GMM Estimation and Testing II

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Hansen, Heaton, and Yaron (1996): In a Monte Carlo example of consumption CAPM, two-step optimal GMM with with many overidentifying restrictions is biased. Continuously updated GMM estimator (CUE) is much less biased.

CUE: Let $\hat{\Omega}(\beta) = \sum_i g_i(\beta) g_i(\beta)'/n.$

$$\hat{\beta}_{CUE} = \arg\min_{\beta} \hat{g}(\beta)' \hat{\Omega}(\beta)^{-1} \hat{g}(\beta).$$

LIML analog.

Altonji and Segal (1996): In Monte Carlo examples of minimum distance estimation of variance matrix parameters, two-step optimal GMM with with many overidentifying restrictions is biased. GMM with an identity weighting matrix is much less is biased.

Give some theory that explains these results.

Higher-order Bias and Variance

Stochastic Expansion: Estimators that come from smooth (four times continuously differentiable) moment conditions have an expansion of the form

$$n^{1/2}(\hat{\beta} - \beta_0) = \tilde{\psi} + Q_1(\tilde{\psi}, \tilde{a})/n^{1/2} + Q_2(\tilde{\psi}, \tilde{a}, \tilde{b})/n + O_p(1/n^{3/2}),$$

where Q_1 and Q_2 are linear in each argument, are smooth, $\psi(z)$, $a(z)$, $b(z)$, are mean zero random vectors, and $\tilde{\psi} = \sum_i \psi(z_i)/n^{1/2}$, $\tilde{a} = \sum_i a(z_i)/n^{1/2}$, $\tilde{b} = \sum_i b(z_i)/n^{1/2}$.

An approximate bias is given by

$$Bias(\hat{\beta}) = E[Q_1(\tilde{\psi}, \tilde{a})]/n = E[Q_1(\psi(z_i), a(z_i))]/n.$$

Simple example:

$$\hat{\beta} = r(\bar{z}), r(z) \text{ smooth, } \bar{z} = \sum_{i=1}^{n} z_i/n.$$

GMM estimator with $g(z, \beta) =$:

Expand around $\mu = E[z_i]$; with $\beta_0 = r(\mu)$:

$$\begin{aligned}\sqrt{n}(\hat{\beta} - \beta_0) &= r'(\mu)\sqrt{n}(\bar{z} - \mu) + r''(\mu)n(\bar{z} - \mu)^2/2\sqrt{n} \\ &\quad + r'''(\mu)n^{3/2}(\bar{z} - \mu)^3/6n + [r'''(\tilde{z}) - r'''(\mu)]n^{3/2}(\bar{z} - \mu)^3/n\end{aligned}$$

where $\tilde{z}$ is between $\bar{z}$ and μ. Here $\psi(z) = a(z) = b(z) = z - \mu$, $Q_1(\psi, a) = r''(\mu)\psi a/2$.

Bias is

$$Bias(\hat{\beta}) = a''(\mu)Var(z_i)/2n.$$

Approximation works quite well in describing how bias depends on number of moment conditions. Breaks down if identification is very, very weak.

This is where bias formula for 2SLS comes from.

Can also get a variance approximation, though does not work as well.

Can interpret as the bias of an approximating distribution, i.e. there is a precise sense in which dropping the higher order terms is OK.

Can use this approximation to select moments to minimize higher-order mean square error; Donald and Newey (2001).

Can also use it to compare different GMM esitmators, like CUE.

Bias of GMM:

Recall GMM notation. Let

$$
\begin{aligned}
g_i &= g_i(\beta_0), G_i = \partial g_i(\beta_0)/\partial\beta, \\
\Omega &= E[g_i g_i{}'], G = E[G_i], \\
\Sigma &= (G'\Omega^{-1}G)^{-1}, H = \Sigma G'\Omega^{-1}, P = \Omega^{-1} - \Omega^{-1}G\Sigma G'\Omega^{-1} \\
a &= (a_1, ..., a_p), a_j \equiv tr(\Sigma E[\partial^2 g_{ij}(\beta_0)/\partial\beta\partial\beta'])/2
\end{aligned}
$$

Bias for GMM has three parts:

$$
\begin{aligned}
Bias(\hat{\beta}_{GMM}) &= B_G + B_\Omega + B_I, \\
B_G &= -\Sigma E[G_i' P g_i]/n, \\
B_\Omega &= HE[g_i g_i' P g_i]/n, \\
B_I &= H(-a + E[G_i H g_i])/n.
\end{aligned}
$$

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Interpretation: B_G is bias from estimating G, B_Ω is bias from estimating Ω, and B_I is bias for GMM estimator with moment functions $G'\Omega^{-1}g_i(\beta)$.

