Behavioral Economics and Finance Spring 2004
Problem-set 1: Prospect Theory
Due: March 3[1]

Reminder: The continous PT value is

$$\mathrm{V} = \int_0^\infty v(x) f(x) \pi'(p(\widetilde{x} \geq x)) dx + \int_{-\infty}^0 v(x) f(x) \pi'(p(\widetilde{x} \leq x)) dx$$

where f is the lottery density, π is the probability weighing function, and

$$v(x) = \begin{cases} x^\beta, x > 0 \\ -\lambda(-x)^\beta, x < 0 \end{cases}$$

1. **Lottery behavior.**

   Lottery tickets win a unique prize of value G with probability p. They are sold at price $C > pG$. So, if you buy n tickets, then the probability to win G is np.

   (a) Write the Prospect-Theory value $V(n)$ of buying n tickets. Give a first order approximation under the assumption $pn << 1$.

   (b) Using the Prelec weighing function $\pi(p) = \exp(-(-\ln(p))^\alpha)$, find under what conditions a Prospect-Theory agent would buy at least one ticket. Compare with an expected utility agent agent. .

   (c) Compute the number of tickets bought, $n^\star$ (under the assumption $pn << 1$). Evaluate numerically for reasonable values of the parameters (e.g. $p = 10^{-6}, G = 10^6$, $C = 2$, $\lambda = 2$, $\alpha = .85, \beta = .65$).

   (d) Give an analytic expression of $V(n)$ for np small. Plot $V(n)$ as a function of n.

   (e) Comment: does Prospect-Theory offer a good explanation of observed lottery behavior? How would you fix the theory?

2. **Portfolio choice and Prospect-Theory.**

   Consider agents who behave as Prospect-Theory utility maximizers under an horizon T. They allocate their wealth between an index fund and a risk free asset. Allocating a proportion θ of their wealth in stocks gives value of the invested portfolio $\theta e^{\widetilde{R}(T)} + (1-\theta)e^{rT}$ where $\widetilde{R}(T) \sim N(\mu T, \sigma^2 T)$.

   For numerical applications, use $\mu = 6\%$, $r = 0$, $\sigma = .17$.

   (a) Write the Prospect-Theory utility associated to θ.

   In what follows, you can for simplicity take a simple loss aversion reduced form of PT: the weighting function is replaced by actual probabilities and $v(x) = \begin{cases} x, x > 0 \\ \lambda x, x < 0 \end{cases}$

   (b) For which horizon $T(\theta_0)$will people start to be willing to put a small amount θ_0 in equity? Compare with an expected utility maximizer. How would your answer change if agents were computing/perceiving gains and losses on 2 separate mental accounts, one for bonds, one for equity?.

   (c) Plot $V(\theta)$ as a function of θ.

   (d) Do you think prospect theory solves the equity premium puzzle (as argued in Bernartzi&Thaler, QJE95)?

3. **Decision rule of a PT agent.** During lecture 2 we assumed that there exists a PT agent who accepts a gamble with normal distribution of mean μ and the standard deviation σ if and only if $\frac{\mu}{\sigma.} > k$ for some parameter k. Was this assumption justified?

4. **Big problem.** Take one of the problems of PT that were discussed during lecture 2, on Thursday Feb 12, and try to solve it.