0$$

but $V_k \leq 0$ and $V_k < 0$ in some state...

- irreversability

  $\Rightarrow$ lower investment

- if $k_t \in \{0,1\} \Rightarrow$ option value intuition [Dixit&Pyndick]

- Q: lower average capital?

  A: NO

- investment rule

  $k_0 \geq k_1^* \rightarrow$ don't invest

  $k_0 < k_1^* \rightarrow$ invest until $k_1 = k_1^*$

  $\rightarrow$ barrier control

- costly reversability

  capital sells at discount

  $\rightarrow$upper barrier control

- comparative statics

  increase uncertainty

4 Bentolila and Bertola

- homogenous return
$$\pi(k,z) = k^\alpha z^{1-\alpha}$$
- Bellman
$$V(k_-, z_-) = \mathbb{E}\left\{\max_{k \geq (1-\delta)k_-}\left[\pi(k,z) - rk + \beta V(k,z)\right]\right\}$$
- guess
$$V(k,z) = zV\left(\frac{k}{z}, 1\right) = zv\left(\frac{k}{z}\right)$$
$$z_- V\left(\frac{k_-}{z_-}, 1\right) = \mathbb{E}\max_{\frac{k}{z} \geq (1-\delta)\frac{z_-}{z}\frac{k_-}{z_-}}\left\{z\left(\pi\left(\frac{k}{z}, 1\right) - r\frac{k}{z}\right) + \beta\mathbb{E}\left[zV\left(\frac{k}{z}, z\right)\right]\right\}$$
- normalized
$$\begin{aligned} v(k_-) &= \mathbb{E}\,\varepsilon \max_k[\pi(k) - rk + \beta v(k)] \\ & \text{s.t.} \quad k \geq \varepsilon^{-1}(1-\delta)k_- \end{aligned}$$
- FOC
$$\pi'(k^*) = r - \beta V'(k^*)$$
- lower investment
$$V'(k) \leq 0 \Rightarrow k^* < k^\#$$
- Example:

  $\varepsilon = \{\epsilon_b, \epsilon_g\}$

  prob. $\lambda, 1-\lambda$
- no adjustment $\Rightarrow$
$$k = (1-\delta)\varepsilon^{-1}k_-$$

  then
$$\epsilon_g\epsilon_b(1-\delta)^2 = 1 \Rightarrow \epsilon_g = \epsilon_b^{-1}(1-\delta)^{-2}$$

  keeps us on a grid
$$\begin{aligned} & k^*, k^*\epsilon_b^{-1}(1-\delta)^{-1}, k^*\epsilon_b^{-2}(1-\delta)^{-2}, ... \\ & k^*(\epsilon_b(1-\delta))^{-n} \qquad n = 0, 1, 2, ... \end{aligned}$$

- invariant

$$\begin{aligned} p_n &= (1-\lambda)\, p_{n+1} + \lambda p_{n-1} \qquad n = 1, 2, ... \\ p_0 &= (1-\lambda)\, p_1 + (1-\lambda)\, p_0 \end{aligned}$$

- solving

$$1 = (1-\lambda)\frac{p_1}{p_0} + (1-\lambda) \Rightarrow \frac{p_1}{p_0} = \frac{\lambda}{1-\lambda}$$

  substituting

$$1 = (1-\lambda)\frac{p_{n+1}}{p_n} + \lambda\frac{p_{n-1}}{p_n} \Rightarrow \frac{p_{n+1}}{p_n} = \frac{\lambda}{1-\lambda}$$

$$\Rightarrow p_n = p_0\left(\frac{\lambda}{1-\lambda}\right)^n$$

- costly reversability:
  regulate k from above as well

- from Bertola-Bentolila

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See Figure 1 on p. 388 in Bentolila, Samuel, and Giuseppe Bertola. "Firing Costs and Labour Demand: How Bad is Eurosclerosis?" *Review of Economic Studies* 57, no. 3 (1990): 381-402.

- Abel and Eberly (1999; JME)

5 Labor: Firing Costs

- effects of firing costs on labor demand?
- Bertola and Bertolila (1990)
- Hopenhayn and Rogerson (1993)

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See Figure 2 on p. 392 in Bentolila, Samuel, and Giuseppe Bertola. "Firing Costs and Labour Demand: How Bad is Eurosclerosis?" *Review of Economic Studies* 57, no. 3 (1990): 381-402.

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See Figure 3 on p. 392 in Bentolila, Samuel, and Giuseppe Bertola. "Firing Costs and Labour Demand: How Bad is Eurosclerosis?" *Review of Economic Studies* 57, no. 3 (1990): 381-402.

Higher uncertainty

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See Figure 4 on p. 393 in Bentolila, Samuel, and Giuseppe Bertola.
"Firing Costs and Labour Demand: How Bad is Eurosclerosis?"
Review of Economic Studies 57, no. 3 (1990): 381-402.

- Hopenhayn-Rogerson → ergodic distribution of shocks
- entry and exit of firms
- ergodic distribution of labor demand
- compare steady states with and without

6 Aggregate Shocks

- concerted shocks in z
- distribution is state variable
- average reaction depends on distribution

7 Fixed Costs

- fixed cost if $i_t > 0$

- don't invest unless its important
  → innaction (fixed cost is irreversable)
- avoid fixed costs:
  lump investment (no small investments)
- Ss rule
- other end: generalized Ss

8 Generalized Hazard

Caballero-Engle

- probability of investing
  depends on distance from preferred point
- previous case: discontinuous hazard
- Calvo: constant hazard
- example 1
  aggregate firms with different sS rules
- example 2
  firms with random fixed cost

9 Aggregation

- state: distribution of firms
- evolution of distribution
  aggregate vs. idiosyncratic shocks
- non-linear response
- history matters

10 General Equilibrium

Julia Thomas $\rightarrow$ small effects in GE

Bachman-Caballero-Engle $\rightarrow$ large in recalibrated model