B_G : Comes from correlation of G_i and g_i; endogeneity is the source; Example:

$$\begin{aligned} g_i(\beta_0) &= Z_i(y_i - X_i'\beta), G_i = -Z_i X_i', g_i = Z_i\varepsilon_i; \Omega = \sigma_\varepsilon^2 E[Z_i Z_i'], \\ E[G_i' P g_i] &= E[X_i\varepsilon]E[Z_i' P Z_i] = E[X_i\varepsilon]tr\left(PE[Z_i Z_i']\right) = E[X_i\varepsilon]\sigma_\varepsilon^{-2}(m-p), \end{aligned}$$

under homoskedasticity. $E[G_i' P g_i]$ nonzero due to correlation of X_i and ε_i. Grows at same rate as m. Consistent with large biases in Hansen, Heaton, and Yaron (1996).

B_Ω : Zero if third moments zero, e.g. $E[\varepsilon_i^3|Z_i] = 0$ in IV setting. Nonzero otherwise, generally grows with m (certainly magnitude of $g_i' P g_i$ does). Consistent with biases in Altonji and Segal (1996) (where G_i is constant so $B_G = 0$), where g_i includes is covariance moment condittions

Continuous updating GMM (CUE):

$$\hat{\beta}_{CUE} = \arg\min_{\beta} \hat{g}(\beta)'\hat{\Omega}(\beta)^{-1}\hat{g}(\beta).$$

Bias of CUE:

$$Bias(\hat{\beta}_{CUE}) = B_{\Omega} + B_I.$$

Eliminates bias due to endogeneity and estimation of Jacobian in optimal linear combination of moments. Explains Hansen, Heaton, and Yaron (1996).

CUE is a Generalized Empirical Likelihood (GEL) estimators. All GEL estimators eliminate B_G; some also eliminate B_{Ω}.

GEL: For concave function $\rho(v)$ with domain an open interval containing zero.

$$\hat{\beta}_{GEL} = \arg\min_{\beta} \sup_{\lambda} \sum_{i=1}^{n} \rho(\lambda' g_i(\beta)).$$

Computation: Concentrate out λ, using analytical derivatives for β.

Special cases of GEL:

CUE: Any quadratic $\rho(v)$ (i.e. $\rho(v) = A + Bv + Cv^2$),

Empirical Likelihood (EL): For $\rho(v) = \ln(1 \_ v)$,

$$\hat{\beta} = \arg \max_{\beta, \pi_1, \ldots, \pi_n} \sum_{i=1}^{n} \ln \pi_i, \sum_{i=1}^{n} \pi_i = 1, \sum_{i=1}^{n} \pi_i g_i(\beta) = 0.$$

Exponential Tilting (ET): For $\rho(v) = -\exp(v)$,

$$\hat{\beta} = \arg \min_{\beta, \pi_1, \ldots, \pi_n} \sum_{i=1}^{n} \pi_i \ln \pi_i, \sum_{i=1}^{n} \pi_i = 1, \sum_{i=1}^{n} \pi_i g_i(\beta) = 0.$$

Bias for GEL: For $\rho_j(v) = \partial^j \rho(v)/\partial v^j$, $\rho_j = \rho_j(0)$, $\rho_0 = 0$, $\rho_1 = -1, \rho_2 = -1$,

$$Bias(\hat{\beta}_{GEL}) = B_I + (1 + \frac{\rho_3}{2}) B_\Omega.$$

For EL, $\rho_3 = \partial^3 \ln(1 - v)/\partial v^3 = -2$, so $Bias(\hat{\beta}_{EL}) = B_I$.

Also, EL is higher-order efficient. Higher-order variance smaller than direct bias correction.

Probabilities: For $\hat{g}_i = g_i(\hat{\beta})$ and $\hat{v}_i = \hat{\lambda}'\hat{g}_i$, GEL probabilities for each observation are given by

$$\hat{\pi}_i = \rho_1(\hat{v}_i)/\sum_{j=1}^{n}\rho_1(\hat{v}_j), (i = 1, ..., n).$$

$\sum_{i=1}^{n}\hat{\pi}_i a(z_i, \hat{\beta})$ efficiently estimates $E[a(z, \beta_0)]$, subject to moment conditions.

Can explain bias results by interpreting first-order conditions. Let $\hat{G}_i = \partial g_i(\hat{\beta})/\partial\beta$ and $\hat{k}_i = [\rho_1(\hat{v}_i) + 1]/\hat{v}_i$ ($\hat{k}_i = -1$ for $\hat{v}_i = 0$).

First-order condition for β: Differentiate $\sum_i \rho(\lambda' g_i(\beta))$ with respect to β

$$0 = \sum_i \rho_1(\hat{v}_i)\hat{G}_i'\hat{\lambda}.$$

First-order condition for λ :

$$0 = \sum_i \rho_1(\hat{v}_i)\hat{g}_i = \sum_i [\rho_1(\hat{v}_i) + 1]\hat{g}_i - n\hat{g}(\hat{\beta}) = \sum_i \hat{k}_i\hat{g}_i\hat{g}_i'\hat{\lambda} - n\hat{g}(\hat{\beta})$$

Solve for $\hat{\lambda} = n\left(\sum_i \hat{k}_i\hat{g}_i\hat{g}_i'\right)^{-1}\hat{g}(\hat{\beta})$. Plug into previous equation and divide,

$$0 = \left(\sum_i \hat{\pi}_i\hat{G}_i\right)\left(\sum_i \hat{k}_i\hat{g}_i\hat{g}_i'\right)^{-1}\hat{g}(\hat{\beta})$$

First-order conditions for GEL:

$$0 = \left(\sum_i \hat{\pi}_i \hat{G}_i\right) \left(\sum_i \hat{k}_i \hat{g}_i \hat{g}_i'\right)^{-1} \hat{g}(\hat{\beta})$$

CUE, $\hat{k}_i = (\hat{v}_i - 1 + 1)/\hat{v}_i = 1$, so first order conditions are

$$0 = \left(\sum_i \hat{\pi}_i \hat{G}_i\right) \left(\sum_i \hat{g}_i \hat{g}_i'/n\right)^{-1} \hat{g}(\hat{\beta}).$$

EL, $\hat{k}_i = (-(1 - \hat{v}_i)^{-1} + 1)/\hat{v}_i = (1 - \hat{v}_i - 1)/\hat{v}_i(1 - \hat{v}_i) = -1/(1 - \hat{v}_i)$, so

$$0 = \left(\sum_i \hat{\pi}_i \hat{G}_i\right) \left(\sum_i \hat{\pi}_i \hat{g}_i \hat{g}_i'\right)^{-1} \hat{g}(\hat{\beta}).$$

All use efficient weighting in Jacobian part of first-order conditions, EL uses efficient weighting in second moment matrix, CUE uses standard weighting.

Efficient weighting removes correlation of matrix with moments, i.e. removes bias.

All GEL have efficient Jacobian. EL has efficient second moment matrix. In Altonji-Segal Monte Carlo design EL bias not as small as one would have hoped.

Higher-Order Efficiency

Let $\hat{B} = E[Q_1(\widehat{\psi(z_i)}, a(z_i))]$. Bias corrected estimator is

$$\tilde{\theta} = \hat{\theta} - \hat{B}/n.$$

Higher order variance of $\tilde{\theta}$ is

$$\begin{aligned} V_n &= \Sigma + \Xi/n, \Sigma = E[\psi(z_i)\psi(z_i)'], \\ \Xi &= \lim_{n\to\infty}\{Var(\tilde{Q}_1) + E[(\sqrt{n}\tilde{Q}_1 + \tilde{Q}_2)\tilde{\psi}'] + E[\tilde{\psi}(\sqrt{n}\tilde{Q}_1 + \tilde{Q}_2)']\} \\ &\quad +Acov(\sqrt{n}(\hat{B} - B), \tilde{\psi}') + Acov(\tilde{\psi}, \sqrt{n}(\hat{B}' - B')) \end{aligned}$$

Famous result, conjectured by Fisher, show later by Rao, Pfanzagl and Wefelmeyer, is that bias corrected MLE is higher order efficient.

Extends to GMM. Bias corrected EL is higher-order efficient. With discrete data, bias corrected EL becomes MLE in large samples. Discrete data can approximate any data.

Choosing Among Instruments (Donald and Newey (2001)).

Linear simultaneous equation, one endogenous explanatory variable:

$$\begin{aligned} y_i &= X_i'\beta_i + \varepsilon_i, X_i = (Y_i, Z_{i1}')', \\ Y_i &= \bar{Y}_i + v_i, \end{aligned}$$

$\bar{Y}_i$ reduced form for variable Y_i. Let j index instrument set, $Z(j)$ a $n \times K_j$ matrix of instrumental variable observations for. $Z(j)'\varepsilon/n \xrightarrow{p} 0$ for each j, so each instrument set is valid. Let $P_j = Z(j)[Z(j)'Z(j)]^{-1}Z(j)'$ be projection matrix on j^{th} instruments. Let

$$X = [X_1, ..., X_n]', Y = (Y_1, ..., Y_n)', y = (y_1, ..., y_n)'$$

j^{th} instrument set the 2SLS and LIML esitmators are respectively

$$\hat{\beta}_j = (X'P_jX)^{-1}X'P_jy, \tilde{\beta}_j = \arg\min_{\beta} \frac{(y - X\beta)'P_j(y - X\beta)}{(y - X\beta)'(y - X\beta)}.$$

Choose among instrument sets $1, ..., J$ by minimizing a large K_j estimate of mean square error (MSE).

Let $\bar{Z}$ be some fixed, large set of instruments that does not vary with j, $\bar{\beta}$ the IV estimator (2SLS or LIML), $\bar{\varepsilon} = y - X\bar{\beta}$, $\bar{v} = (I - \bar{Z}(\bar{Z}'\bar{Z})^{-1}\bar{Z}')Y$,

$$\hat{\sigma}_\varepsilon^2 = \bar{\varepsilon}'\bar{\varepsilon}/n, \hat{\sigma}_v^2 = \bar{v}'\bar{v}/n, \hat{\sigma}_{\varepsilon v} = \bar{\varepsilon}'\bar{v}/n, \hat{\gamma} = \hat{\sigma}_v^2 - \frac{\hat{\sigma}_{\varepsilon v}^2}{\hat{\sigma}_\varepsilon^2}.$$

Let $\hat{v}_j = (I - P_j)Y$, $\hat{R}_j = \hat{v}_j'\hat{v}_j/n + \hat{\sigma}_v^2 K_j/n$.

$$\hat{S}_{2SLS}(j) = \hat{\sigma}_{\varepsilon v}^2 \frac{K_j^2}{n} + \hat{\sigma}_\varepsilon^2 \hat{R}_j, \qquad \hat{S}_{LIML} = \hat{\gamma}\hat{\sigma}_\varepsilon^2 \frac{K_j}{n} + \hat{\sigma}_\varepsilon^2 \hat{R}_j.$$

Choose 2SLS estimator $\hat{\beta}_j$ that minimizes $\hat{S}_{2SLS}(j)$.

Choose LIML estimator $\tilde{\beta}_j$ that minimizes $\hat{S}_{LIML}(j)$.

Note: LIML has smaller MSE for large K_j.

Monte Carlo

Model is joint normal (Gaussian) with

$$y_i = \beta_0 Y_i + \varepsilon_i, Y_i = Z_i'\pi + v_i,$$

and reduced form coefficients

$$\pi_k = c(\bar{K}) \left(1 - \frac{k}{\bar{K}+1}\right)^4 \text{ for } \left(k = 1, .., \bar{K}\right).$$

$\bar{K}$ is maximal number of instruments, $c(\bar{K})$ is chosen so $\pi'\pi = R_f^2/(1 - R_f^2)$, where R_f^2 is the reduced form r-squared. Each $Z(j)$ is first j columns of the matrix of all instrumental variables, so here $K_j = j$. Sample sizes are $n = 100$, where $\bar{K} = 20$ and 5000 replications, and $n = 1000$, where $\bar{K} = 30$ and 1000 replications. Estimators were 2SLS-all and LIML-all which are 2SLS and LIML (respectively) using all $\bar{K}$ instruments. Estimators using the data driven number of instruments are denoted 2SLS-op and LIML-op. Estimates of $\sigma_{\varepsilon v}$ and σ_ε^2 were obtained using estimates of the two equations using the number of instruments that were optimal for estimating the first stage based on cross-validation.

$\sigma_{u\epsilon}$	Estimator	$N=100$ Med. Bias	Med. AD	Dec. Rge	Cov. Rate	$N=1000$ Med. Bias	Med. AD	Dec. Rge	Cov. Rate
0.5	OLS	0.495	0.495	0.226	0.000	0.496	0.496	0.068	0.000
	2SLS-all	0.473	0.473	0.502	0.323	0.377	0.377	0.382	0.278
	2SLS-op	0.398	0.602	2.954	0.825	0.216	0.284	0.888	0.759
	LIML-all	0.370	0.872	4.796	0.912	0.057	0.403	1.846	0.931
	LIML-op	0.400	0.548	1.851	0.904	0.162	0.267	0.937	0.904

Angrist and Krueger (1991) emipirical application.

Instrument Set	K	Cross-Val.	$\hat{S}_{2SLS}$	$\hat{S}_{LIML}$
Q	63	10.154409	4.1435071	4.2338311
Q+Q*Y	90	10.154451	4.1440244	4.2338357
Q+Q*S	213	10.154288	4.1499843	4.2337091
Q+Q*Y+Q*S	240	10.154282	4.1521275	4.2336937

Instrument Set	K	2SLS	LIML
Q	63	0.1077	0.1089
Q+Q*Y	90	0.0869	0.0905
Q+Q*S	213	0.0991	0.1152
Q+Q*Y+Q*S	240	0.0928	0.1064
Optimal	$\hat{K}$	0.1077	0.1064
		0.0195	0.0144

Fuller delivers similar answers.

Many Instrument Asymptotics

Estimates from Angrist and Krueger (1991) with optimal instrument set are:

Optimal	2SLS ($\hat{K} = 63$)	LIML ($\hat{K} = 230$)
	.1077	.1064
	(.0195)	(.0144)

2SLS and LIML estimates similar but LIML (with 180 overidentifying restrictions) seems to have smaller standard errors.

What about problem that asymptotic approximation poor and variance larger with many instruments?

This problem is accounted for in the standard error reported here.

Based on an asymptotic approximation where the number of instruments grows at the same rate as the sample size.

Leads to an approximation to the distribution of t-ratios with error rate not depending on the number of instruments.

$$n^{1/2}(\hat{\beta}_{LIML} - \beta_0) = \tilde{\psi} + Q_1(\tilde{\psi}, \tilde{a})/n^{1/2} + o_p(1),$$

$Q_1(\tilde{\psi}, \tilde{a})/n^{1/2}$ is asymptotically normal and uncorrelated with $\tilde{\psi}$ when number of instruments K grows at the same rate as the sample size.

Requires homoskedasticity: Under heteroskedasticity $\hat{\beta}_{LIML}$ not consistent when K grows as fast as n; more on this below.

First result on this is Bekker (1994).

Gives consistent standard errors under Gaussian (ε_i, v_i), or when have zero third moments and no kurtosis.

Hansen, Hausman, and Newey (2007), Estimation with Many Instrumental Variables, give results for non Gaussian disturbances. Variance matrix formula is different than Bekker (1994) with nonnormality but not much different if K/T is small.

To describe the Bekker (1994) variance estimator, let $P = Z(Z'Z)^{-1}Z'$ be the projection matrix used to form $\hat{\beta}_{LIML}$ and

$$\begin{aligned}\hat{\varepsilon} &= y - X\hat{\beta}_{LIML},\ \hat{\sigma}^2_{\varepsilon} = \hat{\varepsilon}'\hat{\varepsilon}/T,\ \hat{\alpha} = \hat{\varepsilon}'P\hat{\varepsilon}/\hat{\varepsilon}'\hat{\varepsilon},\ \hat{X} = X - \hat{\varepsilon}(\hat{\varepsilon}'X)/\hat{\varepsilon}'\hat{\varepsilon},\\ \hat{H} &= X'PX - \hat{\alpha}X'X,\ \hat{\Sigma} = \hat{\sigma}^2_{\varepsilon}[(1-\hat{\alpha})^2\hat{X}'P\hat{X} + \hat{\alpha}^2\hat{X}'(I-P)\hat{X}]\end{aligned}$$

The Bekker variance estimator is

$$\begin{aligned}\hat{V} &= \hat{H}^{-1}\hat{\Sigma}\hat{H}^{-1}\\ &\sqrt{n}(\hat{\beta}_{LIML} - \beta_0) \xrightarrow{d} N(0, V), n\hat{V} \xrightarrow{p} V.\end{aligned}$$

Can show $\hat{V}$ bigger than usual variance formula and can be substantially so even when K/n is small.

Ex: Monte Carlo based on Angrist and Krueger (1991).

	Bias/β	RMSE	Size
2SLS	-0.1440	0.0168	0.318
LIML	-0.0042	0.0168	0.133
Bekker SE's			0.049

Many Instruments and Heteroskedasticity

LIML is not consistent with heteroskedasticity.

"Jackknife" version is, Hausman, Newey, Woutersen, Chao, and Swanson (2007).

$$\hat{\beta}_{HLIM} = \arg\min_{\beta} \hat{Q}(\beta), \hat{Q}(\beta) = \frac{(y - X'\beta)P(y - X'\beta) - \sum_i P_{ii}(y_i - X_i'\beta)^2}{(y - X'\beta)'(y - X'\beta)}.$$

Heteroskedasticity/many instrument robust standard errors:

Let $\bar{X} = [y, X]$, $\tilde{\alpha}$ be the smallest eigenvalue of $(\bar{X}'\bar{X})^{-1}(\bar{X}'P\bar{X} - \sum_i P_{ii}\bar{X}_i\bar{X}_i')$, and $\tilde{H} = X'PX - \sum_i P_{ii}X_iX_i' - \tilde{\alpha}X'X$. Then

$$\hat{\beta}_{HLIM} = \tilde{H}^{-1}(X'Py - \sum_i P_{ii}X_iy_i - \tilde{\alpha}X'y),$$

For $\tilde{\varepsilon} = y - X\hat{\beta}_{HLIM}$ and $\tilde{X} = X - \tilde{\varepsilon}(X'\tilde{\varepsilon})/\tilde{\varepsilon}'\tilde{\varepsilon}$, let

$$\tilde{\Sigma} = \sum_{i,j=1}^{n} \sum_{k \notin \{i,j\}} \tilde{X}_i P_{ik} \tilde{\varepsilon}_k^2 P_{kj} \tilde{X}_j' + \sum_{i \neq j} P_{ij}^2 \tilde{X}_i \tilde{\varepsilon}_i \tilde{\varepsilon}_j \tilde{X}_j'$$

The variance estimator is

$$\begin{aligned} \tilde{V} &= \tilde{H}^{-1}\tilde{\Sigma}\tilde{H}^{-1}, \\ &\sqrt{n}(\hat{\beta}_{HLIM} - \beta_0) \xrightarrow{d} N(0, V), n\hat{V} \xrightarrow{p} V. \end{aligned}$$

Median Bias

μ^2	K	$LIML$	$HLIM$	$HFUL$	$HFUL\frac{1}{k}$	$JIVE$	CUE
8	0	−0.001	0.050	0.078	0.065	−0.031	−0.001
8	8	−0.623	0.094	0.113	0.096	0.039	0.003
8	28	−1.871	0.134	0.146	0.134	0.148	−0.034
32	0	−0.001	0.011	0.020	0.016	−0.021	−0.001
32	8	−0.220	0.015	0.024	0.016	−0.021	0.000
32	28	−1.038	0.016	0.027	0.017	−0.016	−0.017

Nine Decile Range: .05 to .95

μ^2	K	$LIML$	$HLIM$	$HFUL$	$HFUL\frac{1}{k}$	$JIVE$	CUE
8	0	2.219	1.868	1.494	1.653	4.381	2.219
8	8	26.169	5.611	2.664	4.738	7.781	16.218
8	28	60.512	8.191	3.332	7.510	9.975	1.5E+012
32	0	0.941	0.901	0.868	0.884	1.029	0.941
32	8	3.365	1.226	1.134	1.217	1.206	1.011
32	28	18.357	1.815	1.571	1.808	1.678	3.563